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EFFICIENT COMPUTATIONAL FRAMEWORK FOR UNCERTAINTY MANAGEMENT AND DESIGN OF SAFETY CRITICAL SYSTEMS

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ABSTRACT

Many phenomena as well as complex and safety critical systems can be studied and analysed only by using virtual prototypes and predictive mathematical models. One of the greatest challenges of virtual prototyping is to improve the fidelity of the computational analysis. This can only be achieved by explicitly including variability and uncertainties from different sources. Variability is inherent in many natural systems, and therefore cannot be reduced. Uncertainty is also always present since it is not possible to perfectly model or predict future events for which no real-world data are available.

Although stochastic methods offer a much more realistic approach for analysis and design, their utilization in practical applications remains quite limited. One of the reasons is the difficult to propagate different representation of the uncertainties. Another reason is the computational cost of stochastic analysis that it is often by orders of magnitude higher than the deterministic analysis.

This paper presents a powerful and unified representation of the uncertainty and an efficient computational framework. The computational tools satisfy the industry requirements regarding numerical efficiency, flexibility, scalability and analysis of detailed models that can be used to analyse a wide range of engineering and scientific problems.

INTRODUCTION

Knowledge about the future behaviour of engineered systems is the basis for reaching economical and safety relevant decisions in our society and appears in different fields (e.g. automotive and aerospace industry, financial, environmental science, mechanical and energy sector). In order to predict accurately the behaviour of such systems and/or structures, mathematical models must be constructed and then evaluated. Computer-aided modelling and simulation is now widely recognised as the third `leg` of scientific method, alongside theory and experimentation. Together with observed responses, it provides the basis to broaden the understanding of the system behaviour and the relationship between loads and performance.

Nowadays, in many engineering fields computational approaches and virtual prototypes are used to characterize, predict, and simulate complex systems. These advancements have allowed engineering practitioners to reduce the number of expensive and destructive tests necessary to qualify new products. In fact, the performance of these products can be tested in a simulated environment and the necessary changes introduced before producing a physical item, reducing the overall development cost and time. Many phenomena can be studied only by using computational processes such as complex simulations or analysis of experimental data. One of the greatest challenges of virtual prototyping (i.e. using numerical simulation and computational models to replace or reduce the number of hardware tests), is to improve their accuracy and reliability. For instance, it is essential to implement the scatter, which is always present in those tests (such as loads, material, geometry), into the computational model.

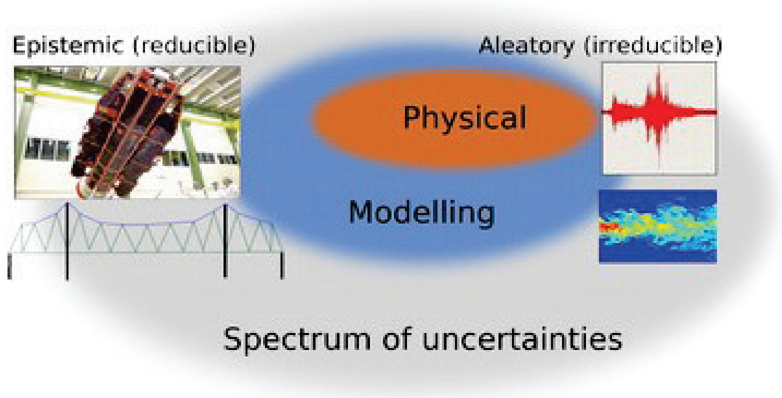

Figure 1: Aleatory and epistemic uncertainties that need to be included in the analysis.

The design of safety critical systems requires the explicit inclusion of varying levels of uncertainty and variability from different sources. Variability is inherent in many natural systems such as e.g. radioactive decay, earthquakes and therefore cannot be reduced. Uncertainty is also always present since it is not possible to perfectly model systems and components or predict real world situations, such as future weather conditions or collect infinite amount of samples (see Figure 1). Although such uncertainty can in theory be reduced in practice it is not possible (e.g. it might be too expensive or too complex to construct, validate and run a detailed model). Progress in virtual response prediction can only be accomplished in case uncertainties are included in the analysis (Nowakowski 2014).

Classical probabilistic approaches are well-established techniques often used to enhance robustness of simulations by accounting for uncertainty from different sources. Application of classical probabilistic frameworks, especially in cases affected by lack of information, may require strong initial assumptions which are often hardly justifiable. The assumptions made to deal with uncertainty due to data unavailability can deeply influence the final results and lead to severe risk misjudgement. Recent reports have clearly shown that the risk assumed by the decision maker is often wrongly estimated due to inadequate assessment of uncertainty. Despite the different level of uncertainty, decision makers still need to take clear choices based on the available information (Crespo,2013). They need to trust the methodology adopted to propagate the uncertainties through multi-disciplinary analysis, in order to quantify the risk with the current level of information and to avoid wrong decisions due to artificial restrictions introduced by the modelling.

Since the nuclear designs are subject to strict safety, reliability, environmental and service requirements, the quantification of uncertainties and risks is a necessity and a challenge. In turn, it requires innovative powerful mathematical approaches and software that allows the inclusion of non-deterministic analysis as a practice standard routing in the virtual prototyping. The availability of such software is particularly important for the analysis and design of resilient structures and systems, see e.g. Schuëller (2009).

GENERALIZED REPRESENTAION OF THE UNCERTAINTY

Multiple mathematical concepts can be used to characterize variability and uncertainty. Probability distributions can be used to represent the relative frequency of a given state of the system (i.e., that the data are distributed a certain way), or they can represent the degree of belief or confidence that a given state of the system exists (i.e., that we have the appropriate data). Often very limited information is available, and collecting more data or samples might be not possible or too expensive. Given the limitations of data, quantification methods often rely on subjective judgement and assumptions and it may not always seem reasonable to characterize the uncertainties in a classical probabilistic way.

To avoid the inclusion of subjective and often unjustified hypothesis, the imprecision and vagueness of the data can be treated by using concepts of imprecise probabilities. Imprecise probability combines probabilistic and set theoretical components in a unified construct allowing the identification of bounds on probabilities for the events of interest, (see e.g. Klir (2006), Beer et al. (2015) and Beer (2013)). Generalized probabilistic approaches are powerful methodologies which could in some cases be coupled to the traditional approaches in order to give a different prospective to the results enhancing outcome robustness. Some of the most intensively applied methodologies are discussed in literature by different mathematical concepts: Dempster-Shafer Evidence theory, interval probabilities, level two probably approach, Fuzzy-based approaches) random set theory as well as Bayesian updating approaches. Explanatory examples of such flexible mathematical frameworks are provided by e.g. Alvarez (2006), Ferson (2003) and Patelli (2015).

Random Set Theory

Random set theory is a general framework specially suited to model naturally the aforementioned representations of uncertainty without making any implicit or explicit assumption at all. Without entering an the mathematical details of the random set theory, the random sets can be understood as random variables that sample sets (i.e. the focal elements) as realizations, not points. A random set, Γ, is a $(\sigma_\Omega - \sigma_F)$-measurable mapping $\Gamma : \Omega \rightarrow F, \alpha \mapsto \Gamma(\alpha)$ and every $\gamma := \Gamma(\alpha) \in F$ is a *focal element* while F is a *focal set*. Analogously to the definition of a random variable, this mapping can be used to define a probability measure on (F,σ_F): an event $\mathrm{R} \in \sigma_F$ has the probability:

$$P_\Gamma(\mathcal{R}) = P_\Omega\{\alpha \in \Omega : \Gamma(\alpha) \in \mathcal{R}\}. \quad (1)$$

When all focal elements of F are singletons, then Γ becomes a random variable X. Hence, $\Gamma(\alpha) = X(\alpha)$ and the probability of occurrence of the event F, is $\mathrm{P}_X(F) := (P_\Omega \circ x^{-1})(F) = P_\Omega\{\alpha : x(\alpha) \in F\}$ for every $\mathrm{F} \in \sigma_X$. In the case of random sets, it is not possible to compute exactly $\mathrm{P}_X(F)$ but its upper and lower probability bounds. Dempster (1967) defined those upper and lower probabilities by,

$$LP_{(F,P_\Gamma)}(F) := P_\Omega\{\alpha : \Gamma(\alpha) \subseteq F, \Gamma(\alpha) \neq \emptyset\} \quad (2)$$

$$UP_{(F,P_\Gamma)}(F) := P_\Omega\{\alpha : \Gamma(\alpha) \cap F \neq \emptyset\} \quad (3)$$

where $\underline{\mathrm{P}}_{(F,P_\Gamma)}(F) \leq P_X(F) \leq \overline{P}_{(F,P_\Gamma)}(F)$. The original definition of random sets is very general. Alvarez (2006) showed that for some copula that contains the dependence information within the joint random set, and using intervals and d-dimensional boxes as elements of F, it is enough to model possibility distributions, probability boxes, intervals, CDFs and Dempster-Shafer structures or their joint combinations. For instance, a CDF (i.e. a random variable) can be represented as the random set $\Gamma : \Omega \rightarrow F, \alpha \mapsto \Gamma(\alpha)$ where F is the system of focal elements $\Gamma(\alpha) := F_X^{-1}(\alpha)$ for $\alpha \in \Omega$ (the inverse of the CDF F_X). Note that the representation of the CDF as a random set only contains an aleatory component, which is given either by α, or by its corresponding sample $\mathrm{x} = \mathrm{F}_X^{-1}(\alpha)$. There is not an epistemic component in this representation. An interval $\mathrm{I} = [\mathrm{l},\ \mathrm{u}]$ can be represented as the random set $\Gamma : \Omega \rightarrow F, \alpha \mapsto \Gamma(\alpha)$ (i.e. (F, P_Γ)) defined on R where the focal set contains the unique focal element $[\mathrm{l},\ \mathrm{u}]$, that is, $\mathrm{F} = \mathrm{I}$ and $\alpha \in (0, 1] \equiv \Omega$. In other words, all samplings of $\alpha \in \Omega$ draw the interval $[\mathrm{l},\ \mathrm{u}]$. Note that the representation of the CDF as a random set does not contain an aleatory component, inasmuch as it does not matter which value α takes, because all α-s map to the same focal element I. Note that the interval is a degeneration of a Dempster-Shafer structure used to represent overlapping sets and subsets of hypothesis, events or propositions as well as individual hypothesis (see Figure 2). A probability box or p-box (Ferson (2003) $\langle \underline{F}, \overline{F} \rangle$ is a set of cumulative distribution functions (CDFs) $\{\mathrm{F}:\underline{\mathrm{F}} \leq F \leq \overline{F}, F \text{ is a CDF}\}$, delimited by upper and lower CDF bounds $\underline{\mathrm{F}}$ and $\overline{F} : R \rightarrow [0, 1]$, which collectively represents the epistemic uncertainty about the CDF of a random variable (Figure 2).

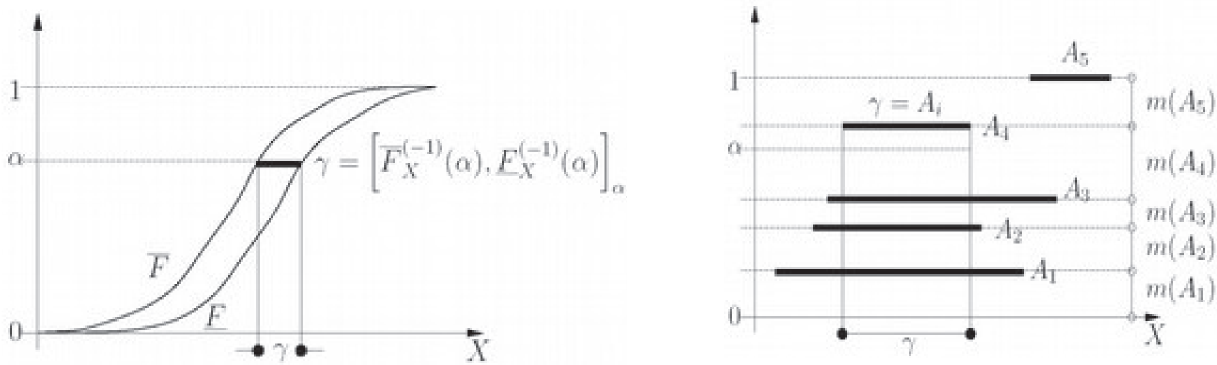

Figure 2 Representations of the uncertain: p-box (left panel) and Dempster– Shafer structure (right panel).

This class of functions may not have additional restrictions (i.e. distributional free p-box) or may belong, as well, to a reduced class of CDFs (distributional p-box). A distribution-free probability box can be represented as the random set $\Gamma : \Omega \rightarrow F, \alpha \mapsto \Gamma(\alpha)$ (i.e. $(\mathrm{F}, \mathrm{P}_\Gamma)$) defined on $\mathbb{R}$ where F is the class of focal elements $\Gamma(\alpha) := \langle \underline{F}, \overline{F} \rangle^{-1}(\alpha) := \left[\overline{F}^{-1}(\alpha), \underline{F}^{-1}(\alpha)\right]$ for $\alpha \in (0,1] \equiv \Omega$ with $\underline{F}^{-1}(\alpha)$ and $\overline{F}^{-1}(\alpha)$ denoting the inverses of $\underline{F}$ and $\overline{F}$. A focal element of the probability box can be represented as the input intervals $\{\mathrm{I}_i : i = 1, 2, \ldots, m\}$ together with the sample of α which is a uniform random variable on $(0,1] \equiv \Omega$. In consequence, it can be represented as the random set $\Gamma : \Omega \rightarrow F, \alpha \mapsto \Gamma(\alpha)$ (i.e. (F, P_Γ)) defined on $\mathbb{R}$ where F is the system of focal elements $\{\alpha \times I_1 \times \cdots \times I_m : \alpha \in \Omega\}$. Observe that each focal element has an aleatory component α and an epistemic component $\times_{i=1}^m I_i$.

Computational challenges

A sample from a random set is simply obtained by generating an α from a uniform distribution on $(0,1]$ and then, obtaining the corresponding focal element $\Gamma(\alpha)$. For example, sampling from a distributional-free probability box, is simply drawing an α uniformly distributed in $(0,1]$ and then obtain its corresponding ``α-cut" $\left[\overline{F}^{-1}(\alpha), \underline{F}^{-1}(\alpha)\right]$. In the case of multivariate random sets, a sample α is drawn from some copula C that models the dependence between the input variables. Then the corresponding marginal focal elements are obtained, and then they are combined as explained in the following. In order to find the image of a focal element, γ_i, through a function g, the extension principle of fuzzy sets is used. This can be done by means of optimization methods (Arora, (2007)), sampling methods (i.e. double Monte Carlo), a vertex method (Dong 1987), a function approximation method or the interval arithmetic method (Zhang2010a) or via advanced Monte Carlo methods (de Angelis, (2015)). One of the main advantage of the random set theory is that it allows to employ the usual reliability theory for estimating the failure probabilities of the two limit state functions $\underline{g}$ and $\overline{g}$, i.e. calculating of bounds on probability $[(\mathrm{F}),(\mathrm{F})]$. Figure 3 shows realizations from the random set. The figure shows also the failure surface, $\mathrm{g(x)=0}$, that defines the safe S and failure F domains. In the Ω-space (b) are defined the regions $\mathrm{F_{LP}}$ and $\mathrm{F_{UP}}$ together with the failure surfaces $\underline{g}(\alpha) = 0$ and $\overline{g}(\alpha) = 0$, where $\underline{g}(\alpha) := \min_{x \in \Gamma(\alpha)} g(x)$ and $\overline{g}(\alpha) := \max_{x \in \Gamma(\alpha)} g(x)$. Those boxes in X which contain at least one point of the failure region F have a corresponding α point in the region $\mathrm{F_{UP}}$; while those boxes in X which are completely contained in the region F have a corresponding α point in the region $\mathrm{F_{LP}}$.

It is clear that significant computational resources are required to propagate the *focal* elements through the system. This is because the analysis of each focal element (that are in general sets and not single number) requires the calculations of bounds of the system response involving and optimization analysis.

For instance, propagating 500 focal elements requires to perform 1000 global optimizations to compute the upper and lower bounds. Efficient computational strategies are therefore required for performing uncertainty quantification on complex and detailed models.

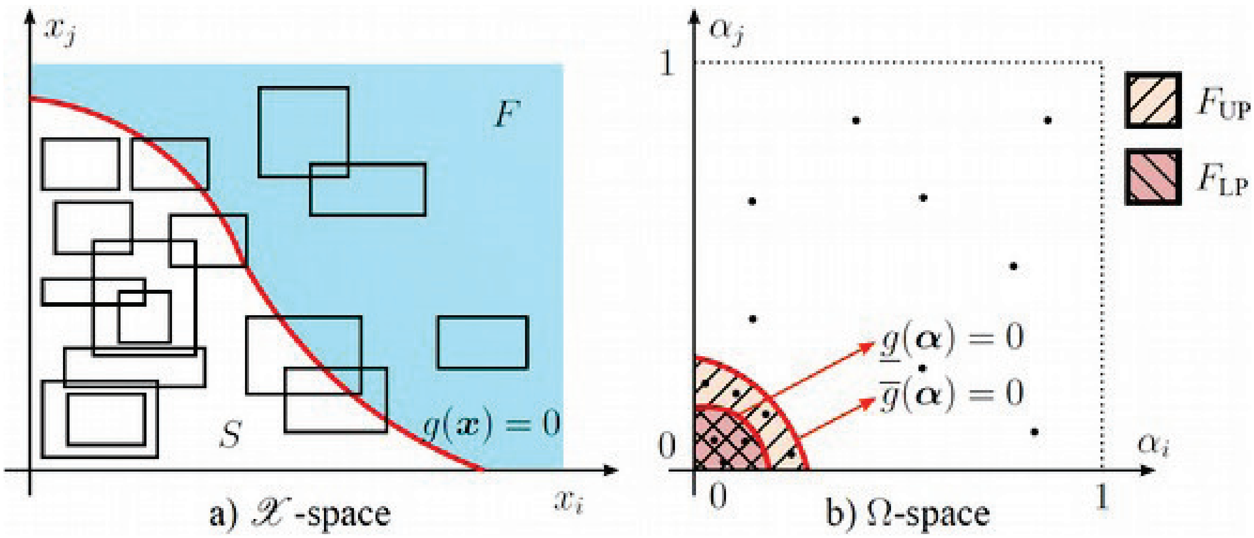

Figure 3: Realizations of the basic variables (Focal elements). In the $\mathbf{X}$-space (physical space) they are (multi-dimensional) boxes (Panel a). In the Ω-space (Panel b), realizations are depicted as the points α .

COMPUTATIONAL FRAMEWORK

The main reason that is limiting the application of the generalized probabilistic approaches is the lack of analysis software and efficient computational tools. Here, a general purpose computational framework that allows to consider explicitly different representation of the uncertainty in presented. The software incorporates the knowledge, understanding and intellectual property from more than 30 years of research in the field of computational stochastic analysis. The software, named COSSAN (Patelli, 2012, Patelli 2014), is based on the original development by the group of Prof. Schuëller at the Institute for Engineering Mechanics, University of Innsbruck, Austria (Pellissetti (2006)). The current version of the software, hosted at the Institute for Risk and Uncertainty at the University of Liverpool, UK, is intended for a wider range of applications in different fields, which includes optimization analysis, life-cycle management, reliability and risk analysis, probabilistic risk assessment, sensitivity and robust design.

In addition, since 2012, an open source version of the software, called OpenCossan is available under the Lesser GNU license. OpenCossan is coded exploiting the object-oriented Matlab® programming environment, where it is possible to define specialized solution sequences, which include reliability methods, optimization strategies, surrogate models and parallel computing strategies. The computational framework is organized in classes, i.e. data structures consisting of data fields and methods. Objects, that are instances of classes can be aggregated forming more complex objects and proving solutions for practical problem in a compact, organized and manageable format. Figure 4 shows a simplified scheme of the computational framework of OpenCossan.

The computation framework allows to characterize the uncertainty as distributional or free p-boxes, random variables, random process and random field, intervals, fuzzy variables, etc. In addition, different optimised algorithms are available to propagate the uncertainty. For instance for the reliability analysis, if the uncertain quantities are defined by means of random variables, the computational framework will estimate the failure probability by using Monte Carlo simulation or advanced Monte Carlo methods (e.g. Importance Sampling, Line Sampling, Subset simulation). On the other hand, if uncertain quantities are defined as intervals or p-boxes, the software will estimate the bounds of the failure probability by means of a double loop Monte Carlo simulation, or by means of tailored solution strategies, e.g. combining an optimization strategy with the Line Sampling method (see e.g. de Angelis (2015)). Furthermore, the developed numerical methods are highly scalable and parallelizable, thanks to its integration with distributed resource management, such as OpenLava and GridEngine.

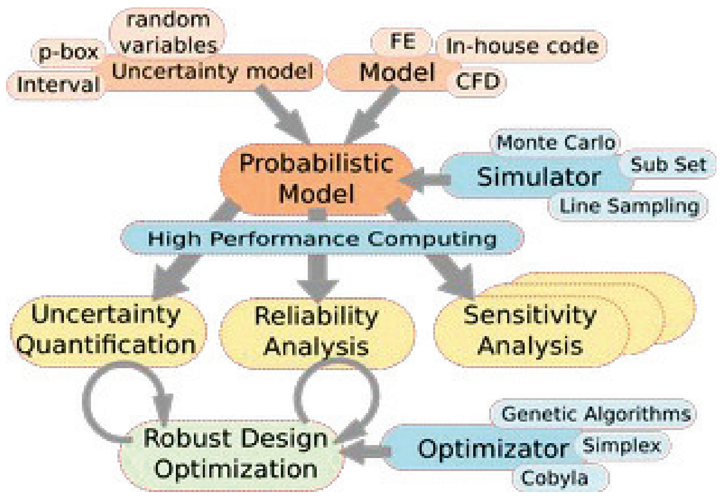

Figure 4: Scheme of the OpenCossan computational framework

APPLICATION

In order to demonstrate the capability and applicability and the proposed approach, selected results from the NASA Uncertainty Quantification challenge problem are presented. The reader is referred to Crespo (2013) for a full description of the challenge problem. The theoretical framework based on Random Set and the computational framework of OpenCossan have been used to solve all the tasks of the challenge problem. Here only a short description of the adopted strategies is reported. A full detailed analysis of the challenge problem can be found in Patelli (2015).

A multidisciplinary model that describes the dynamic of a remotely operated twin-jet aircraft has been developed by NASA Langley Research Center to provide an in-flight validation capability for high risk flight testing beyond the normal flight envelope. The overall aim of the to identify design points that provided optimal worst-case probability performance in the presence of uncertainty, of the control system for the Airborne Subscale Transport Aircraft Research (AirSTAR).

A mathematical model, S shown in Figure 5 that describes the dynamic of a remotely operated twin-jet aircraft is provided as a “Black Box”, contains 21 parameters, p, 8 design variables, d and 8 outputs, g. The input variables were classified into three classes according to their representation of aleatory and epistemic uncertainty: Category I corresponds to random variables with a precisely specified CDF; these variables only posses aleatory uncertainty. Category II represents intervals used to characterize parameters affected by pure epistemic uncertainty. Category III corresponds to input variables whose representation contains both aleatory and epistemic uncertainty and modelled as distributional probability boxes. Furthermore, a set of intermediate variables, x, that can be interpreted as outputs of the fixed discipline analysis, $x=h(p)$, are the inputs of the cross discipline analysis $g=f(x,d)$. The overall aim of the problem proposed is to identify the optimal design parameters, d, without being affected by the uncertainties on the values of the model parameters, p, i.e. perform a robust optimization. More specifically, a robust design point is minimizing the expectation of the maximum of the output,

$J_1 = E[w]$, and minimize the upper bound of the probability of failure $J_2 = P(w < 0))$. This requires to solve a series of subproblems, such as uncertainty characterization, sensitivity analysis, among others, in order to improve the model.

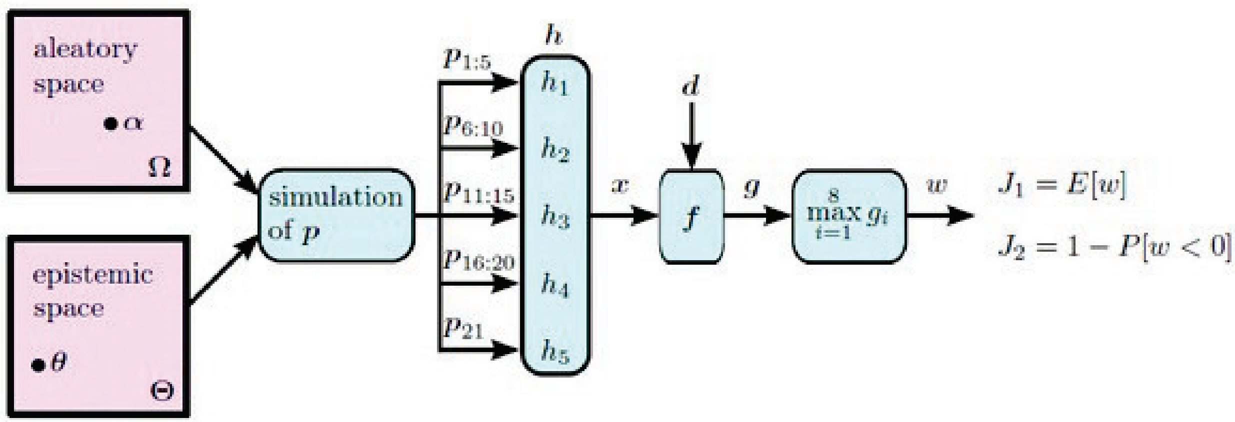

Figure 5: Schematic representation of the NASA Challenge problem

Proposed solution

Different tools and approaches exists for uncertainty quantification and characterization that can be potentially used in the design and safety critical systems. Every method is based on some assumptions and hypothesis that often cannot be verified a priori. Moreover, the simulation strategies are able to produce accurate results only if the right set of parameters are selected and this often cannot be verified. Finally, the numerical implementation might contains errors. For these reasons, it is a necessary to perform the analysis using different strategies and hypothesis in order to be able to *cross-validate* the results. Cross-validation is important to prevent hypotheses suggested by the data specially where further samples are costly or simply impossible to collect. For these reasons each task have been tackled using at least two different approaches.

The first task was to reduced the epistemic uncertainty on the output of the model $x = h(\alpha; \theta)$ based on the availability of a limited set of data (observations) $\{x_k^e : k = 1, 2, \ldots, n_e\}$. These observations of the ``true uncertainty model'' $\theta^* \in \Theta$ can be used to improve the uncertainty model, i.e. to reduce the original intervals of the epistemic uncertainties by excluding those combinations of parameters that fail to describe the observations. A simple and fast approach to improve the uncertainty model is based on the comparison of the CDFs of the observations of the true uncertainty model and those obtained by means of random combinations of the input parameters and then identify the support of the intervals (epistemic uncertainty) that it is in agreement with the observations. In practice, random realizations in the epistemic space are generated $\theta_i \in \Theta$ assuming, for example, a uniform PDF on Θ (in agreement with the Laplace's principle of indifference). Thereafter the set of points $\{\alpha_j, j = 1, 2, \ldots, n\}$ is sampled from the aleatory space Ω according to the copula C, and the inverse transform method is used in order to generate a set of n samples that are distributed following the uncertainty models of the inputs with parameters θ_i. These realizations are mapped through the model h in order to obtain the set of samples $\{ x_j^i, j = 1, 2, \ldots, n\}$ For a single realization θ_i, the Kolmogorov-Smirnov statistic, which is defined as

$$D_i = \sup_x |F_i(x) - F_e(x)|, \quad (4)$$

is used to measure the similarity between the CFDs. It is possible to use some smoother techniques such as the Gaussian Kernel density estimation to compute the experimental CDF. Gaussian kernel density estimates are given by

$$\hat{F}_i(x) = \frac{1}{n\sigma\sqrt{2\pi}} \sum_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{x - x_j^i}{\sigma}\right)^2\right); \tag{5}$$

where σ represents the smoothing parameter, proportional to the so-called bandwidth. A confidence limits on $\mathrm{F}_i(x)$ is obtained by choosing different critical values of the test statistic D_p. This implies that a band of width $\pm D_p$ around $\mathrm{F}_e(x)$ will entirely contain $\mathrm{F}_i(x)$ with probability $1 - \mathrm{p}$. This allows to identify those combinations of epistemic parameters such that $\mathrm{P}(\mathrm{D}_i > D_p) = p$ as shown in Figure 6.

Once a better (reduced) uncertainty model is available, it is possible to quantify the effect of the uncertain model parameters on quantities of interest such as the mean, variance or quantiles of the response of the system or its failure probability. The generalized probabilistic model makes the uncertainty quantification a rather challenging task in terms of computational cost. In fact, the challenge is to compute the lower and upper bounds of the model outputs. Monte Carlo method remains the most versatile tool to propagate epistemic and aleatory uncertainty (e.g. performing a double Monte Carlo loop). However, this approach is extremely inefficient in high dimensional spaces (due to the presence of the stochastic optimization part). A more efficient approach, as shown in Figure 7, consists in performing a global search in the epistemic space Θ (outer loop) to identify the bounds of $\mathrm{J}_1 = E[w]$, and $\mathrm{J}_2 = P(w < 0))$. Each candidate solution proposed by the optimization algorithm define a probabilistic problem (i.e. the inner loop) that can be analysed by means Monte Carlo simulation. In other words, the inner loop propagates the aleatory uncertainty and estimates the statistical quantities of interest. Monte Carlo integration in the aleatory space Ω is insensitive to the dimensionality of the problem although it can be inefficient in case

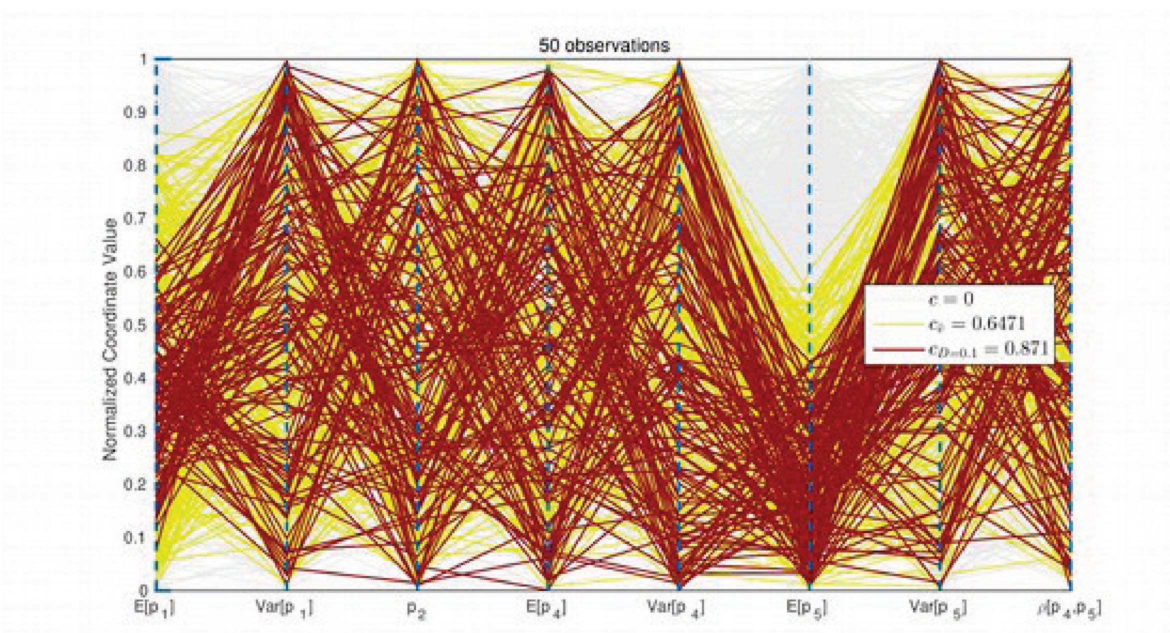

Figure 6: Parallel coordinate plot for the input parameters p for different values of the confidence interval.

of the estimation of very small probability of failure. In such case, the so called advanced Monte Carlo methods such as Importance Sampling, Subset Simulation and Line Sampling should be used.

The final task in the design of a safety critical system is to perform a robust design optimization. The main aim of the robust design is to consider explicitly the effects of the uncertainties in the optimization problem. A possible solution of this problem can be obtained performing an optimization analysis able to identify the design point with improved robustness and reliability characteristics. Generally in robust design only one bound is of interest; for instance we would like to reduce minimize the probability of failure and hence only the upper bound is of interest:

$$d^{opt} = \arg\min_{d \in D} \overline{P_f}(d) \tag{6}$$

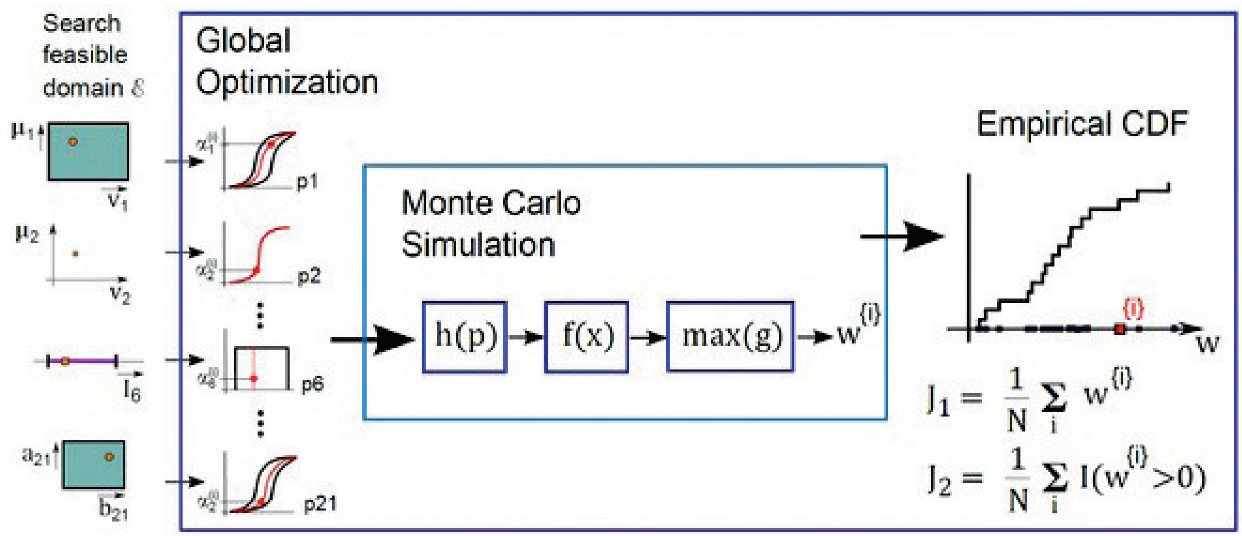

Figure 7: Epistemic and aleatory uncertainty propagation

The robust optimization is a very computational challenging task since for each candidate solution d, the uncertainty propagation task need to be computed leading. Thus, the direct solution is infeasible even for academic problems due to the tremendous numerical costs involved. To solve this problem, the use of surrogate models is necessary. Surrogate-models mimic the behaviour of the original model, by means of an analytical expression with negligible computational cost making the efficiency of the optimization methods almost irrelevant. The approximation is constructed by selecting some predefined interpolation points in the design space, at which the failure probability is estimated; then, a surrogate model is adjusted to the data collected in a least squares sense. As the construction of this approximation over the entire domain can be demanding, it may be easier to generate an approximation of the failure probabilities over a sub-domain, i.e. to generate a local surrogate model. Local surrogate model might require generally less evaluation points to be constructed although they have to be continuously updated in order to follow the current values of the design variables. The most versatile surrogate model are the Artificial Neural Networks although other models such as Kriging, Response Surface can be used as well. Hence, the robust design has been performed.

CONCLUSION

Efficient consideration of uncertainties is the basis for design more competitive, reliable and resilient products on nuclear field and in other sectors as well (e.g. automotive industry, aerospace, mechanical and civil engineering, etc.). This will allow to support economical and safety relevant decisions in our society by means the availability of fast and high fidelity numerical analysis allowing fast prototyping. In particular, the robust design of safety-critical systems requires not only the explicit treatment of different forms and representations of all the known and unknown uncertainty but also, performing a number of different tasks.

Generally, the design of such systems requires inputs and criteria of different disciplines and one of the main challenges in uncertainty management is how to propagate the uncertainty and understand how the uncertainty in one field affects other disciplines. This requires the availability of a mathematical framework and efficient and generally applicable computational tools as presented in this paper. Random set theory allows to represent without any assumption at all different form of uncertainty. Opencossan allows to analyse the effect of the uncertainties adopting a number of different efficient and scalable numerical strategies.

Considering different approaches to solve the same engineering problem might be seen a waste of resources and time. However, all the existing approaches for dealing with epistemic and aleatory uncertainty require fine tuning of their parameters in order to be efficient and accurate. Hence, it is of paramount importance to be able to verified the results against a different procedure. In this respect, the availability of an open, flexible and modular computational framework is essential.

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