PINM—a Fast Running Finite Element Model for Fuel Pin Mechanics Analyses in LMFBR Transients

G. van Goethem, H. Nguyen, J. Reynen
Commission of the European Communities, J.R.C. Ispra Establishment, Information Analysis and Data Handling Division, I-21020 Ispra, Italy

Abstract

The scope of the modular code EAC (European Accident Code) is to describe physical phenomena which contribute to the reactivity balance in LMFBR whole core analysis under hypothetical accident situations. The PIN Mechanics module PINM, in particular, is a simplified plane strain finite element program which predicts the reactivity effects due to thermal dilatation and in-pin molten fuel motion. Thermo-mechanics is actually modelled using a series of "initial strains" which represent, successively, thermal dilatation, restructuring, irradiation swelling, plasticity and creep, together with cracking. In addition, this program predicts the time and location of cladding failure which are introduced in EAC as initial conditions for the subsequent out-of-pin molten fuel motion model. Several numerical results are presented to illustrate the versatility and the efficiency of this module in the framework of a large whole core analysis code such as EAC.

1. Introduction

The European Accident Code (EAC), of which a first version has been released to the member countries, is developed at the Joint Research Centre of the CEC at Ispra (Italy) with the aim of analysing the initiating phase of low-probability whole-core accidents in LMFBRs. It is a multichannel modular computer code able to receive a variety of stand-alone programs originating from national laboratories and covering such diverse fields as single- and two-phase hydrodynamics, fast reactor neutronics, fission-gas behaviour and fuel pin thermo-mechanics. Making these modules work together in a physically and numerically coherent way results in a complete picture of the reactor dynamics which is mainly determined by thermal feedback effects [1]. Neutronic stability, indeed, depends upon the balance between positive (sodium voiding) and negative neutronics feedbacks (Doppler effect, thermal dilatation, in- and out-of-pin fuel motion).

Emphasis is put in this paper on the simple PINM module (PIN Mechanics) developed at Ispra with the aim of predicting not only fuel and cladding expansion (important negative feedback effects) but also time and location of pin rupture. The latter parameters are primordial to predict a possible shut-down phase of the reactor after a primary power excursion induced by sodium voiding: at fuel pin failure, indeed, dispersive forces are produced which are able to move the fuel out of the core towards regions of negative reactivity worths. Cladding rupture depends, in fact, upon fuel thermal expansion and the subsequent development of cracks, together with fission-gas release which all contribute to pressurize the fuel pin up to unsustainable fuel-cladding pressures.

The choice of a finite element formulation in this case is dictated by the requirement of having a modular program able to receive at any time new boundary conditions (e.g. slip condition along the fuel-cladding interface) or new "initial strains" (that is: all kinds of thermo-chemical contributions to volume swelling), as it is shown in the paper.

2. Selection of a Finite Element for Pin Mechanics

The problem of stress distribution in bodies of revolution under axi-symmetric loading is considerably simplified if plane strains are assumed in the axial direction: only the normal stresses in the 3 principal directions (radial, hoop and axial = 1, 2 and 3) are nonzero and there are no shear stresses. As a consequence each element is characterized by 3 unknown quantities (3 degrees of freedom) which are chosen to be: the 2 nodal values of the radial displacement field U_1 and U_2 together with the axial (uniform) strain ε_3. When the fuel-cladding gap is open, each of the 2 materials, fuel and cladding, is characterized by its own uniform axial strain (that is: planes perpendicular to the z-axis remain plane during deformation) so that both the fuel and the cladding may move at a different rate. On the other hand, when the fuel-cladding gap is closed, only 1 common value is considered for the axial strain in fuel and cladding [2].

As far as kinematics is concerned, the true radial displacement field $U^*(r)$ is interpolated over each element using linear shape functions N_1 and N_2 as follows:

$$U^*(r) \cong \frac{r_2-r}{r_2-r_1}U_1 + \frac{r-r_1}{r_2-r_1}U_2 \quad (= N_1U_1 + N_2U_2) \tag{1}$$

According to the definition of the total strain vector:

$$(\varepsilon)^t = (\varepsilon_1,\varepsilon_2,\varepsilon_3) = (dU^*/dr,\ U^*/r,\ \varepsilon_z) \tag{2}$$

one defines the linear transformation matrix [B] which relates the displacement to the total strain:

$$(\varepsilon) = [B](U_{ps}) \tag{3}$$

where (U_{ps}) denotes the "displacement"-like vector of the plane strain element, that is:

$$(U_{ps})^t = (U_1,U_2,\varepsilon_z) \tag{4}$$

The total strain consists actually of many components which may be classified in two broad categories, namely: elastic and initial (or, generally speaking, stress independent) strains, that is:

$$(\varepsilon) = (\varepsilon)_{e\ell} + (\varepsilon)_o \tag{5}$$

which are associated with internal stresses (through Hooke's law $(\sigma) = [D]\{(\varepsilon)-(\varepsilon)_o\}$) and with loading conditions (see Table 1), respectively.

Starting from the virtual work formulation of equilibrium over an arbitrary finite element Ω, written in terms of principal components of stress and strain [3], that is:

$$\int_{\Omega} (\sigma)^t (\delta\varepsilon)\, d\Omega = (F)^t (\delta U) \tag{6}$$

one is now able to derive the desired matrix equation:

$$(\delta U_{ps})^t \underbrace{\left[\int_{\Omega} [B]^t [D][B] d\Omega\right]}_{\substack{\text{stiffness} \\ \text{matrix } [K]}} (U_{ps}) = (\delta U_{ps})^t (F) + (\delta U_{ps})^t \underbrace{\left[\int_{\Omega} [B]^t [D] d\Omega\right]}_{\substack{\text{element stress} \\ \text{matrix } [S]}} (\varepsilon)_o \tag{7}$$

where superscript t denotes transpose of a matrix and (F) is the vector of nodal contact forces.

As the latter energy principle is of application for any virtual displacement (δU_{ps}) compatible with the boundary conditions, one is left with the following matrix equation (still at the level of one single element, regardless of the neighbouring elements):

$$[K](U_{ps}) = (F) + [S](\varepsilon)_o \tag{8}$$

with $[S] = [B]^t[D]$ and $[K] = [S][B] \int_{\Omega} d\Omega$ if [B] is constant. Equation (8), then, is solved in a standard way, taking advantage of the symmetry of the stiffness matrix [K].

3. Cracking Model

The fuel which is brittle at temperatures lower than 1400°C develops a network of cracks along the principal directions of the stress tensor as soon as the fracture strength in tension is exceeded [2]. Because of the parabolic temperature gradient in the fuel, tensile stresses are induced only in the outer region of the fuel whereas the inside of the fuel can sustain appreciable plastic deformation before fracture.

In terms of finite elements a crack in a given direction means a zero stress boundary condition in this direction and, as a consequence, a virtual variation of the "displacement" vector would have an effect on only 2 of the 3 stresses. This means that the corresponding stress and strain (elastic + initial) have to be withdrawn from the original Virtual Work Principle (6). The finite element approach, indeed, takes into consideration only stresses and strains which contribute effectively to the production of virtual work, that is: which are free to vary as virtual displacements are prescribed on the structure.

In fact, three types of cracks are envisaged:

1) the radial cracks which are oriented along horizontal planes perpendicular to the z-axis: only the radial and axial stresses and strains (σ_1, σ_3 and ε_1, ε_3, respectively) are to be considered in the virtual work expression and the modulus matrix [D] of the Hooke's law reduces to

$$[D] = 2G\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{9}$$

with G = shear modulus and $\lambda_1 = 2G\nu$ (ν = Poisson ratio);

2) the axial cracks which are oriented along radial planes passing through the fuel-pin axis: only the radial and hoop stresses and strains (σ_1,σ_2 and ε_1,ε_2, respectively) are to be considered and the [D] matrix is identical as previously;

3) the double cracks which are simultaneous radial and axial cracks: only the radial stress and strain (σ_1 and ε_1, respectively) are allowed to vary freely and the [D] matrix then reduces to Young's modulus.

Each of these cracking patterns has a direct effect on the fuel volume increase, which is described by means of a smeared crack strain, ε_{crack} (see Table 1), and this determines also the crack volume available for fission gas pressurization. Finally, crack healing is also made possible when the corresponding stresses become compressive again.

4. Creep and Plasticity Formulations

Formulation of the permanent strains based on the von Mises assumption that creep and plastic deformations occur only when the stress state deviates from pure hydrostatic tension or compression leads to the stress-strain relations known as the Prandtl-Reuss flow laws:

$$\bar{\varepsilon}_{p\ell} = \mu(s) \cdot \bar{r} \qquad (10)$$

where $\bar{\varepsilon}_{p\ell}$ and $\bar{r}$ denote the principal plastic strain and the deviatoric stress vector, respectively, whereas $\mu(s)$ is the proportionality factor, function of von Mises' equivalent stress s, as it is required by Drucker's postulate of normality.

To determine μ in a coherent manner, it has been found useful to start from the equality of energy dissipation [4], which is the basis to define the equivalent plastic strain $e_{p\ell}$:

$$s \cdot de_{p\ell} = (\sigma)^t (d\varepsilon_{p\ell}) \qquad (11)$$

where (σ) denotes the vector of principal stresses as in the virtual work equation (6). After some algebraic manipulations using Hooke's law together with the yield stress correlation:

$$\sigma_y = f(T, e_{p\ell}) \quad \text{(with T = temperature)} \qquad (12)$$

one derives the desired quantity $\mu(s)$ for the two cases:

a) plasticity: $$\mu = \frac{1 - \sigma_y/s}{6G + 2H} \qquad (13)$$

where H is the strain hardening coefficient, that is $H = \partial f/\partial e^p$;

b) creep: $$\mu = \frac{\Delta t \cdot \dot{\varepsilon}_{creep}}{2s} \quad \text{(with } \Delta t = \text{time step)} \qquad (14)$$

where $\dot{\varepsilon}_{creep}$ represents the creep rate law, $\dot{\varepsilon}_{creep} = g(T, \sigma_2)$ with σ_2 being the hoop stress as it is measured in the standard tube-burst tests. Note that these tests are characterized by a biaxial stress rate ($\sigma_1 = 0$ and $\sigma_3 = 1/2\ \sigma_2$, thin-wall tube formulas) so that the creep law can be easily transformed into a function of the von Mises stress s, as follows:

$$\dot{\varepsilon}_{creep} = g(T, \sqrt{4/3}\ s) \qquad (15)$$

and, as a consequence, the proportionality factor μ becomes a function of s, as in the case of plasticity.

In both the plasticity and the creep calculations, the loading can then be applied progressively in terms of increments of the equivalent stress, which accelerates the numerical convergence process.

TABLE 1 - Components of strain

Component		Symbol	Isotropic	Permanent	Calculated during
$(\varepsilon)_{e\ell}$	elastic	$\varepsilon_{e\ell}$	no	no	SS + TS
$(\varepsilon)_o = (\alpha\Delta T)$	thermal	ε_{th}	yes	no	SS + TS
$+ (\varepsilon)_{perm}$	dishing	ε_{di}	no	yes	input
	restructuring	ε_{rest}	yes	yes	SS(+TS)
	swelling	ε_{swell}	yes	yes	SS
	plastic	ε_{pl}	no	yes	SS + TS
	creep	ε_{creep}	no	yes	TS
$+ (\varepsilon)_{crack}$	cracking	ε_{crack}	no	no	SS + TS

5. Results and Conclusions

As a test it has been found useful to run PINM in the framework of an unprotected loss-of-coolant exercise involving both irradiated and non-irradiated channels. Global results are presented in Fig. 1 which are of interest for those who are in charge of predicting the energetics coming out from such hypothetical accidents. It is shown in particular that a primary power excursion can take place because of the dominating positive sodium voiding effect in the first phase of this accident, but that shut-down follows immediately when the negative reactivity effects due to fuel dilatation and in-pin motion enter into play. Fig. 2, on the other hand, shows that the fuel and the cladding dilatate axially with the same amount of plane strain along the edges without gap, contrarily to the regions with a gap characterized by a zero contact pressure.

As a conclusion it may be argued that a simplified computer program such as PINM is quite feasible and can provide sufficient insight into the problems associated with fuel pin mechanics in both nominal reactor operation and hypothetical accident situations. Obviously a more detailed analysis, focusing for example on bambooing of fuel elements, would require the use of more sophisticated 3-D models which are out of scope for the current whole core accident codes.

References

[1] VAN GOETHEM, G., "EAC - analysis of hypothetical LOF and TOP accidents with emphasis on the ICE-type finite element hydrodynamics module EACMAND", 7th Int. Conf. on Structural Mechanics in Reactor Technology, paper E 2/8, Chicago, USA, August 22-26, 1983.

[2] OLANDER, D.R., "Fundamental aspects of nuclear reactor fuel elements", Technical Information Centre, Oak Ridge, ERDA, TID-26711-PI, 1976.

[3] ZIENKIEWICZ, O.C., "The finite element method in engineering science", McGraw-Hill, London, 1971.

[4] REYNEN, J. et al., "Stress analysis of a bimetallic cylindrical joint", 1st Int. Conf. on Structural Mechanics in Reactor Technology, paper D1/3, Berlin, Germany, Sept. 20-24, 1972.

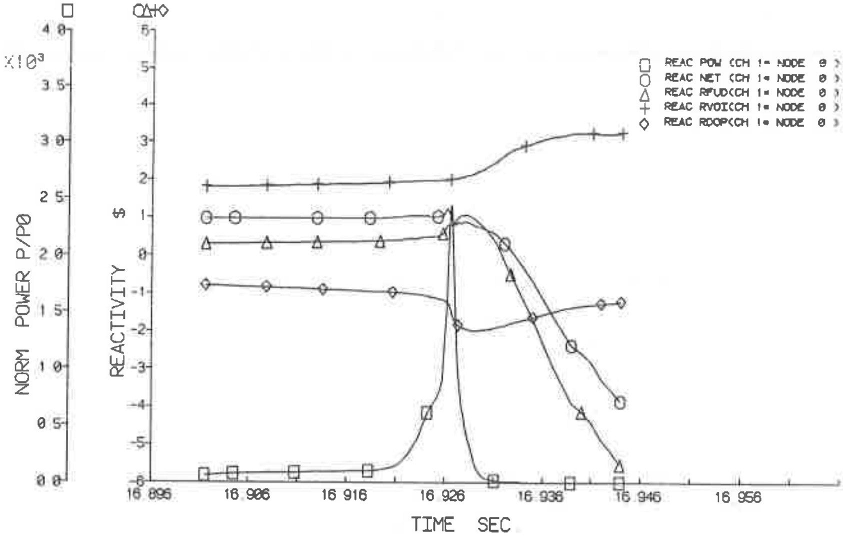

FIGURE 1 - FUEL DILATATION REACTIVITY AND REACTOR SHUTDOWN

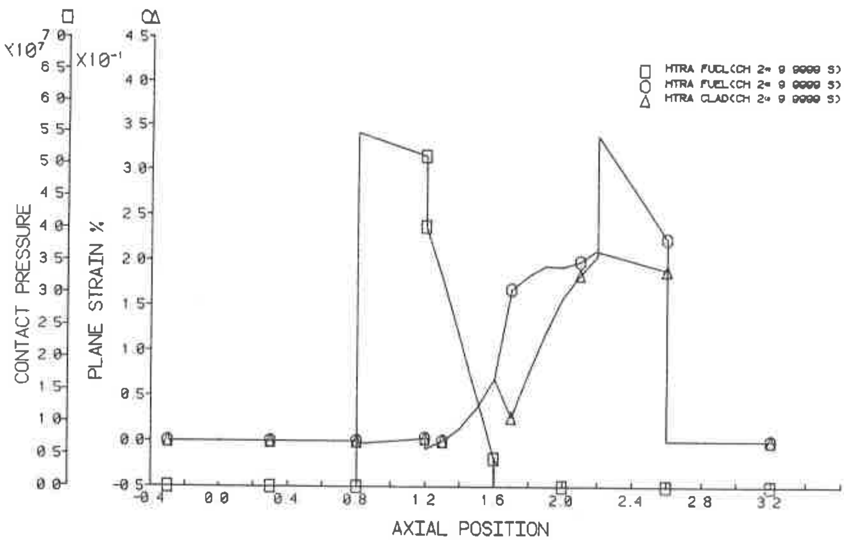

FIGURE 2 - FUEL/CLADDING MECHANICAL INTERACTION