

BELL-RING VIBRATION RESPONSE OF NUCLEAR CONTAINMENT VESSEL WITH ATTACHED-MASS UNDER EARTHQUAKE MOTION

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SUMMARY

There are two types of vibrations, such as "Beam-type vibration" and "Bell-Ring type vibration", of nuclear containment vessels as axisymmetric thin shell structures.

formed only to the "Beam-type vibration". Because, in fact, the response participation factor for the "Bell-Ring type vibration" under seismic motion is zero when the shell structure is in perfectly axisymmetric shape.

However, as in nuclear containment vessels, when the thin axisymmetric shell has several attached heavy masses such as the equipment hatch or the man holes, seismic responses of "Bell-Ring type vibration" are caused unexpectedly large and become remarkably more important than the "Beam type vibration".

For the seismic analysis of "Bell-Ring type vibration", there was the method of analysis advocated by H. Shibata. This method was the approximate uncoupled analysis using the natural mode shapes of unweighted perfect axisymmetric shell, on the assumption that the effect of the attached mass might be very small to their natural modes.

However, when we applied this method to some models, the responses of "Bell-Ring type vibration" calculated were noticeably smaller than the experimentally obtained results.

In this paper, we show the seismic response analysis of the "Bell-Ring type vibration" coupled with the "Beam type vibration" through the attached masses with the new consideration.

These results show good agreement between the theoretical calculation and the experiment.

1. Introduction

It is of vital importance to know before hand, from the standpoint of safety of the nuclear power station, the dynamic response of a nuclear containment vessel in case of an earthquake. In the past, study has been made mainly on the beam-type vibration when discussing the aseismic strength of the nuclear containment vessel. This can be attributed, it seems, to the fact that the bell-ring type vibration in which the section looks corrugated was not so important since the nuclear containment vessel can be regarded as axisymmetric thin cylindrical shell. As a matter of fact, if in a completely axisymmetric shell, the participation factor that excites bell-ring type vibration under an earthquake will become zero theoretically if integrated along the periphery, so much so that bell-ring type vibrations are not excited to allow only the beam-type vibrations to be excited. However, strictly speaking, the nuclear containment vessel is not perfectly axisymmetric. That is because, the vessel has on it a few local attached masses such as equipment hatch or air lock etc. which are quite heavy. As a result, these local attached masses will appreciably excite bell-ring type vibrations of the axisymmetric shell.

There is Shibata's Method (*1) as a response analysis under an earthquake when the axisymmetric shell has attached masses, but this method is an approximate analysis with using the free vibration mode shapes of the unweighted shell, on the assumption that the effects of attached masses are negligible. We, the authors, applied this method to the earthquake response test by using the model of the nuclear containment vessel, but it was discovered that the calculated value tends to be considerably smaller than the tested value.

For this reason, in consideration of the possible effect of the attached masses, we analysed the normal vibration of non-axisymmetric shell as employing the series of normal mode function of axisymmetric shell.

The response value calculated by this new analysis showed very good agreement to the experimented value, which convinced us of the propriety of this analysis.

What can be said from the result of this analysis is that in the response to the earthquake on a axisymmetric thin cylindrical shell with attached masses such as nuclear containment vessel, the bell-ring type vibration is more predominant than the beam-type vibration, and the importance of bell-ring type vibration has been established in theory and practice.

2. Analysis of Vibration Response of Nuclear Containment Vessel

The normal mode function in a case where concentrated masses are attached to the axisymmetric cylindrical shell, we assume, can be expressed by the superimposition of normal mode function of the axisymmetric cylindrical shell in a case where the concentrated masses are not attached.

The strain and the kinetic energy of the shell and also the kinetic energy

of the attached masses can be obtained by using these generalized modal function, and apply to the Lagrange's relationship. Fig. 1 shows the coordinates of the axisymmetric cylindrical shell.

2.1 Displacement Function of Shell

The displacement functions u, v and w of the shell in a case where the masses are attached on the axisymmetric cylindrical shell can be expressed by the superimposition of the generalized modal function, and are expressed as follows:

$$\left. \begin{aligned} u &= \sum_{s=1}^n \{ q_{1s} U_s \cos n_s \theta + q_{2s} U_s \cos (n_s \theta + \frac{\pi}{2}) \} \\ v &= \sum_{s=1}^n \{ q_{1s} V_s \sin n_s \theta + q_{2s} V_s \sin (n_s \theta + \frac{\pi}{2}) \} \\ w &= \sum_{s=1}^n \{ q_{1s} W_s \cos n_s \theta + q_{2s} W_s \cos (n_s \theta + \frac{\pi}{2}) \} \end{aligned} \right\} \dots\dots\dots (1)$$

where, S : Order of generalized eigen value
 U_s, V_s, W_s : S-th normal mode function
 q_{1s}, q_{2s} : S-th uncoupled modes depend on Time
 n_s : S-th circular frequency

The first terms on the right side of the equation (1), that is term of ($U_s \cos n_s \theta$, $V_s \sin n_s \theta$ and $W_s \cos n_s \theta$), are general expression of the generalized modal function, and the second are the terms which as a phase of $\pi/2$ against the first terms, and the phase between each generalized modal function can be expressed by the relation of q_{1s} and q_{2s} .

2.2 Strain Energy of Shell

The relationship between the strain and the displacement for the axisymmetric cylindrical shell is expressed as follows:

$$\left. \begin{aligned} \epsilon_x &= \frac{\partial u}{\partial \xi} - \frac{w}{a_2} - Z \left(\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{a_2} \frac{\partial u}{\partial \xi} \right) \\ \epsilon_y &= \frac{1}{a_1} \left\{ \frac{\partial v}{\partial \theta} - w - \frac{Z}{a_1} \left(\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta} \right) \right\} \\ \gamma_{xy} &= \frac{\partial v}{\partial \xi} + \frac{1}{a_1} \frac{\partial u}{\partial \theta} - \frac{2Z}{a_1} \left(\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \xi} + \frac{1}{a_2} \frac{\partial u}{\partial \theta} \right) \end{aligned} \right\} \dots\dots\dots (2)$$

The strain energy U of the axisymmetric cylindrical shell is given by:

$$U = \frac{1}{2} \frac{E}{1-\nu^2} \int_0^L \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} (\epsilon_x^2 + \epsilon_y^2 + 2\nu \epsilon_x \epsilon_y + \frac{1-\nu^2}{2} \gamma_{xy}^2) a_1 dz d\theta d\xi \dots\dots\dots (3)$$

If the equation (2) is substituted into equation (3), using the orthogonality for each order of the generalized modal function, equation (3) can be expressed as follows:

$$U = \sum_{s=1}^n U_s (q_{1s}^2 + q_{2s}^2) \dots\dots\dots (4)$$

where,

$$U_s = \frac{1}{2} \frac{E}{1-\nu^2} \int_0^L \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} \{ (\epsilon_x)_s^2 + (\epsilon_y)_s^2 + 2\nu (\epsilon_x)_s (\epsilon_y)_s + \frac{1-\nu^2}{2} (\gamma_{xy})_s^2 \} a_1 dz d\theta d\xi \dots\dots\dots (5)$$

$(\epsilon_x)_s$, $(\epsilon_y)_s$ and $(\gamma_{xy})_s$ in equation (5) contains only S-th component of displacement of equation (1).

U_s , V_s and W_s required for the equation (5) can be computed generally by using F. E. M. computer code of the axisymmetric cylindrical shell.

In this case, if equation (5) is expressed by using the stiffness matrix

(K_i) of each element in the computer code, the result will be:

$$U_s = \sum_{i=1}^J \left\{ \frac{1}{2} (U_{si}, V_{si}, W_{si}, \frac{\partial W_{si}}{\partial X}) (K_s)_i (U_{si}, V_{si}, W_{si}, \frac{\partial W_{si}}{\partial X})^T \right\} \dots \dots \dots (6)$$

where, J = number of element
i = index for element

2.3 Kinetic Energy of Shell

The kinetic energy is given by the sum of those by shell and those by attached masses.

2.3.1 Kinetic Energy of Shell Itself

The kinetic energy of the shell itself, neglecting the effect due to rotatory inertia, is given as follows:

$$K = \frac{1}{2} \int_0^L \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) a_1 dz d\theta d\xi \dots \dots \dots (7)$$

Substituting equation (1) into (7), and using the orthogonality of the generalized modal function, then the kinetic energy of the shell can be obtained as follows.

$$K = \sum_{s=1}^n K_s (q_{1s}^2 + q_{2s}^2) \dots \dots \dots (8)$$

where,

$$K_s = \frac{1}{2} \int_0^L \int_0^{2\pi} h \rho (u_s^2 \cos^2 \eta_s \theta + v_s^2 \sin^2 \eta_s \theta + w_s^2 \cos^2 \eta_s \theta) d\theta d\xi \dots \dots (9)$$

The K_s in the above equation can be calculated by using

$$U_s = \omega_s^2 K_s \dots \dots \dots (10)$$

where, ω_s : S-th natural circular frequency

2.3.2 Kinetic Energy of Attached Mass

The kinetic energy of the attached masses are given in the following relationship.

$$K_a = \frac{1}{2} \sum_{k=1}^m \frac{\Delta W_k}{g} (\dot{u}_k^2 + \dot{v}_k^2 + \dot{w}_k^2)_{\xi=\xi_k, \theta=\theta_k} \dots \dots \dots (11)$$

where,

$$\left. \begin{aligned} \dot{u}_k &= \sum_{s=1}^n \{ \dot{q}_{1s} u_s \cos \eta_s \theta_k - \dot{q}_{2s} u_s \sin \eta_s \theta_k \} \\ \dot{v}_k &= \sum_{s=1}^n \{ \dot{q}_{1s} v_s \sin \eta_s \theta_k + \dot{q}_{2s} v_s \cos \eta_s \theta_k \} \\ \dot{w}_k &= \sum_{s=1}^n \{ \dot{q}_{1s} w_s \cos \eta_s \theta_k - \dot{q}_{2s} w_s \sin \eta_s \theta_k \} \end{aligned} \right\} \dots \dots \dots (12)$$

where _kU_s, _kV_s, _kW_s : S-th generalized normal function of mass point k.
ξ_k, θ_k : coordinates of mass point k.
ΔW_k : attached weight of point k.
m : number of attached mass.

2.4 Exciting Energy

Since the exciting energy is given by the ground acceleration, the exciting energy will be given as follows:

$$F = a(t) \left(\int_0^L \rho h a_1 \int_0^{2\pi} \{ w \cos(\theta - \varphi) - v \sin(\theta - \varphi) \} d\theta d\xi \right. \\ \left. + \sum_{k=1}^m \frac{\Delta W_k}{g} \{ W_k \cos(\theta_k - \varphi) - v_k \sin(\theta_k - \varphi) \} \right) \dots \dots \dots (13)$$

where, a(t) : acceleration of seismic ground motion

Substituting equation (1) into above equation, then

$$F = \alpha(t) \times \sum_{s=1}^n (f_{1s} q_{1s} + f_{2s} q_{2s}) \quad \text{----- (14)}$$

where,

$$\left. \begin{aligned} f_{1s} &= \int_0^L \rho h a_1 \int_0^{2\pi} \{ w_s \cosh \eta_s \theta \cos(\theta - \varphi) - v_s \sinh \eta_s \theta \sin(\theta - \varphi) \} d\theta d\xi \\ &+ \sum_{k=1}^m \frac{\Delta W_k}{g} \{ {}_k w_s \cosh \eta_k \theta_k \cos(\theta_k - \varphi) - {}_k v_s \sinh \eta_k \theta_k \sin(\theta_k - \varphi) \} \\ f_{2s} &= \int_0^L \rho h a_1 \int_0^{2\pi} \{ w_s \sinh \eta_s \theta \cos(\theta - \varphi) + v_s \cosh \eta_s \theta \sin(\theta - \varphi) \} d\theta d\xi \\ &+ \sum_{k=1}^m \frac{\Delta W_k}{g} \{ {}_k w_s \sinh \eta_k \theta_k \cos(\theta_k - \varphi) + {}_k v_s \cosh \eta_k \theta_k \sin(\theta_k - \varphi) \} \end{aligned} \right\} \text{----- (15)}$$

The first term of equation (15) has the following relationship:

$$0 \quad \dots \text{ for } \eta_s = 1$$

$$v_s = -w_s \quad \dots \text{ for } \eta_s = 1$$

So, the above equation is altered as follows:-

$$\left. \begin{aligned} f_{1s} &= f_{0s} \cos \varphi + \sum_{k=1}^m \frac{\Delta W_k}{g} \{ {}_k w_s \cosh \eta_k \theta_k \cos(\theta_k - \varphi) - {}_k v_s \sinh \eta_k \theta_k \sin(\theta_k - \varphi) \} \\ f_{2s} &= f_{0s} \sin \varphi - \sum_{k=1}^m \frac{\Delta W_k}{g} \{ {}_k w_s \sinh \eta_k \theta_k \cos(\theta_k - \varphi) + {}_k v_s \cosh \eta_k \theta_k \sin(\theta_k - \varphi) \} \end{aligned} \right\} \text{----- (16)}$$

$$\left. \begin{aligned} \text{where, } f_{0s} &= \int_0^L 2\pi \rho h a_1 d\xi \quad \text{for } \eta_s = 1 \\ &= 0 \quad \text{for } \eta_s \neq 1 \end{aligned} \right\} \text{----- (17)}$$

2.5 Vibration and Response Analysis

2.5.1 Free Vibration Analysis

The strain energy (U) and kinetic energy (K + Ka) are substituted into the Lagrange's equation.

$$\frac{d}{dt} \left\{ \frac{\partial K}{\partial \dot{q}_s} \right\} + \left\{ \frac{\partial U}{\partial q_s} \right\} = 0 \quad \text{----- (18)}$$

$$\text{where, } q_s = q_{1s}, q_{2s}$$

This result, indicated by the matrix, is given in the form of the following equation.

$$\left\{ \begin{pmatrix} \mathbf{K}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_s \end{pmatrix} + \begin{pmatrix} \Delta M_1 & \Delta M_2 \\ \Delta M_3 & \Delta M_4 \end{pmatrix} \right\} \left\{ \begin{matrix} \ddot{q}_{1s} \\ \ddot{q}_{2s} \end{matrix} \right\} + \begin{pmatrix} \mathbf{U}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_s \end{pmatrix} \left\{ \begin{matrix} q_{1s} \\ q_{2s} \end{matrix} \right\} = 0 \quad \text{----- (19)}$$

where, $\mathbf{K}_s, \mathbf{U}_s$: diagonal matrix

Meanwhile, each M_1 of the above equation will take the following form :

$$\left. \begin{aligned} (\Delta M_1)_{ij} &= \sum_{k=1}^m \frac{\Delta W_k}{g} \{ {}_k u_i {}_k u_j \cos \eta_i \theta_k \cos \eta_j \theta_k \\ &+ {}_k v_i {}_k v_j \sinh \eta_i \theta_k \sinh \eta_j \theta_k + {}_k w_i {}_k w_j \cosh \eta_i \theta_k \cosh \eta_j \theta_k \} \\ (\Delta M_2)_{ij} &= \sum_{k=1}^m \frac{\Delta W_k}{g} \{ -{}_k u_i {}_k u_j \cos \eta_i \theta_k \sinh \eta_j \theta_k \\ &+ {}_k v_i {}_k v_j \sinh \eta_i \theta_k \cosh \eta_j \theta_k - {}_k w_i {}_k w_j \cosh \eta_i \theta_k \sinh \eta_j \theta_k \} \\ \Delta M_3 &= (\Delta M_2)^T \\ (\Delta M_4)_{ij} &= \sum_{k=1}^m \frac{\Delta W_k}{g} \{ {}_k u_i {}_k u_j \sinh \eta_i \theta_k \sinh \eta_j \theta_k \\ &+ {}_k v_i {}_k v_j \cosh \eta_i \theta_k \cosh \eta_j \theta_k + {}_k w_i {}_k w_j \sinh \eta_i \theta_k \sinh \eta_j \theta_k \} \end{aligned} \right\} \text{----- (20)}$$

Since q_{1s} and q_{2s} have amplitude of q_{1s} and q_{2s} which represent harmonic function in time of the circular frequency ω_c , equation (19) becomes as following expression,

$$-\omega_c^2 \left\{ \begin{pmatrix} \mathbf{K}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_s \end{pmatrix} + \begin{pmatrix} \Delta M_1 & \Delta M_2 \\ \Delta M_3 & \Delta M_4 \end{pmatrix} \right\} \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix} + \begin{pmatrix} \mathbf{U}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_s \end{pmatrix} \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix} = 0 \quad (21)$$

By solving the above equation, the coupled eigen value ω_c ($C = 1 \sim 2n$) and the normal vector q_{1s} , q_{2s} are obtained. In this case, C-th normal mode function of coupled system will be as follows:

$$\left. \begin{aligned} \bar{U}_c &= \sum_{s=1}^n \{ c \lambda_s U_s \cos(n_s \theta + c \nu_s) \} \\ \bar{V}_c &= \sum_{s=1}^n \{ c \lambda_s V_s \sin(n_s \theta + c \nu_s) \} \\ \bar{W}_c &= \sum_{s=1}^n \{ c \lambda_s W_s \cos(n_s \theta + c \nu_s) \} \end{aligned} \right\} \dots (22)$$

$$\left. \begin{aligned} \text{where, } c \lambda_s &= (c \bar{q}_{1s}^2 + c \bar{q}_{2s}^2)^{1/2} \\ c \nu_s &= \tan^{-1} (c \bar{q}_{2s} / c \bar{q}_{1s}) \end{aligned} \right\} \dots (23)$$

As an limiting case where there are no attached masses, that is $\Delta W_k/g = 0$, equation (21) reduce to diagonal matrix and n - uncoupled solution can be made.

2.5.2 Seismic Response Analysis

In this paper, only the horizontal acceleration is considered for seismic response analysis.

Applying Lagrange's equation again, by using the exciting energy shown in equation (14), the relations of the following equation can be obtained, as same as 2.5.1.

$$\left\{ \begin{pmatrix} \mathbf{K}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_s \end{pmatrix} + \begin{pmatrix} \Delta M_1 & \Delta M_2 \\ \Delta M_3 & \Delta M_4 \end{pmatrix} \right\} \begin{Bmatrix} \ddot{q}_{1s} \\ \ddot{q}_{2s} \end{Bmatrix} + \begin{pmatrix} \mathbf{U}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_s \end{pmatrix} \begin{Bmatrix} q_{1s} \\ q_{2s} \end{Bmatrix} = -\alpha(t) \begin{Bmatrix} f_{1s} \\ f_{2s} \end{Bmatrix} \quad (24)$$

By applying the modal analysis to the above equation, term of $\begin{Bmatrix} q_{1s} \\ q_{2s} \end{Bmatrix}$ can be expressed in the following form:

$$\begin{Bmatrix} q_{1s} \\ q_{2s} \end{Bmatrix} = \sum_{c=1}^{2n} Q_c \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix} \quad (25)$$

where, Q_c : C-th normal function of coupled system

Applying the orthogonality of the normal function, equation (24) will be as follows:

$$\ddot{Q}_c + \omega_c^2 Q_c = -\beta_c \alpha(t) \quad (26)$$

In the above equation, damping of the system is not considered, but, even in consideration of damping term, if orthogonality is presumed to be established approximately, the result will generally be as follows:

$$\ddot{Q}_c + 2h_c \omega_c \dot{Q}_c + \omega_c^2 Q_c = -\beta_c \alpha(t) \quad (27)$$

where,

$$\omega_c^2 = \frac{1}{m_c} \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}^T \begin{pmatrix} \mathbf{U}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_s \end{pmatrix} \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}_c$$

$$\left. \begin{aligned}
 2h_c \omega_c &= \frac{1}{m_c} \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}^T \left[\mathbf{C} \right] \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}_c \\
 \beta_c &= \frac{1}{m_c} \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}^T \begin{Bmatrix} f_{1s} \\ f_{2s} \end{Bmatrix} \\
 m_c &= \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}_c^T \left(\begin{Bmatrix} \mathbf{K}_s & \mathbf{O} \\ \mathbf{O} & \mathbf{K}_s \end{Bmatrix} + \begin{Bmatrix} \Delta M_1 & \Delta M_2 \\ \Delta M_3 & \Delta M_4 \end{Bmatrix} \right) \begin{Bmatrix} \bar{q}_{1s} \\ \bar{q}_{2s} \end{Bmatrix}_c
 \end{aligned} \right\} \text{----- (28)}$$

C : damping coefficient

In the above equation, m_c is C-th modal mass and β_c is C-th participation factor and each of them can be expressed in the following formula.

$$\begin{aligned}
 m_c &= \sum_{s=1}^n (K_s c \lambda_s^2) + \sum_{s=1}^m \frac{\Delta W_k}{g} \left\{ \left\{ \sum_{s=1}^n c \lambda_s k_s w_s \cos(n_s \theta_k + c \nu_s) \right\}^2 \right. \\
 &\quad \left. + \left\{ \sum_{s=1}^n c \lambda_s k_s v_s \sin(n_s \theta_k + c \nu_s) \right\}^2 + \left\{ \sum_{s=1}^n c \lambda_s k_s u_s \cos(n_s \theta_k + c \nu_s) \right\}^2 \right\} \text{----- (29)}
 \end{aligned}$$

$$\begin{aligned}
 \beta_c &= \sum_{s=1}^n \left(\frac{f_{0s} c \lambda_s}{m_c} \right) \cos(\varphi + c \nu_s) + \sum_{s=1}^m c \lambda_s \left\{ \sum_{k=1}^m \frac{\Delta W_k}{g m_c} \left\{ k w_s \cos(\theta_k - \varphi) \cos(n_s \theta_k + c \nu_s) \right. \right. \\
 &\quad \left. \left. + k v_s \sin(\theta_k - \varphi) \sin(n_s \theta_k + c \nu_s) \right\} \right\} \text{----- (30)}
 \end{aligned}$$

In equation (30), the first term on the right is by the shell itself and the second term by the attached masses. As can be seen from this equation, the participation factor of the coupled mode connected with the beam-type mode of the shell, as it is easily understood, will grow tremendously. In this paper, the analysis of the seismic response was made by the response spectrum method. Suppose the acceleration response value of the C-th order read from the response spectrum is S_{ac} , the maximum acceleration response value of the shell is given as follows from the modal analysis:

$$\left. \begin{aligned}
 (\ddot{U}_c)_{\max} &= \beta_c S_{ac} \sum_{s=1}^n c \lambda_s U_s \cos(n_s \theta + c \nu_s) \\
 (\ddot{V}_c)_{\max} &= \beta_c S_{ac} \sum_{s=1}^n c \lambda_s V_s \sin(n_s \theta + c \nu_s) \\
 (\ddot{W}_c)_{\max} &= \beta_c S_{ac} \sum_{s=1}^n c \lambda_s W_s \cos(n_s \theta + c \nu_s)
 \end{aligned} \right\} \text{----- (31)}$$

Accordingly, over all response values are given by the root mean square of the response values of each mode, and the result will be:

$$\left. \begin{aligned}
 \ddot{U}_{\max} &= \left(\sum_{s=1}^{2n} (\ddot{U}_c)_{\max}^2 \right)^{1/2} \\
 \ddot{V}_{\max} &= \left(\sum_{s=1}^{2n} (\ddot{V}_c)_{\max}^2 \right)^{1/2} \\
 \ddot{W}_{\max} &= \left(\sum_{s=1}^{2n} (\ddot{W}_c)_{\max}^2 \right)^{1/2}
 \end{aligned} \right\} \text{----- (32)}$$

3. Vibration and Seismic Response Test by Model

In order to confirm the propriety of the theoretic analysis above mentioned, vibration test and seismic response test were made by using model.

3.1 Test Model

A model for vibration test of the nuclear containment vessel were made in such a manner that it satisfies the dynamic similarity law. Here, we do not consider any specific plant as object and the model was standard in scale and construction. Although a dome is found at the top in the actual vessel, but we dispensed with it in the model and used instead a simple cylindrical shell with a stiffner ring at the top designed to restrict the sectional deformation.

In consideration of the easiness of handling in the test, we limited the size of the model to about 1/25 of the actual standard vessel, and we used hard vinyl chloride as material. Fig. 2 shows the rough dimension of the model. Usually, the nuclear containment vessel has an air lock, equipment hatch and a hatch mounted on it as attached masses, which correspond with about 1/10 of the weight of the vessel's cylindrical part. In the test, we devised to mount these attached masses on the vessel in the form as approximate as possible to the "point mass" for better possible correspondence with analysis. The fitting position of the attached masses are as shown in Fig. 3.

The similarity law of the model thus made has the ratio as shown in Table 1.

3.2 Vibration Characteristics Test

In the present test, we fixed the model on the test floor and excited a part of the vessel by a electro-magnetic shaker. The vibration response curve of an arbitrary point of the shell were obtained against frequencies and the frequency was kept constant at each resonant points and measured the vibration modes of the containment vessel in a traverse method by using a small accelo-pick up. In this test, we sought the natural frequencies, vibration modes, and damping constants of the cylindrical shell model with or without attached masses, looked into the effects of the attached masses on the natural frequencies and vibration modes to have them serve as the basic data of analysis of the subsequent response tests.

3.3 Sinusoidal Vibration Response Test

The model was fixed on the vibration table to sweep frequencies by the sinusoidal wave to obtain the response acceleration of an arbitrary point of the model, and the value of acceleration response was divided by the acceleration of the vibration table to indicate the value in the non-dimensional value as "acceleration magnification factor." In this test, we changed the vibration direction to the model and looked into the relationship between the vibration direction and the response of shell.

3.4 Seismic Response Test

The model was set up on the vibration table and the exciting input equivalent to the seismic motion of the ground was applied to the vibration table and sought the acceleration response and stress response of the model.

The seismic waves used here were the El-centro earthquake and random wave.

For the former, we changed the time axis according to the model similarity law.

4. Comparison of the Analysis and the Test Result

Let us compare the result of the theoretical analysis mentioned in Chapter 2 and the test results. The calculation of the natural frequencies and their modes of the axisymmetric cylindrical shell without attached mass in the theoretical analysis were obtained by the analytical code of the finite element method. The calculated values and the tested values are as shown in Fig. 4. The bell-ring type vibrations in them are in good accord.

However, the beam type vibration in $n = 1$ are not well in accord. It became clear later that this was due to the lack of fitting-rigidity of base stiffener of the shell to the table. Therefore, for the theoretical analysis of the coupled vibration in case where there are attached masses, we employed the measured value for beam type vibration frequency as positive.

4.1 Vibration Characteristics

In the natural frequency with the attached masses, the calculated value and the tested value correspond well. The attached masses are comparatively small as compared with the total weight of the shell, there are virtually little difference in frequencies from a case where there are no attached masses. However, the fundamental normal modes in a case where the attached masses are absent, as shown in Fig. 5, indicates localized predominant mode in the vicinity of the attached mass point, and the calculated values and the tested values are well in accord. Also with regard to other modes, as shown in Fig. 6, there are slight differences but these values correspond very well. In the vicinity of the beam-type natural frequency, as can be seen from equation (30), the beam-type vibrations and bell-ring type vibrations are coupled and it is clear that the bell-ring type vibration mode is largely excited. This coincides with the result of the response test to be mentioned later.

4.2 Sinusoidal Vibration Response

An example of comparison of tested values and calculated values of frequency response, when we excited the shell with sinusoidal wave in the direction in which the vibration response of the shell with attached mass appears to be the greatest are shown in Fig. 7 to Fig. 9. The vibration of shell shows predominant response in the neighborhood of the natural frequency of the lowest mode of bell-ring type vibration and that of the natural frequency of the beam-type vibration.

In the case of uncoupled analysis, such phenomena does not obtain and the calculated value does not coincide with tested one.

When two-masses are attached on the shell, the natural frequencies appears inverse phase modes like 17.3 Hz, 17.7 Hz and 29.5 Hz, 30.0 Hz and 35.5 Hz, 35.7, so much so that each natural modes are superimposed for the measurement. As compared with a case of one attached mass, the difference of

response of the shell to be received from the direction of excitation will become less according to the increase in number of attached mass.

4.3 Seismic Response

Seismic response analysis was carried out based on the response spectrum concept. Fig. 10 shows the ratio between the maximum value of the seismic acceleration response on the circumference of the shell near the height (el. 75 cm) of the attached masses and the maximum value of the input earthquake on the table. The amount of response at each point on the circumference is random, with no specific tendency of response to the exciting direction. This phenomena indicate that the response of the bell-ring type vibration is large as compared with the beam-type vibration.

As can be seen from this test result, the response in the vicinity of attached mass is not necessary to predict the position where the response will be greatest on the circumference, and we can understand that what is expected is to seek the maximum value enveloping the responses to whole exciting directions. Fig. 11 shows the comparison of the maximum values on the circumference.

In the seismic response also, the test value and the analyzed one shows good agreement as can be seen in this test results. It is quite evident that in the conventional uncoupled analysis the response of the shell tends to be underestimated.

5. Conclusion

When an nuclear containment vessel is suffered from an earthquake, how would that affect the vibration response of the bell-ring type of the vessel ? Our study of this subject can be summerized as follows.

- (i) When the shell is suffered from an earthquake, both the "bell-ring type vibration close to the natural frequency of beam-type in which the beam-type vibration and bell-ring type vibration are coupled" and "lowest frequency of bell-ring type vibration (with the local vibration mode by the attached mass)" are remarkably predominate.
- (ii) In this case, the response of the bell-ring type vibration will become fairly larger than the response of beam-type one.
- (iii) The response distribution in the direction of the shell is random, and it does not necessarily become large near the attached mass point.
- (iv) In the conventional "uncoupled analysis (Shibata's Method)" the bell-ring type response tends to be underestimated, and its application to the design is not appropriate.
- (v) In the "coupled analysis" we advocate in this paper, it is able to obtain the seismic response of the containment vessel, including the bell-ring type vibration, with sufficient accuracy.
- (vi) In the past, the resist-earthquake design for the nuclear containment vessel has been mainly done with emphasis on the beam-type vibrations, but, in the case of vessels equipped with equipment hatch, air lock, etc. the bell-ring type vibration become more important rather than the beam-type vibrations.

References

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Table 1 Similarity

Quantity	Symbol	Dimension	Scale Ratio
Length	l	L	$l_p / l_m = N$
Sectional Area	A	L^2	$A_p / A_m = N^2$
Density	ρ	ML^{-3}	ρ_p / ρ_m
Young's Modulus	E	$ML^{-1}L^2$	E_p / E_m
Poisson's Ratio	ν	-	$\nu_p / \nu_m = 1$
Strain	ϵ	-	$\epsilon_p / \epsilon_m = 1$
Stress	σ	$ML^{-1}T^{-2}$	$\sigma_p / \sigma_m = E_p / E_m$
Displacement	x	L	$x_p / x_m = N$
Force	P	MLT^{-2}	$P_p / P_m = E_p / E_m \cdot N^2$
Frequency	f	T^{-1}	$f_p / f_m = \sqrt{E_p / E_m \cdot \rho_m / \rho_p} \cdot 1/N$
Acceleration	\ddot{x}	LT^{-2}	$\ddot{x}_p / \ddot{x}_m = E_p / E_m \cdot \rho_m / \rho_p \cdot 1/N$
Time	t	T	$t_p / t_m = \sqrt{E_m / E_p} \cdot \rho_p / \rho_m \cdot N$
Damping Coeff	h	-	(*) $1/\sqrt{h_p / h_m}$
Weight	W		$W_p / W_m = \rho_p / \rho_m \cdot N^3$
Suffix : m for model, P for Prototype (*) Correct term for Random Wave Input			

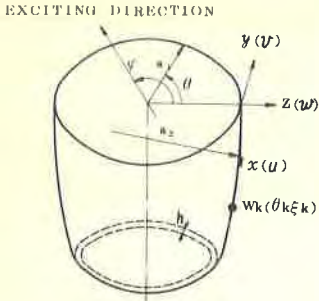


Fig. 1 The Shell geometries And coordinate directions.

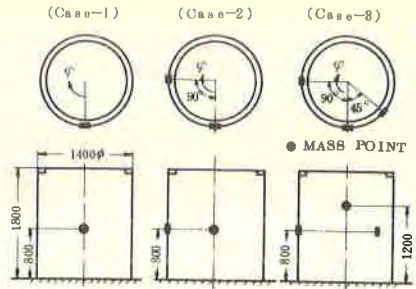


Fig. 3 The Locations of the attached mass

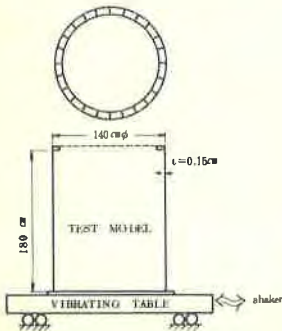


Fig. 2 Experimental equipment set-up for earthquake response

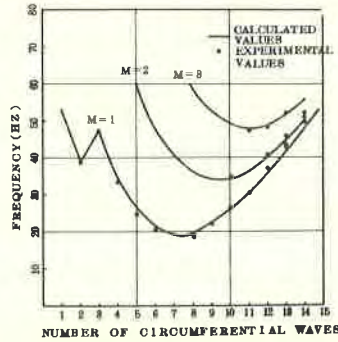


Fig. 4 The free vibration frequencies of unweighted cylinder

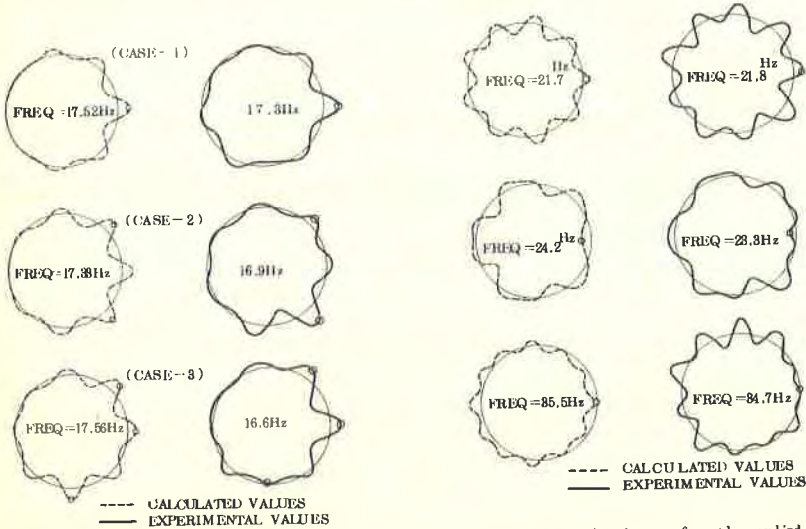


Fig. 5 The fundamental mode shapes

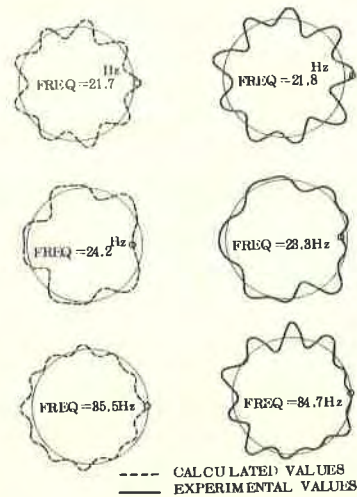


Fig. 6 The mode shapes for the cylinder with one attached mass

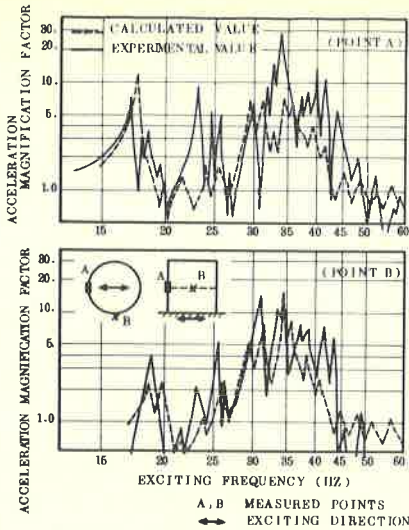


Fig. 7 The frequency response for 1-mass weighted cylinder (CASE-1)

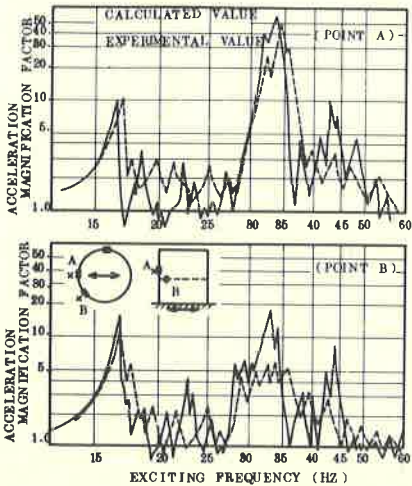


Fig. 9 The frequency response for 8-masses weighted cylinder (CASE-3)

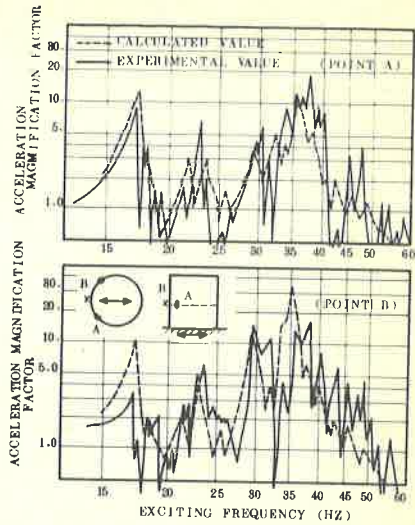


Fig. 8 The frequency response for 2-masses weighted cylinder (CASE-2)

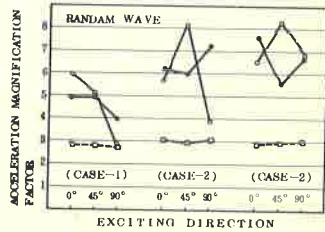
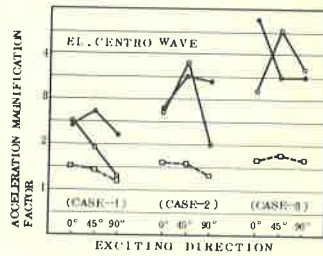


Fig. 11 Max response acceleration at EL. 75cm due to Earthquake waves.

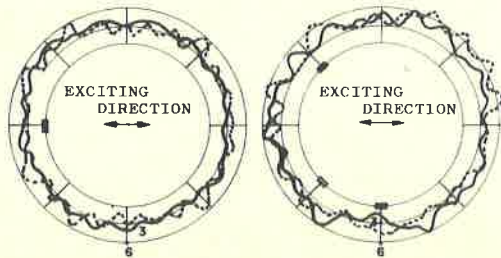


Fig. 10 The circumferential distribution of maximum response acceleration at EL. 75cm due to random wave.

Fig. 10 The circumferential distribution of maximum response acceleration at EL. 75cm due to random wave.

