

ABSTRACT

CHOI, BYOUNGHOAN. TOPICS IN RISK-BASED DESIGN AND PERFORMANCE EVALUATION OF STRUCTURES. (Under the direction of Abhinav Gupta)

This dissertation focuses on three topics that are critical to the development of a risk-based criteria for life-cycle design, analysis, and operation of power plant piping systems. One of three topics describes an exploratory study conducted on the application of risk-based load and resistance factor design approach to Section III of ASME Boiler and Pressure Vessel Codes for piping design. This study considers a straight pipe subjected to pressure and seismic load for service load D in class 2 and class 3 piping systems. The failure criterion needed to characterize the limit-state is considered as plastic instability.

The second topic in this dissertation relates to the analysis of coupled building-piping systems subjected to seismic loads. More specifically, it describes the effect of uncertainties in the modal properties of individual primary and secondary systems on the seismic response of coupled systems. Monte Carlo simulation and First-order reliability methods are used to study the problem in detail and identify critical aspects of this problem. Simple formulations are then developed for incorporation in a response spectrum method so that the design response is consistent with a desired probability of non-exceedence.

The third topic of this dissertation focuses on developing a probabilistic framework for structural condition assessment by using the frequency and mode shape data before and after degradation. While the results obtained using the proposed framework have shown considerable promise in applications to simple systems, significant room exists for improvement. Applications to additional examples with greater complexities are needed to identify the inconsistencies of the framework and develop improvements.

**TOPICS IN RISK-BASED DESIGN AND PERFORMANCE
EVALUATION OF STRUCTURES**

by
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DEDICATION

To my father

BIOGRAPHY

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PART I

INTRODUCTION

1. Introduction

In recent years, significant emphasis has been placed on developing methods for risk-based design, operation, and inspection for power plant structural systems including equipment and piping. This research addresses the following three problems with emphasis on seismic loads, i.e., (i) Reliability-based load and resistance factor design of piping components, (ii) Consideration of the effects of uncertainties on piping response in a coupled building-piping system analysis, and (iii) Probabilistic framework for condition assessment of piping systems using recorded vibration data. Each of these three topics is discussed in detail below.

(a) Reliability-based load and resistance factor design of piping components

Current piping design codes such as Section III of American Society of Mechanical Engineers' (ASME) Boiler and Pressure Vessel and ASME B31 codes [2] rely on the traditional Working Stress Design (WSD) format in which the safety factors are prescribed. These deterministic safety factors are based on experiences and observations from the test data. But, the reliability of the structural component designs based on WSD can vary considerably leading sometimes to excessively conservative designs. In recent years, consistency in reliability of structural components has been achieved by formally addressing uncertainty and randomness using a probabilistic analysis [7,8]. Probabilistic methodologies (design codes) in the form of Load and Resistance Factor Design (LRFD) have now been incorporated by several organizations such as American Institute of Steel Construction (AISC) and the American Concrete Institute (ACI). Ayyub et al. [4,5] and Assakkaf et al. [3] proposed the LRFD methodology for the structural design of ship structures and Mirza [11] that for reinforced concrete structures. The LRFD takes the form $\gamma L \leq \phi S$. S and L are nominal values of resistance and load which are considered as

random variables and γ and ϕ are respective partial factors of load and resistance determined for a given reliability (or probability of failure). A design code developed using LRFD concept provides risk consistency, is likely to result in more economical use of materials, provides compatibility in design for different structural materials, and permits future modifications due to increased knowledge of failure mechanisms, material characterization, and loading environment. The WSD based ASME Section III rules address the design of various piping components such as straight pipes, elbows, bends, supports, valves, etc. The design rules, however, vary depending upon the loading and failure criteria. In this research, as a first step, we study the LRFD approach in the seismic design of a straight pipe segment subjected to seismic loads for failure by plastic instability. The design equations are used to arrive at an acceptable design for the piping components such as straight pipes and elbows. The adequacy of the design is then checked by analyzing the complete piping system consisting of various components and supports. As studied in detail by Gupta and Gupta [9], the seismic behavior of a piping system depends not only upon its own dynamic characteristics but also upon the interaction with the building on which it is supported. For consistency with the risk-based design, it is important to consider the effect of uncertainties in such analyses. The effect of uncertainties in the input earthquake and structural properties of the building and the piping systems on the seismic response of coupled building-piping systems is discussed below.

(b) Consideration of uncertainties in seismic analysis of coupled building-piping systems

In seismic analysis of coupled primary and secondary systems with tuned or nearly tuned frequencies between them, it has been shown that the response of the secondary system is significantly affected by mass interaction and non-classical damping. Therefore, an uncertainty

in the primary system characteristics due to material properties, soil-structure interaction or modeling techniques can lead to a significant variation in the secondary system response. In conventional analysis, uncertainties in frequency of primary system are taken into account by broadening or shifting peaks associated with floor spectra at the location of secondary system supports. Reed et. al. [12] proposed three alternative methods to account for these uncertainties, the probabilistic simulated method, approximate probabilistic method, and the simplified method. These methods provide a floor spectra corresponding to 84% non-exceedance probability (NEP). However, these methods are not suited for the coupled primary-secondary system analysis as the floor response spectra are neither generated nor required. Traditionally, the analysis of a system subjected to earthquake input is handled using the response spectrum method. In the response spectrum method, the design spectra are typically defined at 84% NEP resulting in modal responses that correspond to 84% NEP. This only accounts for uncertainty in the earthquake input. It is considered that the total response obtained after combining individual modal responses will, in general, be close to the corresponding 84% NEP. Such a consideration represents upper bound and has served its purpose well. However, it cannot be extended directly to a situation where uncertainties in the primary and secondary system properties are also included in the evaluation of modal responses. The modal responses defined at 84% NEP can give excessively conservative values of total response. Simple formulations are developed in this research that can be incorporated in the response spectrum method as well as be consistent with the evaluation of total response corresponding to 84% NEP.

(c) Probabilistic framework for condition assessment

For nuclear power plant piping, in-service inspection with nondestructive examination is used to ensure continued operability and safety against degradation related failures. Presently used in-service inspection methods are deterministic and require that the general location of damage be known a priori. Most of the methods use ambient or forced vibration data to evaluate the changes in dynamic properties for condition assessment [6, 10, 13]. These methods can be characterized as either deterministic or probabilistic depending upon the computational approaches used in condition assessment. The concept of estimating degradation of likelihood is more achievable than damage detection in an absolute sense due to unavoidable uncertainties and omissions as well as due to the difficulty in defining absolute damage [1]. A probability-based framework for condition assessment using changes in vibration data can be useful for non-destructive condition assessment. While some recent studies have used probabilistic methods for condition assessment [14,15], these methods often suffer from inconsistency in identifying the likelihood of degradation primarily because they are based on perturbation and on the simplifying assumption of Gaussian distribution for characterizing the various random variables. In this study, we propose modifications to the existing probabilistic frameworks within the context of simulation-based decision making. The proposed approach is not based on perturbation and does not make any assumption regarding the distribution of probability density functions. While this framework has been shown to work well for simple systems, additional work is needed before meaningful conclusions can be reached.

2. Proposed Research

The specific tasks required to achieve the objectives of this research are described below.

(a) Reliability-based load and resistance factor design of piping components

- Define/ Select an appropriate performance function for class 2 and 3 piping in accordance with Equation (9) of ND3600, ASME Section III given below.

$$B_1 \frac{PD}{2t} + B_2 \frac{M}{Z_e} \leq 3.0S_m \quad \text{for Level D}$$

where, B_1, B_2 = primary stress indices for the specific piping component.

P = design pressure, psi

D = outer diameter of the pipe, in

Z_e = elastic section modulus of the pipe, in³

M = resultant moment due to a combination of mechanical loads, in-lb.

S_m = allowable stress, psi

t = thickness, in

The various loads in ASME B&PVC Section III are categorized into four service levels, Levels A through D, respectively. Service level D, in addition to other mechanical loads, corresponds to the loads resulting from a Design Basis Earthquake (DBE).

- Characterize the random variables and their statistical properties associated with the loads and resistance.

- Evaluate the minimum reliability associated with Equation (9) of ND3600, ASME Section III conditioned upon the occurrence of a DBE. This process is often referred to as code calibration.
- Non-dimensionalize the performance function for evaluating the partial load and strength factors in the framework of LRFD
- Evaluate load and resistance factors for specified target reliability values.
- As a check, compare the total safety factors associated with the presently used and the modified equations for the same reliability level.
- Validate the results using Monte Carlo simulation.
- Perform sensitivity analysis to evaluate the effects of changes in probabilistic distribution characterizing the random variables and their statistical properties.

(b) Consideration of uncertainties in seismic analysis of coupled building-piping systems

- Characterize the performance function for combining modal responses in accordance with response spectrum method.
- Non-dimensionalize the performance function for evaluating percentile values of each modal response corresponding to 84% NEP of the total response.
- Evaluate the percentile values of modal responses that give 84%NEP of the total response for uncertainties in base excitation and the properties of the primary and secondary system.
- Investigate the effects of variations in correlation between modal responses.
- Validate the results using Monte Carlo simulation.
- Develop the simplified methods for incorporation in a response spectrum method.

(c) Probabilistic framework for condition assessment

- Use the knowledge gained in conducting uncertainty analysis for evaluating the dynamic properties of secondary systems and in risk-based design method to conduct an exploratory study for developing a probability-based framework for condition assessment using changes in recorded vibration data.
- Evaluate the consistency and accuracy of some recently proposed approaches that also consider uncertainties in the frequency and mode shape data before and after degradation.
- Consistent with existing studies, characterize degradation in terms of changes in rigidities associated with the element stiffness matrices.
- Unlike existing studies that are based on perturbation of modal properties, use changes in response to harmonic excitations for evaluating the changes in rigidity values. To do so, evaluate the steady-state responses analytically using modal properties before and after degradation.
- Unlike existing studies that are based on the assumption of Gaussian distribution, evaluate the probability density functions for changes in rigidity values before and after degradation numerically using Monte Carlo simulation within a simulation-based decision making framework.
- Since the probability density functions evaluated in this approach are highly dependent upon the frequency of input excitation, use a sine-sweep type of input and characterize the likelihood of degradation statistically over the complete range of input frequencies.
- Conduct sensitivity analysis to study the accuracy and consistency of the proposed framework by varying the degree of uncertainties in frequency and mode shape data along with the effect of number of modes considered.

3. Organization

This dissertation consists primarily of three manuscripts. The first manuscript, part II of the dissertation, describes the methodology used and the results obtained from our study related to reliability-based load and resistance factor design of piping components. The specific tasks related to the objective of this work are listed above under section 2(a). This manuscript has been accepted for publication in the international journal of *Nuclear Engineering and Design*. This work has also been presented on more than one occasion to the ASME committee on piping design that has recently setup a working group to further explore the development of a reliability-based piping design code.

The second manuscript, part III of the dissertation, gives the details of our work and the corresponding results for consideration of uncertainties in seismic analysis of coupled building-piping systems. The specific tasks related to the objective of this work are listed above under section 2(b). Certain aspects of this work were presented at the 16th *International Conference on Structural Mechanics in Reactor Technology (SMiRT-16)* and the corresponding paper was published in the conference proceedings. Another paper describing additional results has been accepted for publication and presentation at SMiRT-17 to be held later this year. We anticipate sending the manuscript contained in this dissertation for review and possible publication in the *International Journal of Nuclear Engineering and Design* soon after the completion of the requirements for this dissertation.

The third manuscript, part IV of this dissertation, gives a detailed description of the work conducted on developing a probabilistic framework for condition assessment. The specific tasks related to the objective of this work are listed above under section 2(c). While this manuscript is

comprehensive, additional work is needed before this manuscript can be considered for review by a technical journal. It is so because the results presented in this manuscript are based on very simple systems. While the simplicity of these systems was extremely useful in understanding the complex problem and in developing the framework, additional systems with somewhat greater complexities are needed to evaluate the accuracy and consistency of the proposed framework.

Part V of this dissertation summarizes the work conducted in all the three areas as well as the conclusions reached in these studies. It also gives recommendations for future work in these areas of study.

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PART II

RELIABILITY-BASED LOAD AND RESISTANCE

FACTOR DESIGN FOR PIPING:

AN EXPLORATORY CASE STUDY

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RELIABILITY-BASED LOAD AND RESISTANCE FACTOR DESIGN FOR PIPING: AN EXPLORATORY CASE STUDY

Abhinav Gupta and Byounghoan Choi

Abstract: This paper presents an exploratory case study on the application of LRFD approach to the Section III of ASME Boiler and Pressure Vessel code for piping design. The failure criterion for defining the performance function is considered as plastic instability. Presently used design equation is calibrated by evaluating the minimum reliability levels associated with it. If the target reliability in the LRFD approach is same as that evaluated for the presently used design equation, it is shown that the total safety factors for the two design equations are identical. It is observed that the load and resistance factors are not dependent upon the diameter to thickness ratio. A sensitivity analysis is also conducted to study the variations in the load and resistance factors due to changes in (a) coefficients of variation for pressure, moment, and ultimate stress, (b) ratio of mean design pressure to mean design moment, (c) distribution types used for characterizing the random variables, and (d) statistical correlation between random variables. It is observed that characterization of random variables by lognormal distribution is reasonable. Consideration of statistical correlation between the ultimate stress and section modulus gives higher values of the load factor for pressure but lower value for the moment than the corresponding values obtained by considering the variables to be uncorrelated. Since the effect of statistical correlation on the load and resistance factors is insignificant for target reliability values, the effect of correlated variables can be neglected.

1. Introduction

Development of structural design methodology involves consideration of safety factors to account for uncertainty in loading, material characteristics, geometrical properties, modeling, analysis, etc. Management and control of risk due to uncertainties through proper design is a major engineering goal. The design codes and standards address uncertainties through safety factors that may be defined either implicitly such as those used for the Working Stress Design (WSD) format or explicitly such as those used for the Load and Resistance Factor Design (LRFD) format. Currently used piping design codes such as the Section III of the American Society of Mechanical Engineers' (ASME) Boiler and Pressure Vessel and ASME B31 codes rely primarily on the traditional WSD format in which the safety factors are prescribed deterministically. These deterministic safety factors are based on several years of experience and supporting observations from the test data. While the Section III rules have worked very well over several years, the reliability of these designs can vary considerably leading sometimes to excessively conservative designs.

In recent years, consistency in design has been achieved by formally addressing uncertainty and randomness through a probabilistic analysis (Freudenthal 1947, Galambos and Ravindra 1973, Hasofer and Lind 1974, Haugen and Wirsching 1975, McGregor 1976, Ravindra and Galambos 1978, Ellingwood et al. 1982, Nowak 1993). Significant advances in computing resources and the availability of probabilistic design tools have also facilitated the application of these methods to complex design problems. Probabilistic design methodologies as a basis for design codes have now been incorporated by several organizations that include the American Institute of Steel Construction (AISC) and the American Concrete Institute (ACI). Ayyub et al.

(1995) and Assakkaf et al. (2000) proposed the LRFD methodology for the structural design of surface ship structures and Mirza (1996) that for reinforced concrete columns. The ASME Section XI and Operation & Maintenance codes have adopted risk-informed methodologies for decisions involving in-service inspection, preventive maintenance, repair, and replacement. An effort is underway to evaluate the benefits of LRFD concept in the ASME-Section III piping design methodology (ASME 2002). Consistency in Sections III and XI is likely to provide a risk-informed lifecycle approach that would encompass construction which includes design, operation, and maintenance.

A design code developed using LRFD concept provides risk consistency, is likely to result in more economical use of materials, provides compatibility in design across different structural materials, and permits future modifications due to increased knowledge of failure mechanisms, material characterization, and loading environment. It also provides a framework to account for the time-dependent degradation within a risk-based framework. Such a framework is useful in developing strategies for not only inspection and maintenance but also for life extension and license renewal. Other advantages include but are not limited to simplifications in system reliability analysis, and management of uncertainty in strength, loads and analytical models. Specific to piping design, consistency by using the LRFD concept in Section III is expected to reduce the quantity of seismic restraints or supports needed in a piping system. The consequent savings over the plant lifetime are likely to be significant due to reduced inspection and maintenance costs.

The Section III rules address the design of various piping components such as straight pipes, elbows, bends, supports, valves, etc. The design rules vary depending upon the loading and failure criteria. In this paper, we explore the role of risk-based LRFD approach in the design

of a straight pipe segment for failure by plastic instability. In recent years, significant progress has been made towards using risk-informed design and regulation for power plant systems. Several studies have suggested frameworks for a system-based approach to characterize the acceptable risk associated with a particular piping or equipment. In such a scenario, it is desirable to evaluate the load and resistance factors as a function of reliability and not just a value corresponding to the specified single target reliability as is the case in AISC and ACI codes. The discussion in this paper focuses on: (1) definition of a performance function and random variables to characterize the intended limit-state, (2) characterization of different random variables with appropriate probability distributions, (3) evaluation of the reliability with respect to the defined limit-state for piping designed using the existing code equations, (4) identification of a range for reliability levels to compute the partial safety factors, (5) computation of load and resistance factors (also known as partial safety factors), and (6) sensitivity analysis and the effect of correlation between the random variables. It is anticipated that this study will provide useful input to the work being undertaken by the ASME working group on piping design (ASME 2002).

2. Reliability-Based Limit State Design

A design equation that is based on the LRFD principles is very similar in appearance to the one that is based on deterministic factors of safety. However, the deterministic factors for the loading and the strength terms are selected depending upon a level of reliability acceptable to professional engineers and experts. To develop risk-based design equations, the first step lies in the definition of a performance function corresponding to a design limit-state for the specified

failure modes. In the AISC code for the design of steel structures and the ACI code for the design of concrete structures, limit states are grouped into two categories. First, the ultimate limit states are used to characterize the strength requirements for preventing collapse. Second, the serviceability limit states are used to characterize the requirements for functional use of the structure. In general, if S denotes the strength and L the load, the reliability R can be defined as the probability when $S > L$. Mathematically,

$$R = P(S > L) \quad (1)$$

where $P(-)$ denotes the probability of an event. The performance function can be written as

$$Z = g(S, L) = L - S \quad (2)$$

or in general,

$$Z = g(X_1, X_2, \dots, X_n) \quad (3)$$

where X_i represents probabilistically defined variables for the loads and the strength. The function $g(-)$ is a limit state function that describes the failure criterion. That is,

$$g(-) > 0 \quad \text{failure state} \quad (4a)$$

$$g(-) = 0 \quad \text{limit state} \quad (4b)$$

$$g(-) < 0 \quad \text{survival state} \quad (4c)$$

The probability of failure P_f is then given by the joint probability distribution of X_i 's.

$$P_f = \int_{g(-) > 0} \dots \int f_{X_1 X_2 \dots X_n}(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (5)$$

where $f_{X_1 X_2 \dots X_n}(x_1, x_2, \dots, x_n)$ is the joint probability density function of the random variables X_i 's. In general, the joint density function is unknown and the evaluation of the integral is a formidable task. Alternatively, the reliability of a particular design or the partial safety factors

corresponding to a target reliability index are evaluated using the Advanced First-Order Reliability Method (AFORM) or also known as the Hasofer-Lind method. In AFORM, the performance function Z is expressed by a linear function obtained from a Taylor series expansion about the most probable failure point $(x_1^*, x_2^*, \dots, x_n^*)$ at which $Z = g(x_1^*, x_2^*, \dots, x_n^*) = 0$. As shown in Figure 1 for the simple case of $X_1=S$ and $X_2=L$, the reliability is then expressed in terms of the reliability index β such that $P_f = 1 - \Phi(\beta)$, where Φ is the cumulative distribution function of the standard normal variate (Ang and Tang 1990). For a specified target reliability index, the probability distributions of the load and strength variables are used to evaluate the mean load and resistance factors. To do so, an iterative procedure such as that given by Ayyub and McCuen (2002), by Ang and Tang (1990), or by Haldar and Mahadevan (2000) is usually employed. The mean load factors γ_i and the mean resistance factor ϕ_s are expressed as

$$\gamma_i = \frac{x_i^*}{\mu_{X_i}} \quad , \quad \phi_s = \frac{x_s^*}{\mu_{X_s}} \quad (6)$$

where X_i 's represent the random variables for loads and X_s that for the strength with respective means denoted by μ_{X_i} and μ_{X_s} . The load and resistance factors together are also referred to as partial safety factors. The design equation based on LRFD concept can then be written as

$$\gamma_1 \mu_{L1} + \gamma_2 \mu_{L2} + \dots + \gamma_n \mu_{Ln} \leq \phi_s \mu_s \quad (7)$$

As in the AISC and ACI codes, the above equation can be modified further using the appropriate "bias factors" and express the load and the strength in terms of the nominal values (Ravindra and Galambos 1978) as

$$\gamma_{n1} L_1 + \gamma_{n2} L_2 + \dots + \gamma_{nm} L_n \leq \phi_{ns} S \quad (8)$$

$$\gamma_{ni} = \frac{\gamma_i}{V_i}, \quad \phi_{ns} = \frac{\phi_s}{V_s} \quad \text{and} \quad v_i = \frac{L_i}{\mu_{Li}}, \quad v_s = \frac{S}{\mu_s}$$

where v_i and v_s are bias factors corresponding to load and strength, respectively. For simplicity, the bias factors are not used in the exploratory case study presented here and the partial safety factors are expressed in terms of their mean values.

3. Performance Function for Plastic Instability of Piping

The piping design criteria in Section III of the ASME Boiler and Pressure Vessel Code (ASME 2001) gives allowable stress limits for protecting the structural integrity of the pressure boundary. There are two main aspects of piping design, the pressure design and the design of piping components for other mechanical loads. Pressure design involves sizing of the pipe for minimum wall thickness requirements. Design of piping components such as straight pipe or elbows involves evaluating the combined effects of pressure and moment. The various loads in Section III are categorized into four service levels, Levels A through D, respectively. Service level D, in addition to other mechanical loads, corresponds to the loads resulting from a Design Basis Earthquake (DBE). Further, piping supports and components in Section III are classified as Class 1, Class 2, or Class 3 depending upon the consideration of their importance to safety. The piping design rules for each class are given in the sub-articles NB-3600, NC-3600, and ND-3600 for Class 1, Class 2, and Class 3, respectively. For exploratory case study presented in this paper, we do not consider any thermal effects. In accordance with ND-3600, the maximum allowable design pressure for a pipe with outer diameter D and thickness t is defined by

$$P_a = \frac{2S_m t}{D - 2yt} \quad (9)$$

in which $y = 0.4$ and S_m is the allowable stress that is defined as the smaller of (i) one-fourth of the ultimate stress ($S_U/4$), or (ii) two-third of the yield stress ($2S_y/3$). Piping components are designed according to Equation (9) of ND-3600 for the stresses due to pressure and moment.

$$\begin{aligned}
 B_1 \frac{PD}{2t} + B_2 \frac{M}{Z_e} &\leq 1.5S_m && \text{for Level A} \\
 &\leq 1.8S_m && \text{for Level B} \\
 &\leq 2.25S_m && \text{for Level C} \\
 &\leq 3.0S_m && \text{for Level D}
 \end{aligned} \tag{10}$$

where,

B_1, B_2 = primary stress indices for the specific piping component.

P = design pressure, psi

Z_e = elastic section modulus, in³

M = resultant moment due to a combination of mechanical loads, in-lb.

For a straight pipe, B_1 and B_2 are 0.5 and 1.0, respectively. The above equation is primarily concerned with plastic instability of the piping components. As stated earlier in the context of working stress method of design, Equation 10 considers safety factors on the strength to specify allowable stresses for various load levels. No safety factors are considered for the loads – pressure and the moment terms. In the LRFD concept, the ultimate strength limit-state corresponding to Equation 10 is given by the equation for the formation of a plastic hinge at a particular cross-section. Assuming the material behavior to be elastic-perfectly plastic, we get

$$\frac{PD}{4t} + \frac{M}{Z_p} = S_U \qquad Z_p = 1.27Z_e \tag{11}$$

where Z_p is the plastic section modulus for a straight pipe cross-section. The corresponding performance function is given by rearranging the above equation as

$$g(-) = S_U - \frac{PD}{4t} - \frac{M}{Z_p} = 0 \quad (12)$$

Selection of the above performance function does not indicate that the formation of a plastic hinge is permitted in the new designs. Instead, the new designs would correspond to a specified acceptable reliability against the formation of the plastic hinge. It should also be noted that the formation of a plastic hinge does not mean a loss of integrity in the pressure boundary. The final design equations are arrived at by probabilistically determining the load and the resistance factors corresponding to a specified level of target reliability. If γ_P and γ_M represent the load factors for the pressure and the moment, respectively, and ϕ_S that for the strength, the design equation is written as

$$\gamma_P \frac{PD}{4t} + \gamma_M \frac{M}{Z_p} \leq \phi_S S_U \quad (13)$$

It should be noted that the moment term in the above equation can be further separated into individual parts for moments due to each mechanical load such as the deadweight and the earthquake. For simplicity in illustrating the LRFD concept, we consider the combined moment and not the individual terms corresponding to each mechanical load. Further, we consider only the service Level D and also assume that the moment due to deadweight is much less than that due to earthquake such that the effect of deadweight moment is neglected. Finally, the stress indices B_1 and B_2 for a straight pipe are implicitly included in Equation 13. For other piping components such as elbows, Equation 13 can be rewritten as

$$\gamma_P B_I \frac{PD}{2t} + \gamma_M B_2 \frac{M}{Z_P} \leq \phi_S S_U \quad (14)$$

Note that Z_P is the plastic section modulus for a straight pipe and not the piping component.

At this point in the discussion we would like to mention that a performance function can also be specified corresponding to the design for internal pressure which is dictated by Equation 9. One may argue that the design for both the internal pressure and the plastic hinge should correspond to the same reliability levels for risk-consistency. However, it is extremely important to consider the nature of structural failure modes before doing so. Excessive internal pressure can cause, among other things, a sudden pipe-burst type of failure. Unlike plastic hinge formation which must progress to the formation of plastic hinge mechanism before failure in which the integrity of pressure boundary is not violated and a ductile failure mode provides sufficient response time, pipe-burst is a sudden (brittle) failure that is highly undesirable. Parallels to this situation can be drawn from the building design codes such as the ACI code for concrete structures in which a bending failure of the structural members is ductile and a shear failure is brittle. The brittle failure mode is avoided by overdesigning in shear relative to bending. Likewise, it would be desirable to avoid a failure due to excessive internal pressure by overdesigning for it. One way to achieve this would be to use a much higher value of reliability level in the probabilistic analyses. In this paper, we do not apply LRFD concept to the design for internal pressure. The premise is that Equation 9 which is based on the traditional WSD format with deterministic safety factors has worked well over the years and provides a good basis for overdesigning against excessive internal pressure. It should also be noted that overpressure relief is required for pressured piping systems which also limits the potential for failure due to overpressure.

4. Characterization of Random Variables and Code Calibration

Calculation of partial safety factors requires probabilistic characterization of the load and strength variables in the limit-state equation. Such a characterization of these variables requires quantification of uncertainty and variability in them. Therefore, means, coefficients of variation, and distribution types need to be specified for each variable. This process is based on collection of relevant data and its statistical analysis. In this exploratory case study, meaningful estimates of the relevant quantities are taken based on those used previously by the ASME working group on piping design (Stevenson et al. 2000) with minor modifications based on communication with industry professionals. Stevenson et al. (2000) and Barnes et al. (2000) provide one such ASME sponsored study in which the various variables in Equation 10 are characterized probabilistically for evaluating the failure probabilities of piping systems. Based on this study, a lognormal distribution is first used to characterize the random variables. The effect of other distribution types on the partial safety factors is also studied later. We calculate the partial safety factors by considering the diameter to thickness ratio (D/t) as constant and the pipe material is taken as ASME SA 106 Gr B Carbon Steel pipe having a minimum specified ultimate strength S_U equal to 60 ksi and a minimum yield strength S_y equal to 35 ksi. According to Stevenson et al. (2000), the minimum strength values are defined at mean minus two standard deviation level. For a particular value of D/t ratio, the maximum allowable design pressure is calculated in accordance with Equation 9. Even though we considered several different values for the coefficient of variation of pressure, we give results primarily for a value of 0.35 in this paper unless otherwise specified. A value of 0.35 is relatively much higher than a value of 0.15 taken by Stevenson et al. (2000). A high value is selected to conduct a sensitivity analysis. For a lower value, the load

factor for the pressure term is relatively insensitive to the changes in the quantities like the mean design pressure and the type of probability distribution. The distributions, means, and coefficients of variation for various random variables used in this study are summarized in Table 1. Later, the sensitivity of the partial load factors is studied by considering deviations in these estimates. To begin with, the random variables are considered to be statistically independent. The effect of correlated variables is studied later.

Once the random variables representing the load and the strength are characterized, the partial safety factors can be evaluated corresponding to a specified target reliability level using the advanced second moment method (Ayuub and McCuen 2002, Haldar and Mahadevan 2000, Ang and Tang 1990). The target reliability levels are typically selected based on the reliability levels that are implicit in the current designs. Such a process of developing LRFD guidelines to meet the target reliability levels that are implicit in the current practices is called code calibration. The first step in code calibration, therefore, lies in determining the reliability levels implicit in the current design practice. One way to do so is to identify several representative designs and evaluate their reliability with respect to the performance function considered which in this case is given by Equation 12. For a particular D/t ratio and the piping material, the allowable design pressure is given by Equation 9 whereas the moment is calculated from a seismic analysis. Depending upon how close is the left-hand side of Equation 10 to the allowable stress term on the right-hand side, the reliability of a piping system can vary over a large range. The moment, calculated from seismic analysis for a given earthquake, can vary significantly depending upon the method of analysis used (Gupta and Gupta 1995). The presently used methods for seismic analysis of piping systems are generally considered excessively conservative. The primary interest in code calibration is the minimum reliability levels

associated with the present practice, i.e. Equation 10. If the maximum allowable design pressure P_a for a piping with known D/t and S_m is calculated from Equation 9, piping can be designed to withstand a maximum value of seismic moment that can be calculated as

$$M_{code} = 3.0S_m Z_e - \frac{PDZ_e}{4t}; \quad 0 \leq P \leq 2P_a \quad (15)$$

The maximum value of allowable moment calculated from the above equation corresponds to 84 percent non-exceedence probability conditioned upon a given design earthquake because the DBE response is typically calculated at the corresponding probability of non-exceedence (Kennedy 1999). For convenience, the performance function can be rewritten in a non-dimensional form by defining a mean stress ratio \bar{r}

$$\bar{r} = \frac{\mu_M / Z_p}{\mu_p D / 4t} \quad (16)$$

Figure 2 shows the minimum conditional reliability index β (reliability index for a given design earthquake) calculated by the AFORM for different values of the mean stress ratio \bar{r} . It should be noted that for mean design pressure μ_p less than equal to $2P_a$ and the corresponding mean design moment μ_M calculated from an 84 percent non-exceedence value given by Equation 15, the minimum value of \bar{r} that satisfies the limit-state equation is equal to 0.9. For $\bar{r} < 0.9$, the mean design pressure μ_p is greater than $2P_a$. As shown in Table 2, the reliability values are almost identical for different D/t ratios for a given value of the design pressure indicating that the reliability value is independent of the D/t ratio. These values also indicate that the minimum reliability values of practical interest are less than 2.25. Once again, we would like to emphasize that the reliability levels of actual piping system designs are much higher than these values.

5. System-Based Target Reliability

Conventionally, the partial safety factors are calculated for a target reliability value that is identified based on the minimum reliability levels calculated in the previous section. Specific to power plant piping, such an approach would, however, result in piping designs that may either be conservative or unconservative with respect to their contribution to the overall plant risk calculated from a plant wide Probabilistic Risk Assessment (PRA). In other words, the contribution of different piping systems to the overall plant risk varies significantly. Therefore, selection of a single target reliability for all piping systems in a plant will be governed by the reliability of a piping that has the maximum impact on the overall risk. Consequently, this target value is likely to be excessively conservative for piping that does not contribute significantly to the overall risk.

In a recent project funded under the Department of Energy's Nuclear Energy Research Initiative (NERI) grant, a PRA-based framework has been proposed for risk-informed regulatory requirements with respect to the design and operation of new nuclear power plants (Apostolakis et al. 2001, Apostolakis 1985) The framework is based on a top-down hierarchy for defining the risk goals and for defense in depth. Development of a master logic diagram (MLD) is suggested for implementation of the framework (Duran et al. 2000, Apostolakis et al. 2001). An MLD is used to identify the safety functions, systems, structures, and components that are required to maintain safety and to identify the accident initiators as well as the system response failures that could compromise safety. Identification of the critical structures and components through an MLD can then be used to develop appropriate design strategies. In this context, consistency of piping reliability with the plant reliability assessment can be achieved by selecting partial safety

factors corresponding to the reliability levels identified for a particular piping in the plant-wide PRA. Such an approach would also provide a basis of determining the degree of redundancy and diversity needed in a design. To do so, it would be desirable to calculate partial safety factors for a range of target reliability index β_0 . To strike a balance with the current practice, the range can be chosen based on the minimum reliability levels calculated in the previous section and given in Table 2. Based on the β values in Table 2 and Figure 2, we take the lower bound as 1 and the upper bound as 3, i.e., $1 \leq \beta_0 \leq 3$. Even though the highest value in Figure 2 is less than 2.5, the upper limit is taken as 3 primarily for academic purpose. Note that these β_0 values represent reliability index conditioned upon a given design earthquake.

6. Computation of Partial Safety Factors

To begin with, we compare the partial safety factors calculated using the advanced second moment method with those calculated using the Monte Carlo simulation. The purpose is to evaluate the effect of non-linearity in the performance function on the computation of partial safety factors using the AFORM. For calculating the partial safety factors corresponding to a target reliability level, the AFORM does not require prior knowledge of the mean moment μ_M because the mean values for the design pressure, the plastic section modulus, and the strength are known. However, this is not the case in a Monte Carlo simulation wherein the mean for the moment term is needed to calculate the failure point and the partial safety factors. Such a limitation is typically overcome by using AFORM within the Monte Carlo to evaluate approximate values of μ_M . For illustration purposes, let us consider a linear performance function that is written as

$$g(x) = a_o + \sum_i a_i X_i \quad (17)$$

in which a_i are constants and X_i are the random variables. According to the AFORM, we can write

$$\beta_0 = \frac{\mu_g}{\sigma_g} = \frac{a_o + \sum_i a_i \mu_{X_i}}{\sqrt{\sum_i (a_i \Omega_i \mu_{X_i})^2}} \quad (18)$$

where μ , σ , and Ω represent the mean, standard deviation, and coefficient of variation, respectively. If a particular μ_{X_i} is unknown, Equation 18 can be solved directly to evaluate its value corresponding to a given value of target reliability β_0 . For non-linear performance function, a direct solution is not possible and the unknown mean is evaluated by an iterative solution. The iterative solution used for this purpose can be like the one suggested by Rackwitz and Fiessler (1978) with a slight modification that the equations are solved for the unknown mean corresponding to a given value of β_0 and not vice versa. The mean calculated by such a procedure is then used in the Monte Carlo simulation for evaluating the partial safety factors. Approximately one million simulation sets are used for computing γ and ϕ factors corresponding to each value of β_0 . Figure 3 shows that the partial safety factors calculated using the AFORM and those calculated using the Monte Carlo simulation are close to each other.

Next, we illustrate that the design equation in the LRFD format corresponds to the same total safety factor as the original design equation based on WSD. For this purpose, we compare the total safety factor corresponding to Equation 10 with that corresponding to Equation 13 in which the partial safety factors correspond to the minimum target reliability levels calculated earlier and given in Table 2. Since $Z_p = 1.27Z_e$, the total safety factor, SF , corresponding to Equation 13 can be calculated as follows.

Step 1. Express the design equation as

$$\gamma_L \left(\frac{\mu_P D}{4t} + \frac{\mu_M}{Z_e} \right) = \gamma_P \frac{\mu_P D}{4t} + \gamma_M \frac{\mu_M}{Z_P} \leq \phi_s \mu_S \quad (19)$$

Step 2. Define a mean stress ratio $\bar{r} = \frac{\mu_M / Z_P}{\mu_P D / 4t}$

Step 3. Calculate the total safety factor

$$SF = \frac{\gamma_L}{\phi_s} = \frac{\mu_S}{\frac{\mu_P D}{4t} (1 + \bar{r})} \quad (20)$$

The total safety factors for various values of design pressure and \bar{r} are compared in Table 3 which shows that the two sets of results are same, as they should be. As in case of Table 2, the safety factors are same irrespective of the value of D/t . Consequently, we consider only a single value of D/t in the rest of this study.

7. Sensitivity Analysis

In this section, we present the results of a parametric study conducted to explore the sensitivity of the partial safety factors to the changes in the characterization of various random variables summarized in Table 1. We study the effect of variability in the following four quantities: (i) coefficient of variation values for the P , M , and S_U , (ii) mean stress ratio \bar{r} , (iii) distribution types used for characterizing the random variables, and (iv) the statistical correlation between the random variables.

(i) *Coefficients of Variation*: We consider the effect of variability in the quantities for only the pressure, moment, and ultimate stress, i.e., Ω_P , Ω_M , and Ω_S , respectively. A variability in Ω_Z is

ignored as its value is very small relative to the corresponding values for other random variables.

Each of the coefficient of variation is varied between $\pm 20\%$ of the values given in Table 1.

Figures 4 to 12 give the partial load factors for each such case. As anticipated, a high value of Ω_M controls the calculation of load factors and a variation in its value significantly influences γ_M .

The effect of variations in Ω_P , Ω_M , and Ω_S is relatively much smaller on γ_P and on ϕ_S .

(ii) *Mean Stress Ratios*: For different values of \bar{r} , Figures 13 to 15 show the effect on partial safety factors. For a given value of target reliability index β_0 , it is observed that the load and the resistance factors γ_M , γ_P and ϕ_S are insensitive to values of \bar{r} when $\bar{r} \geq 0.9$. Even though the safety factors are highly sensitive to the value of \bar{r} and β_0 for $\bar{r} < 0.9$, such small values of \bar{r} correspond to a mean design pressure greater than $2P_a$ that is highly impractical.

(iii) *Distribution Types*: Selection of the distributions for characterizing the random variables can be critical in the probabilistic analysis using AFORM. As stated earlier and given in Table 1, the random variables in this study are characterized as log-normal based on the existing studies conducted by the ASME working group on piping design (ASME 2002). We study the effect of distribution types by considering two additional cases. In one case all the random variables are characterized by normal distributions. In the second case, the pressure and the section modulus are characterized by log-normal distributions whereas the moment and ultimate strength are characterized by extreme value distributions, the Type II extreme value distribution for the moment and the Weibull distribution for the ultimate strength. Table 4 summarizes all the three cases and the effect on partial safety factors is shown in Figures 16 to 18. It is observed that characterization of variables by normal distribution, case I, gives conservative estimates for γ_P and ϕ_S . On the other hand, γ_M is significantly higher for cases II and III, i.e. characterization of variables by lognormal distribution or a combination of lognormal and extreme value

distributions. It is important to note that the differences in γ_M for the lognormal and the Type II extreme value distributions are negligible for β_0 less than 2.25. As discussed in detail earlier, the minimum reliability values associated with the current code are on the order of 2.25 or less. The upper limit, $\beta_0 = 3$, is chosen primarily for academic purposes. Further, the uncertainty in moment is much larger than that in the pressure. Therefore, case II in which all the random variables are characterized as lognormal appears to be suitable in calculating the safety factors.

(iv) *Correlated Variables*: Next, we study the effect of statistical correlation that may exist between certain random variables. For example, selection of a material with higher ultimate strength would result in a reduction in the lower section modulus. Therefore, the two variables appear to be negatively correlated. However, no statistical correlation exists between the design pressure and any other variable. Similarly, the moment is not correlated with any other variable. If ρ represents the statistical correlation and the subscripts p , s , m , and z the random variables for pressure, ultimate stress, moment, and section modulus, then we can write $\rho_{ps} = \rho_{pz} = \rho_{zm} = \rho_{pm} = \rho_{sm} = 0$. In this study the maximum value of correlation coefficient ρ_{sz} is taken as -0.3. Figures 19 to 21 illustrate the effects of statistical correlation for $\rho_{sz} = 0.0, -0.1, -0.2,$ and -0.3 . As shown in these figures, the effect of statistical correlation on load and resistance factors is insignificant for reliability values.

8. Summary and Conclusions

An exploratory case study on the application of the LRFD approach to the Section III of ASME Boiler and Pressure Vessel Code for piping design is presented in this paper. The objective is to provide useful input to the similar but more comprehensive study being undertaken by the

ASME working group on piping design. While a complete piping system consists of several components such as straight pipes, elbows, branch connections, etc., only a cold straight pipe section is considered in this study. The performance function is defined with respect to a failure mode that is defined by plastic instability. For simplicity, only Service level D is considered and the effects of pressure and seismic moment are considered. Since the effect of dead weight is insignificant with respect to the DBE loading in service level D, it is neglected in the present study. As a first step in this process, the presently used design equation that is based on the working stress method of design is calibrated by calculating the minimum reliability levels associated with it for various values of design pressure and the diameter to thickness D/t ratio. It is observed that the minimum reliability index varies between a narrow range of 1.86 and 2.21 when mean design pressure is less than equal to $2P_a$. The D/t ratio has no influence on the minimum reliability levels. It is also shown that the D/t ratio has no influence on the partial safety factors calculated using LRFD approach. Monte Carlo simulation is used to verify the computation of partial safety factors using the First Order Reliability Method. It is illustrated that the total safety factor for the presently used design equation is same as that for a design equation based on the LRFD format in which the target reliability is equal to the minimum reliability of the presently used design equation.

The partial safety factors are calculated for a range of target reliability values. The purpose is to provide consistency with a systems-based approach such as that proposed by Apostolakis et al. (2001). In such an approach, a plant wide PRA may be used to identify critical structural systems and components and to allocate the target reliabilities for different piping systems based on the outcome from a Master Logic Diagram (MLD) that is generated using the top-down hierarchical approach. Identification of critical systems and components from such an

approach also assists in the appropriate allocation of redundancies for defense in depth. Finally, the sensitivity of the partial safety factors is studied by (i) considering a variability in the values of the coefficient of variation for various random variables, (ii) varying the ratio of mean design pressure to mean design moment, (iii) considering different distribution types for each random variables, and (iv) considering the statistical correlation between specific random variables. The observations made from the sensitivity studies show that corresponding to a specified target reliability level β_0 the safety factors are insensitive to the values of mean stress ratio when the mean design pressure is less than equal to $2P_a$. The load factor γ_M is quite sensitive to the value of coefficient of variation for the moment especially for larger values of β_0 . This is so because a large uncertainty in moment characterized by a higher value of its coefficient of variation governs the design. The effect of changes in the coefficient of variation for the pressure and the strength is relatively negligible on γ_P and ϕ_S . It is also observed that the load factors can vary significantly for different types of distributions considered. For reliability index values corresponding to those encountered in practical applications, consideration of a lognormal distribution for the moment gives almost similar values of γ_M as those given by the consideration of the Type II extreme value distribution. Significant differences exist for target β_0 values greater than 2.25. Characterization of variables by normal distribution gives higher values for γ_P and ϕ_S but much lower values for γ_M . Consideration of lognormal distribution for all the variables appear to be a reasonable choice. Statistical correlation has no influence on partial safety factors.

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Table 1. Statistical characteristics of random variables

| Variable | Distribution Type | Mean Value | Coefficient of Variation |
|----------|-------------------|------------------|--------------------------|
| P | Log-normal | $\leq 2P_a^*$ | 0.35 |
| Z_p | Log-normal | From D/t value | 0.05 |
| M | Log-normal | N/A | 0.50 |
| S_u | Log-normal | 75 ksi | 0.10 |

* P_a is calculated from equation 8

Table 2. Minimum reliability index β corresponding to equation 10

| D/t | Mean Design Pressure(μ_P) | | |
|-------|---------------------------------|-------|--------|
| | 0 | P_a | $2P_a$ |
| 20 | 1.86 | 2.02 | 2.21 |
| 40 | 1.86 | 2.01 | 2.20 |
| 60 | 1.86 | 2.01 | 2.20 |
| 80 | 1.86 | 2.01 | 2.20 |

Table 3. Total safety factors

| Mean Design Pressure (μ_P) | Mean Stress Ratio (\bar{r}) | Current Design Equation | LRFD Based Equation |
|-------------------------------------|------------------------------------|----------------------------|------------------------|
| P_a | 2.0 | 3.20 | 3.20 |
| | 3.0 | 2.40 | 2.40 |
| $2P_a$ | 0.5 | 3.20 | 3.20 |
| | 1.5 | 1.92 | 1.92 |

Table 4. Type of distribution of parameters

| Variable | Distribution Type | | |
|----------|-------------------|------------|-----------------------|
| | CASE I | CASE II | CASE III |
| P | Normal | Log-normal | Log-normal |
| Z | Normal | Log-normal | Log-normal |
| M | Normal | Log-normal | Type II extreme value |
| S_u | Normal | Log-normal | Weibull |

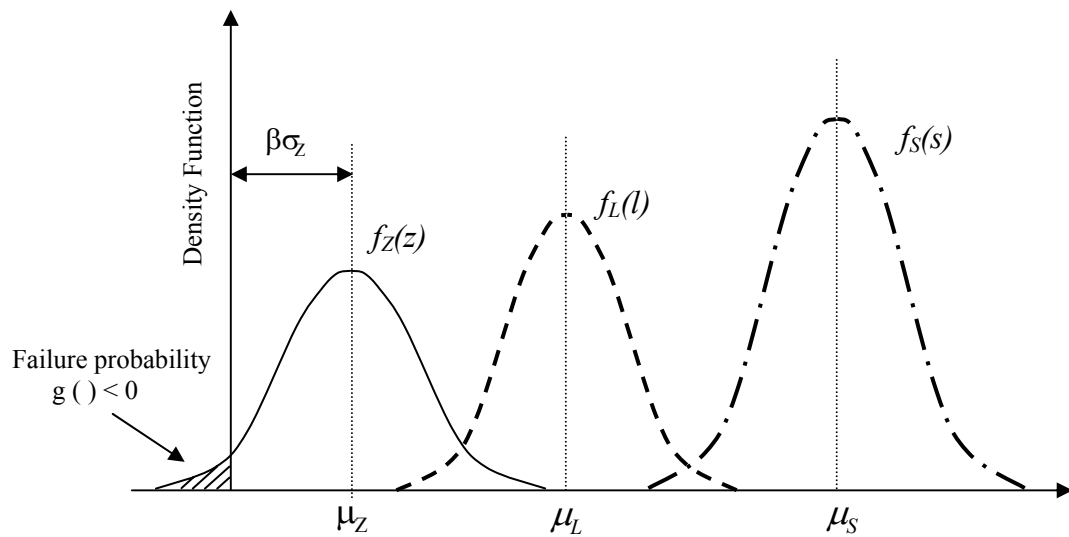


Figure 1. Probability density functions for S , L , and Z

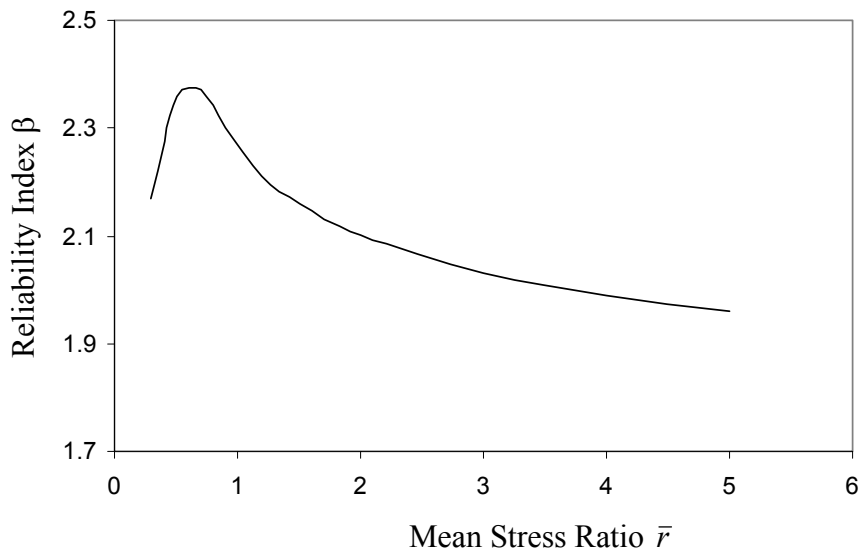


Figure 2. Variation of reliability index with mean stress ratio \bar{r}

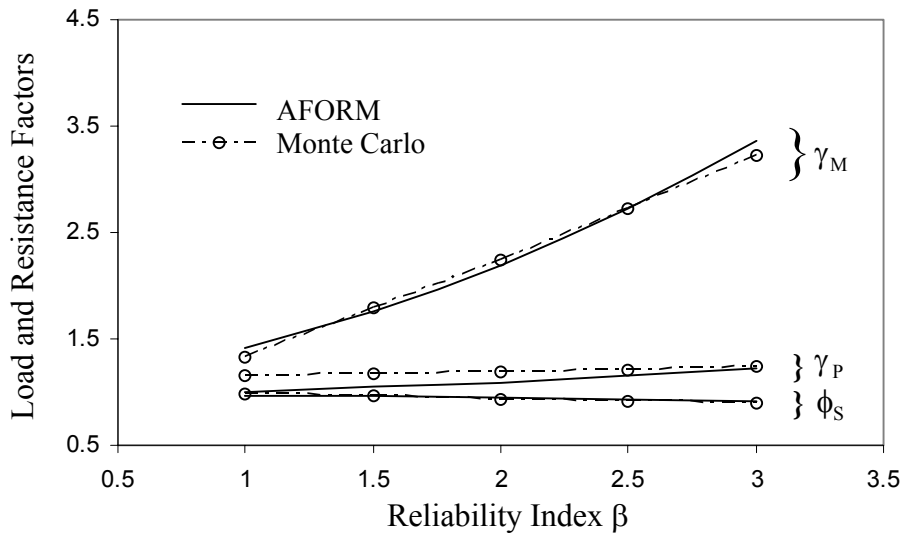


Figure 3. Verification with Monte Carlo simulation

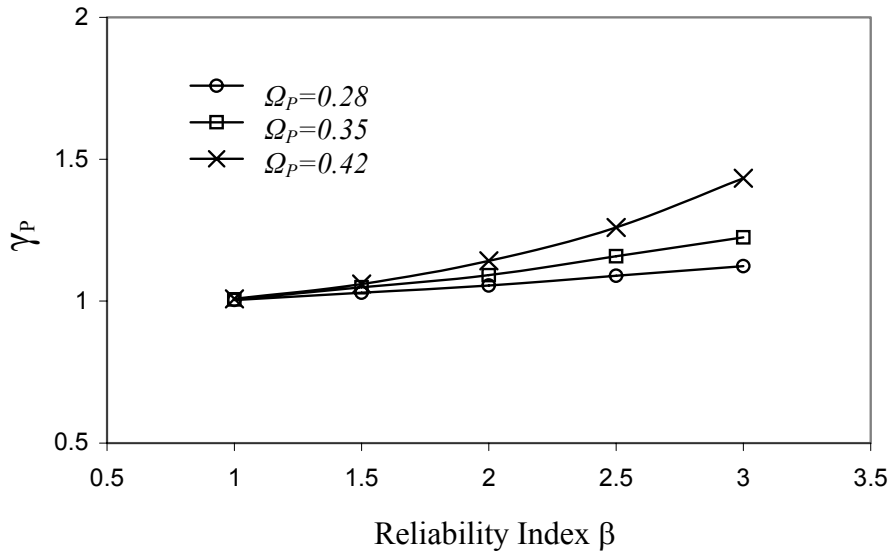


Figure 4. Effect of Ω_P on load factor γ_P

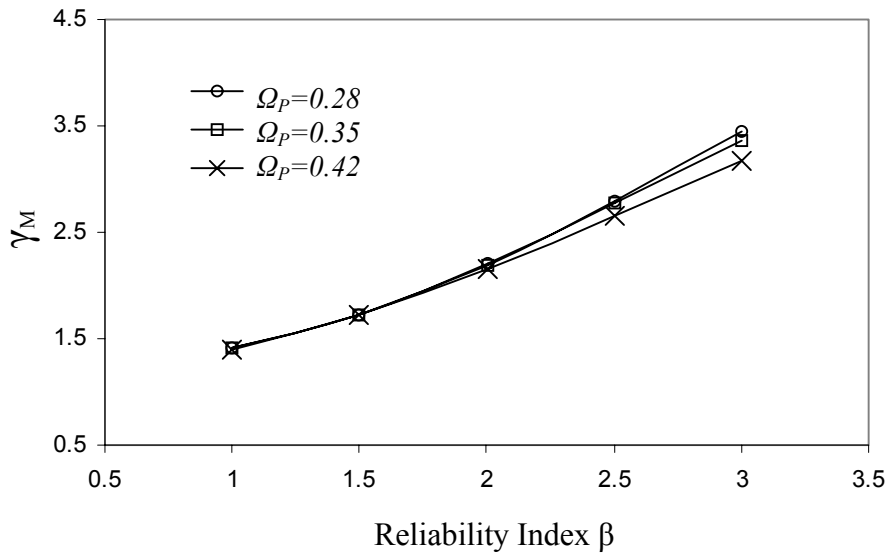


Figure 5. Effect of Ω_p on load factor γ_M

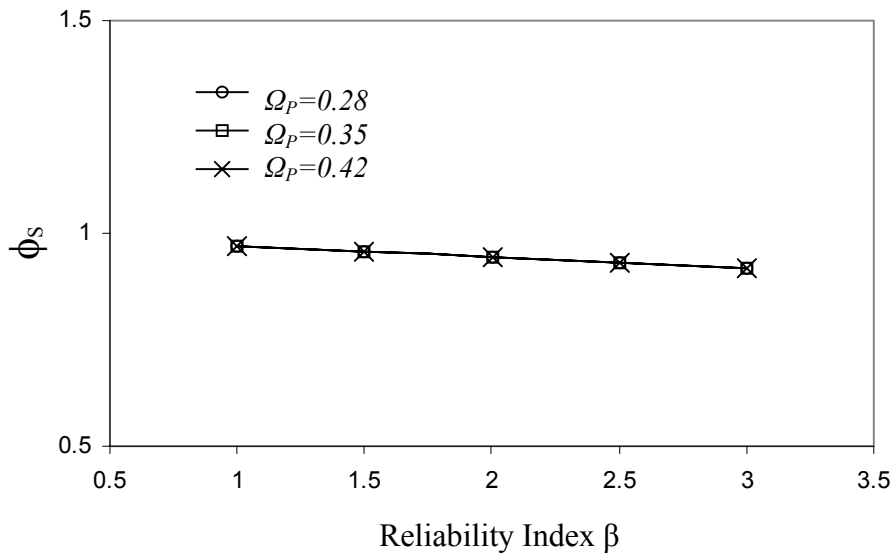


Figure 6. Effect of Ω_p on resistance factor ϕ_S

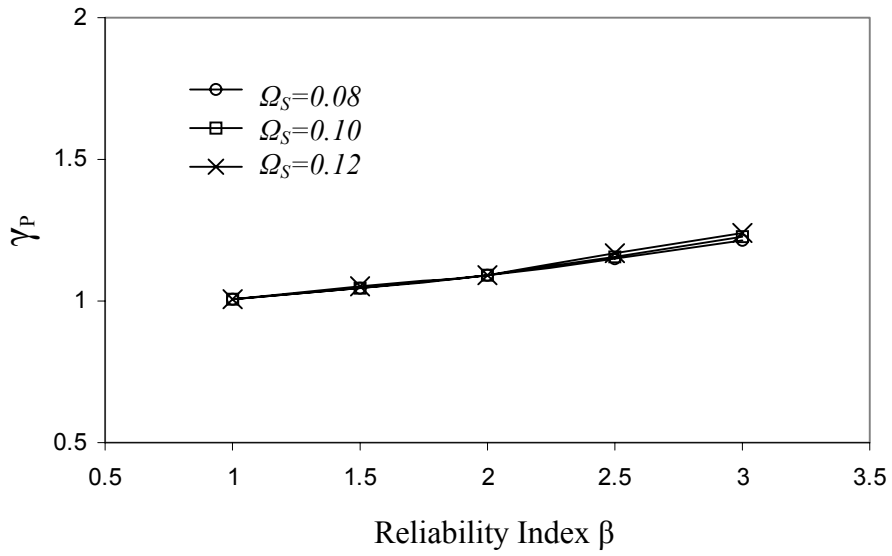


Figure 7. Effect of Ω_S on load factor γ_P

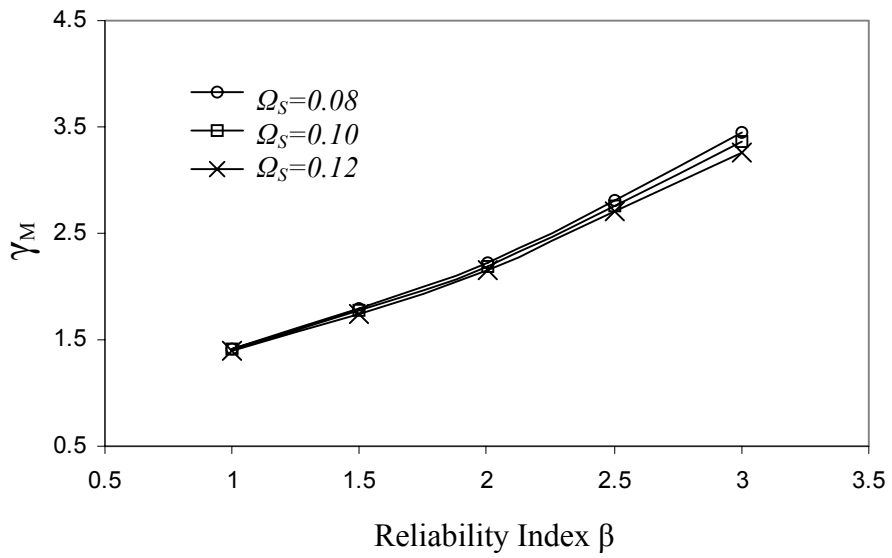


Figure 8. Effect of Ω_S on load factor γ_M

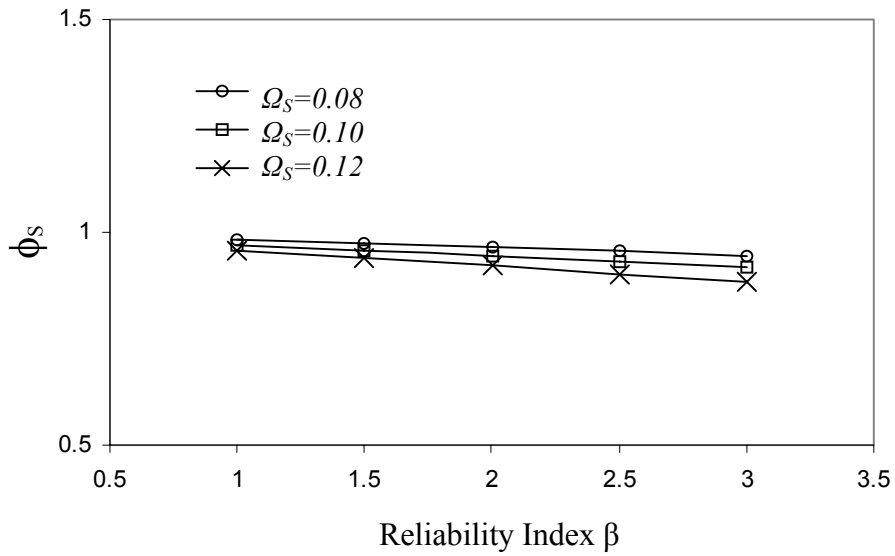


Figure 9. Effect of Ω_S on resistance factor ϕ_S

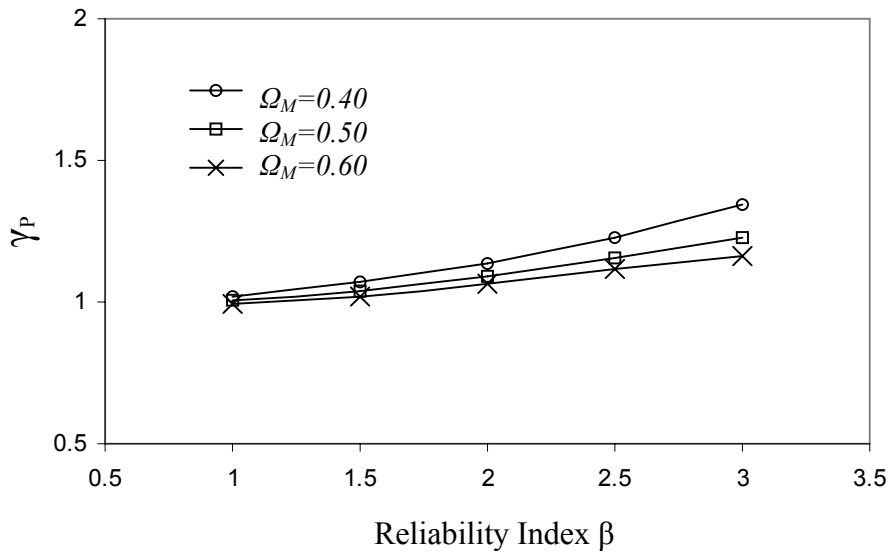


Figure 10. Effect of Ω_M on load factor γ_P

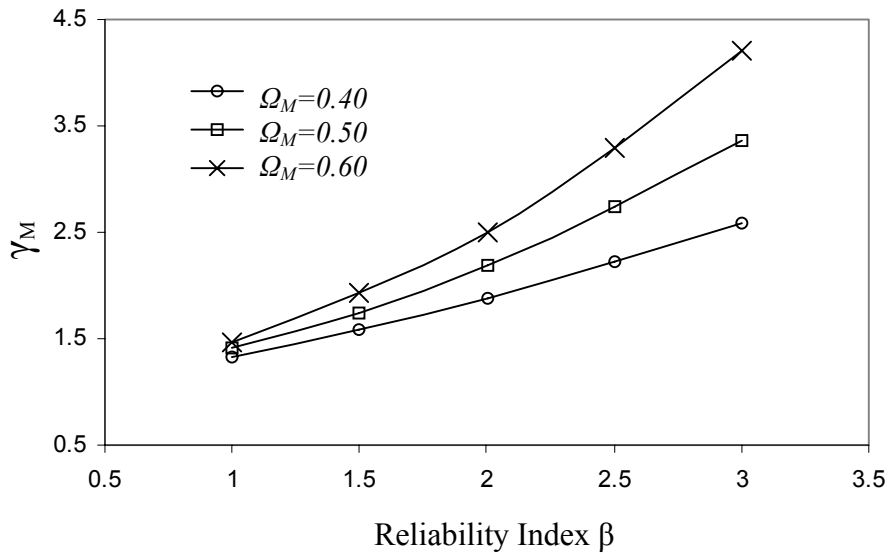


Figure 11. Effect of Ω_M on load factor γ_M

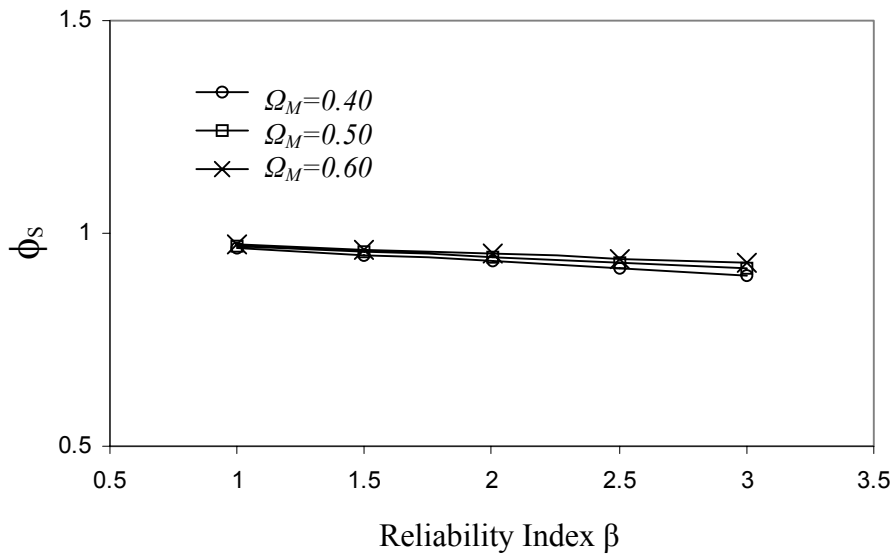


Figure 12. Effect of Ω_M on resistance factor ϕ_S

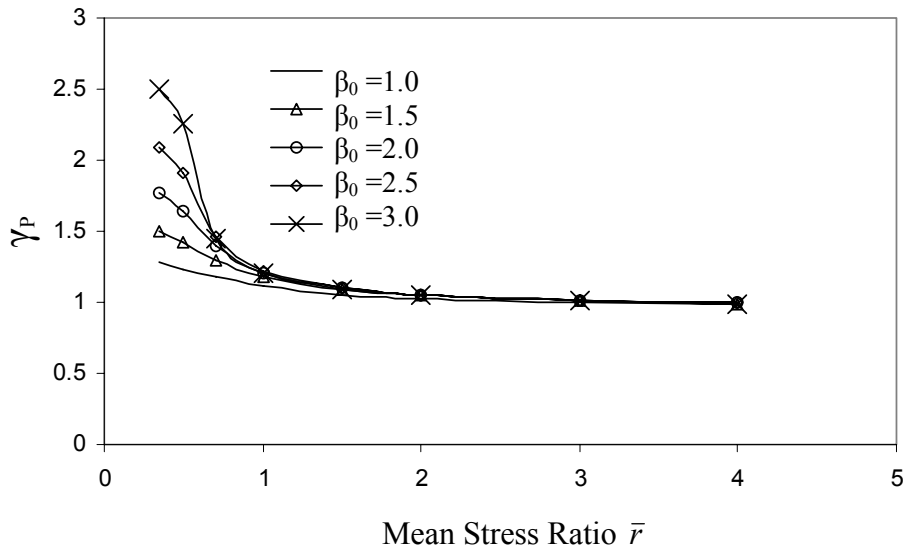


Figure 13. Effect of \bar{r} on load factor γ_P

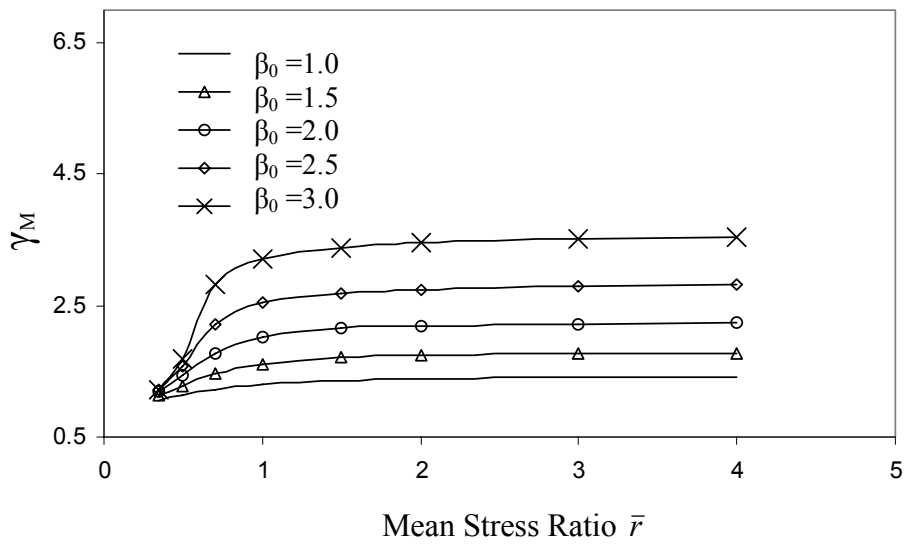


Figure 14. Effect of \bar{r} on load factor γ_M

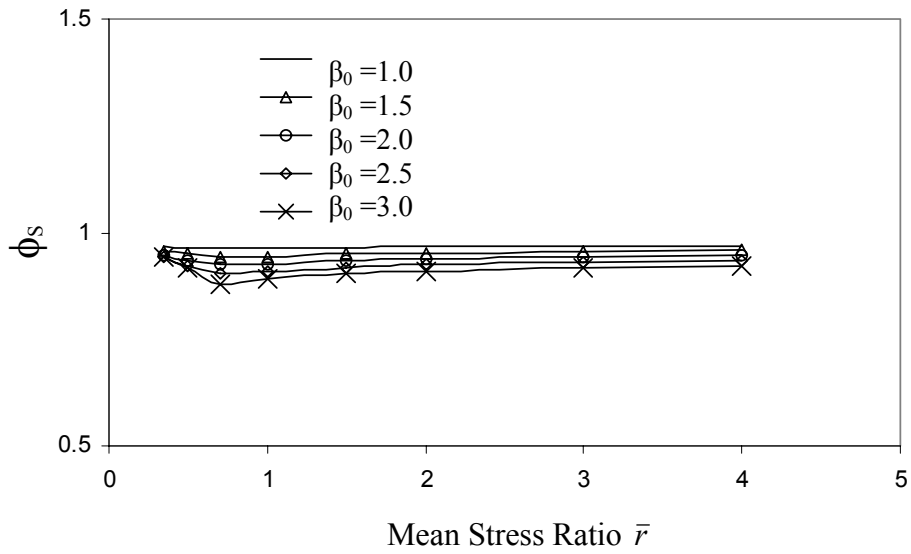


Figure 15. Effect of \bar{r} on resistance factor ϕ_s

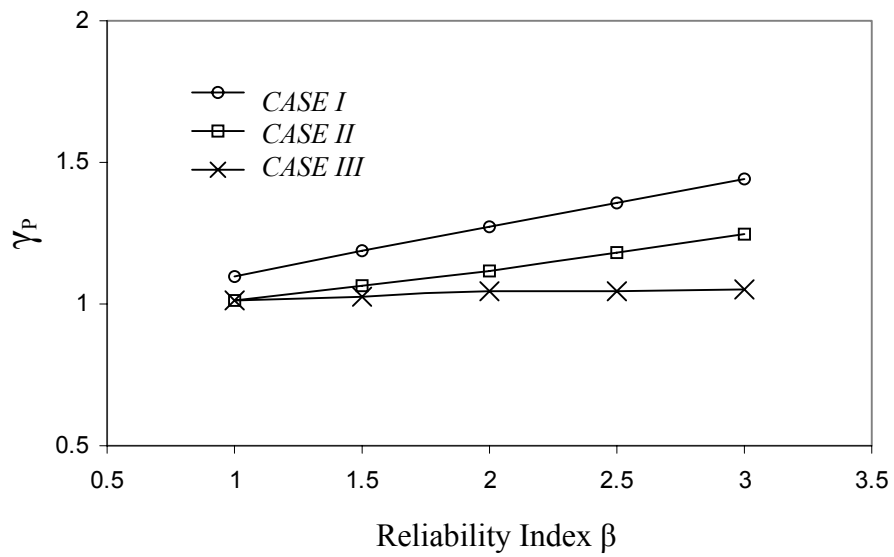


Figure 16. Effect of distribution types on load factor γ_P

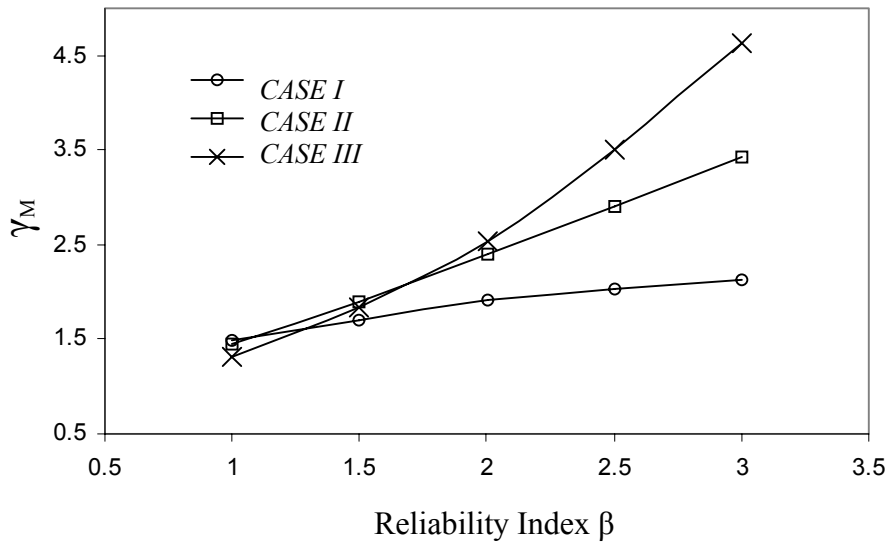


Figure 17. Effect of distribution types on load factor γ_M

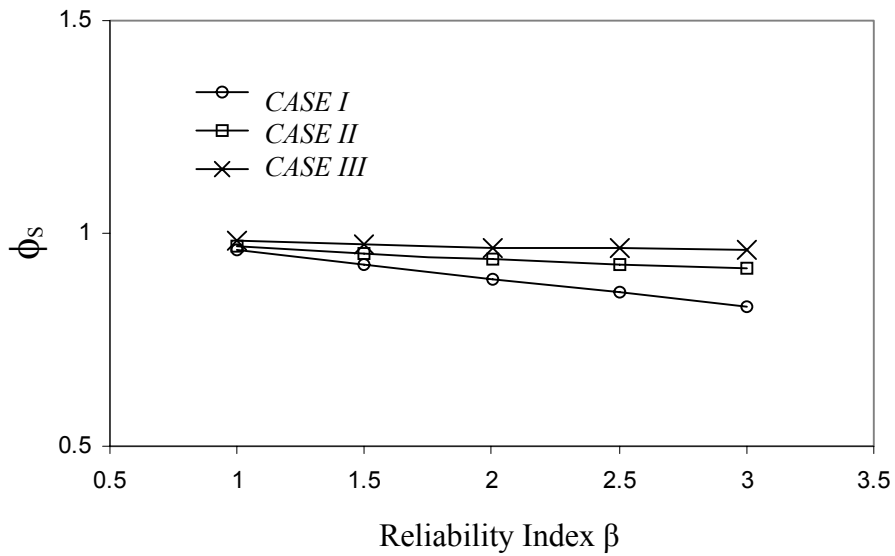


Figure 18. Effect of distribution types on resistance factor ϕ_s

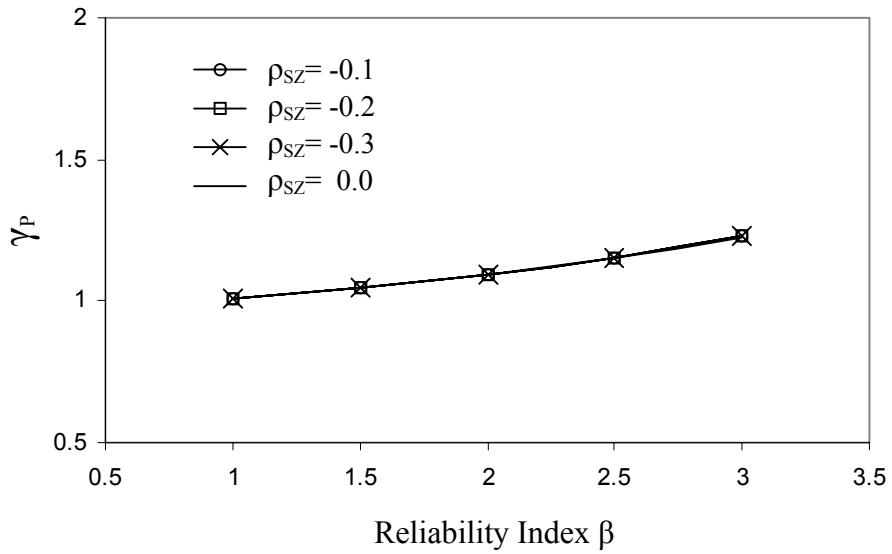


Figure 19. Effect of statistical correlation on load factor γ_P

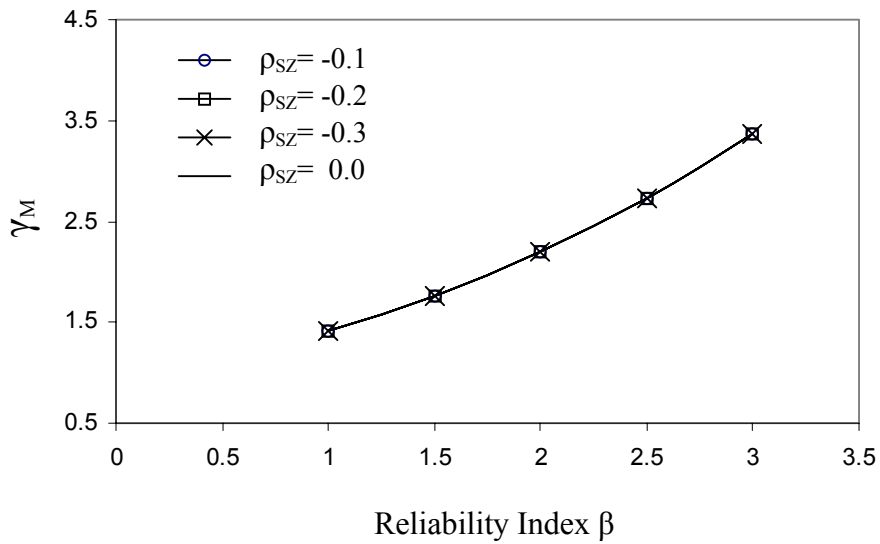


Figure 20. Effect of statistical correlation on load factor γ_M

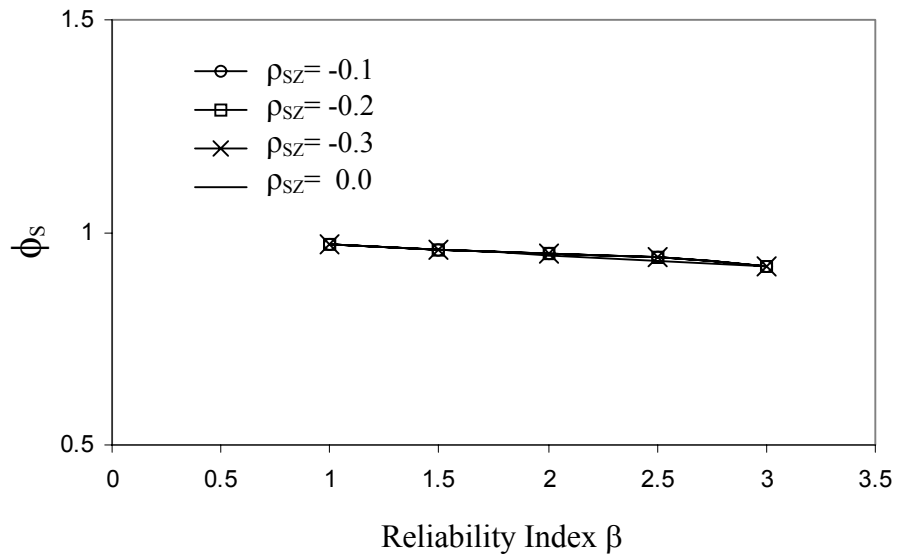


Figure 21. Effect of statistical correlation on resistance factor ϕ_S

PART III

CONSIDERATION OF UNCERTAINTIES IN SEISMIC ANALYSIS OF COUPLED BUILDING-PIPING SYSTEMS

Byounghoan Choi and Abhinav Gupta

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CONSIDERATION OF UNCERTAINTIES IN SEISMIC ANALYSIS OF COUPLED BUILDING-PIPING SYSTEMS

Byounghoan Choi and Abhinav Gupta

Abstract: This study presents the effect of uncertainties in modal properties of uncoupled primary and secondary systems in the seismic analysis of non-classically damped coupled system. It is shown that if two uncoupled system is tuned or nearly tuned, an uncertainty in the frequencies and modal damping ratios of the two uncoupled systems significantly influences the secondary system response. However, this effect can be ignored in detuned systems. It is also shown that the design response when defined at 84% NEP of the responses obtained by performing multiple response spectrum analyses is excessively higher than the design response at 84% NEP obtained from multiple time history analyses. The percentile values of the individual modal responses needed to evaluate a combined design response at 84% NEP are evaluated by using a first order reliability method. This study shows that the modal responses when defined at 84% NEP would give relatively accurate values of design response only if the two modes are perfectly correlated or only a single mode contributes to the particular response quantity of interest. It is impractical to directly use a first order reliability method in response spectrum method for evaluating the design response due to computational complexity and inefficiency. Alternatively, the simplified methods based on total probability theorem are presented to evaluate the design response. It is shown that these methods give the design response very closer to the true value.

1. Introduction

Seismic response of secondary systems, in addition to their own dynamic properties, depends upon the interaction with the primary structure supporting them. A variation in the primary structure properties due to uncertainties in material characteristics, soil-structure interaction or modeling techniques can cause a significant variation in the secondary system response. In the conventional uncoupled analysis, the effect of these uncertainties is considered by broadening the peaks of floor response spectra or conducting multiple analyses by shifting the floor response spectra in a specified range of frequencies. According to the USNRC [13], this frequency region is obtained by considering a $\pm 15\%$ variation in the frequencies associated with the spectral peaks. Unlike a coupled primary-secondary system analysis, an uncoupled analysis can be excessively conservative as it does not account for the effects of non-classical damping and mass interaction between the two sub-systems [3,4]. Peak Broadening and Peak Shifting methods increase the conservatism in the conventionally evaluated responses [1]. Reed et. Al. [12] proposed alternative methods to account for uncertainties by generating floor spectra corresponding to 84% non-exceedance probability. However, these methods cannot be directly used in the coupled primary-secondary system analysis as the floor response spectra are neither generated nor required. One way to account for the effect of these uncertainties in a coupled primary-secondary system analysis is to conduct multiple analyses by varying the primary system frequencies and evaluating the final response statistically. Aradhya and Gupta [3] numerically studied the effect of uncertainties in the earthquake input and the frequency of uncoupled primary system. Igusa and Der Kiureghian [9,10] investigated the effect of uncertainties in primary and secondary system properties on secondary system response and presented a procedure to determine the

reliability of systems with uncertain parameters subjected to stochastic input using first or second order reliability methods. Jensen and Iwan [11] developed a method for dynamic analysis of linear systems with uncertain parameters subjected to stochastic input. While these studies make similar observations, no procedure exists to account for an uncertainty in the primary and secondary system properties in a coupled modal synthesis by a response spectrum method. In this paper, we present a detailed discussion on incorporating the effect of uncertainties in the modal properties (frequencies and modal damping ratios) of both the uncoupled primary and the uncoupled secondary system in a coupled system analysis by response spectrum method.

2. Coupled System Analysis

The equation of motion for an N -DOF coupled primary-secondary system is given by

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{U_b\}\ddot{u}_g \quad (1)$$

in which $\{U\}$ is the displacement vector relative to the fixed base and $\{U_b\}$ is the static displacement vector of the coupled system when the base of the primary system undergoes a unit displacement in the direction of the earthquake. Gupta and Gupta [5,6] introduced a transformation in which the displacement at primary system DOF are expressed relative to its fixed base and those at the secondary system DOF relative to the primary system connecting DOF. The corresponding equation of motion in the transformed coordinates is written as

$$[\overline{M}]\{\ddot{\overline{U}}\} + [\overline{C}]\{\dot{\overline{U}}\} + [\overline{K}]\{\overline{U}\} = -[\overline{M}]\{\overline{U}_b\}\ddot{u}_g \quad (2)$$

The complex eigenvector pair evaluated by solving Eq. (2) together give two real response vectors $\{\overline{\Psi}_i^d\}$ and $\{\overline{\Psi}_i^v\}$ corresponding to the i^{th} coupled mode.

$$\overline{U} = \sum_{i=1}^N \overline{U}_i = \sum_{i=1}^N \overline{U}_i^d - \overline{U}_i^v = \sum \{\overline{\Psi}_i^d\} y_i - \{\overline{\Psi}_i^v\} \dot{y}_i \quad (3)$$

where y_i is the relative displacement and \dot{y}_i the relative velocity of an equivalent SDOF oscillator. In practice, the design responses are calculated using the response spectrum method in which the earthquake input is defined by a design spectrum corresponding to a specified level of non-exceedence probability (NEP). In this method, the modal responses are calculated as follows and combined in accordance with an appropriate rule to obtain the design response [7].

$$\{\overline{U}_i^d\} = \{\overline{\Psi}_i^d\} S_{Di}^d \quad \{\overline{U}_i^v\} = \{\overline{\Psi}_i^v\} S_{Vi}^v = \omega_i \{\overline{\Psi}_i^v\} S_{Di}^v \quad (4)$$

where S_{Di}^d and S_{Di}^v are the spectral displacements corresponding to the frequency and damping ratio of the i^{th} coupled mode as obtained from the relative displacement and the relative velocity spectrum, respectively.

3. Relative Significance of Uncertainty in Earthquake Input and Modal Properties

Gupta and Gupta [3,4] have shown that the secondary system response can be greatly influenced by the mass interaction and non-classical nature of damping in a coupled system analysis.

Therefore, uncertainties in the modal properties of primary and secondary systems can lead to

significant variation in the secondary system response. To illustrate the effect of uncertainties in modal properties of primary and secondary systems, we consider two different coupled (SDOF primary – SDOF secondary) systems of the type shown in Fig. 1. One of the two coupled systems comprises of detuned primary and secondary oscillators while they are perfectly tuned in the other coupled system. Table 1 gives the characteristics of the uncoupled systems in each of the two 2-DOF coupled systems. The relative effects of uncertainties in the earthquake input, the frequencies, and the modal damping ratios of the uncoupled primary and secondary systems is evaluated by conducting multiple time history analyses for each of the two coupled systems. First, the secondary system responses are evaluated by considering a variation in only the input excitation. We consider 75 real earthquake records normalized to the same value of ZPA for this purpose [2]. Table 2 lists the primary characteristics of these earthquake records. Next, the responses are calculated using only a single normalized earthquake record but considering a variation in the frequencies of the uncoupled primary and secondary systems. Finally, the secondary system responses are calculated by considering a variation in only the modal damping ratios of the uncoupled systems. Fig. 2 shows the relative values of displacements for each of the three analyses types in the detuned as well as tuned cases. For ease of comparison, the secondary system displacements in Fig. 2 are normalized with respect to the maximum displacement evaluated in a particular type of analysis. As evident from Fig. 2 for a tuned system, the effects of uncertainties in frequencies and input excitation have relatively greater significance than that due to uncertainties in only the damping. However, the effect of uncertainties in the modal damping ratios of the two uncoupled systems cannot be neglected as these uncertainties can also influence the secondary system response significantly. On the other hand, the effect of uncertainties in the input excitation dominates the response in a detuned system thereby

rendering the effect of uncertainties in frequencies and modal damping ratios of the uncoupled systems meaningless. Consequently, a greater emphasis is placed on consideration of tuned or nearly tuned primary and secondary systems in the rest of this paper.

4. Monte Carlo Simulation: Time History Analysis versus Response Spectrum Analysis

In response spectrum method, the design spectrum is defined at 84% NEP. The primary reason for characterizing the design spectrum at 84% NEP is to account for uncertainty in the earthquake input. Consequently, the design response is implicitly defined at the same NEP, i.e. if a design spectrum corresponds to the 84% NEP over multiple earthquake time histories that are normalized to the same value of ZPA, then an alternative to evaluating the design response would be to conduct multiple time history analyses of the structure using individual time histories and to select the 84th percentile value as the design response [12]. Such an approach would also facilitate the evaluation of design response when uncertainties in the frequencies and modal damping ratios of uncoupled systems are considered. Considering the random variables representing the frequencies of the uncoupled systems, their modal damping ratios, and the earthquake input to be independent, a Monte Carlo simulation is conducted in which multiple time history analyses are performed. For each analysis, the variables are sampled from a set of randomly generated values for the frequencies and modal damping ratios together with a collection of normalized earthquake time histories that were used to arrive at the design spectrum. Once again, the design response is selected as that corresponding to the 84% NEP. This approach cannot, however, be extended directly to the response spectrum method due to reasons discussed below.

As stated earlier, characterization of design spectrum at 84% NEP is intended to account for uncertainty in the earthquake input. Such a characterization ensures that the individual modal responses correspond to the same NEP and that the design response obtained after combining modal responses is close to the intended value of 84% NEP. When an uncertainty in the modal properties is also considered, this process is likely to give satisfactory results for detuned primary - secondary systems wherein uncertainty in input excitation dominates the response of secondary system as shown in Fig. 2. Therefore, one may conduct a Monte Carlo simulation wherein multiple response spectrum analyses are conducted by considering a single design spectrum together with a variation in the frequencies and modal damping ratios of the uncoupled primary and secondary systems. The design response can then be specified as that corresponding to the 84% NEP over all the responses. Contrary of the behavior exhibited by detuned primary – secondary systems, the secondary system response in a tuned case is not dominated by the uncertainty in input excitation alone as is shown in Fig 2. Therefore, selection of a design response corresponding to 84% NEP is likely to be excessively conservative. This is so because the design spectrum is itself specified at 84% NEP and the modal responses evaluated at 84th percentile value by considering a variation in only the modal properties will give values that are much higher than the corresponding values obtained from multiple time history analyses. These excessively high values of modal responses then result in design responses that are much higher than the true design response evaluated from a Monte Carlo simulation using multiple time history analyses.

Next we use numerical illustration to support the above discussion by considering four different cases of coupled primary – secondary systems whose characteristics are given in Table 3. Case I represents a detuned system whereas the other three cases represent tuned or nearly

tuned systems. Once again, we consider 75 real earthquake records normalized to a unit value of peak ground acceleration. To consider the effect of uncertainties in the modal properties of primary and secondary systems, it is assumed that the random variables representing frequencies and modal damping ratios of the uncoupled systems are Gaussian with coefficient of variation equal to 0.15 for each. A total of 7500 responses are evaluated by considering combinations of 75 earthquakes and 100 sets of randomly sampled frequencies and modal damping ratios. The secondary system design response is then defined as that corresponding to 84% NEP over these 7500 responses. To conduct a Monte Carlo simulation with multiple response spectrum analyses, a design spectrum corresponding to 84% NEP is evaluated using the individual response spectrum for each of the 75 time histories normalized to unit value of ZPA. These spectra are shown in Fig. 3. The design spectrum is then used as a single specified input in conducting multiple response spectrum analyses by considering variations in the frequencies and modal damping ratios of the uncoupled primary and secondary systems. Fig. 4 compares the secondary system response corresponding to 84% NEP over these multiple responses with those evaluated from multiple time history analyses. It is evident that in a tuned or nearly tuned primary – secondary systems, specification of design response at 84% NEP can result in excessively high values especially when the effect of uncertainties are considered. Fig 4 also shows that the two responses are close to each other for the detuned system. The above discussion and illustration necessitates the development of a method to accurately account for the effect of uncertainties in the modal properties of the primary and secondary systems in a seismic analysis of the coupled system by response spectrum method.

5. First Order Reliability Method (FORM) Approach

For modes with closely spaced frequencies such as those encountered in tuned or nearly tuned primary – secondary systems, the double sum combination rule is used to obtain the design response [7].

$$R^2 = \sum_i \sum_j \varepsilon_{ij} R_i R_j \quad (5)$$

in which R is the design response whereas R_i , and R_j are the modal responses in the i^{th} and j^{th} modes, respectively. The modal correlation coefficient is denoted by ε_{ij} . When multiple response spectrum analyses are conducted by varying the frequencies and modal damping ratios of the uncoupled primary and secondary systems, each analysis gives a unique set of values for modal responses. Evaluation of design response R then requires that the modal responses be defined at specified NEP values so that a combination of these values in accordance with Eq. (5) gives a design response close to 84% NEP. The problem at hand is same as that of evaluating the load and resistance factors in a reliability-based design. To evaluate the NEP at which modal responses are needed to be defined, we consider FORM. The first step lies in characterizing the performance function. The performance function corresponding to Eq. (5) can be written as

$$g(-) = R^2 - \sum_i \sum_j \varepsilon_{ij} R_i R_j = 0 \quad (6)$$

The above equation would, however, require statistical properties of not only R_i and R_j but also those of R which are difficult to quantify. Therefore, we re-formulate Eq. (6) into a non-dimensional form as follows.

$$g(-) = 1 - \sum_i \sum_j \varepsilon_{ij} X_i X_j = 0 \quad X_{i,j} = R_{i,j} / R \quad (7)$$

Eq. (7) facilitates the evaluation of partial load factors for R_i and R_j by considering only the coefficient of variations for X_i and X_j together with different values for the mean of X_i / X_j . It should be noted that for all $X_j \geq X_i$, this ratio varies between 0 and 1, i.e., $0 \leq X_i / X_j \leq 1$. If γ_i and γ_j are the partial load factors for modal responses in the i^{th} and j^{th} modes, respectively, we can write

$$R^2 = \sum_i \sum_j \varepsilon_{ij} (\gamma_i R_i)(\gamma_j R_j) \quad (8)$$

The partial load factors give the corresponding values of NEP. Once again, we consider the SDOF primary – SDOF secondary systems described earlier in this paper to study the partial load factors. The limit-state equation (performance function) corresponding to Eq. (7) becomes

$$g(-) = 1 - X_1^2 - X_2^2 - 2\varepsilon_{12} X_1 X_2 = 0 \quad (9)$$

For simplicity and consistency with the current practice adopted in conventional uncoupled analysis, we assume that the probability distribution and the coefficient of variation for each modal response is same. It is also assumed that correlation coefficient ε_{12} is constant in a particular application of FORM to Eq. (9). However, the correlation coefficient is varied between 0 and 1 to study the nature of variation in partial load factors corresponding to different values of ε_{12} . Figs. 5 and 6 show the values of NEP (corresponding to the evaluated partial load factors) for each modal response as evaluated using FORM. In these figures, the mean modal response ratio is defined as

$$\bar{r} = \frac{\mu_{X_1}}{\mu_{X_2}} = \frac{\mu_{R_1}}{\mu_{R_2}} \quad (\mu_{R_2} \geq \mu_{R_1}) \quad (10)$$

in which μ_{R_1} and μ_{R_2} are mean values of modal responses R_1 and R_2 , respectively. As shown in Figs. 5 and 6, the NEP values vary from 50% to 84% for each modal response depending upon the values of \bar{r} and ε_{12} . These figures show that the modal responses should be defined at 84% NEP in systems that have perfectly correlated modal responses ($\varepsilon_{12} = 1$) and in systems where only a single mode contributes to the particular response quantity of interest ($\bar{r} = 0$). In other cases, characterizing modal responses at 84% NEP would give design response that is higher than its true value. To validate FORM, percentile values of modal responses are evaluated by considering three different cases that are described in Table 4. Table 5 compares percentile values of each modal response selected from Figs. 5 and 6 corresponding to \bar{r} and ε_{12} given in

Table 5 with those corresponding to 84% NEP evaluated from time history analyses. As listed in Table 5, the two sets of results are close to each other.

6. Proposed Methods

While the study using FORM provides a better understanding of the problem and gives relatively accurate results, its implementation in a response spectrum method would be computationally inefficient for real-life applications wherein several hundreds of response quantities are evaluated by combining many more than two modal responses. Therefore, a simple method is needed for incorporating the understanding gained from the study using FORM into the response spectrum method. To do so, let us subdivide the modal property variations into n disjointed sets each of which contains mutually exclusive frequencies and modal damping ratios. The design response $R_{0.84}$ corresponding to 84% NEP can then be expressed as

$$P(R \leq R_{0.84}) = \int_{-\infty}^{R_{0.84}} f_R(r) dr = \sum_{i=1}^n \int_{-\infty}^{R_{0.84}} f_{R/Z_i}(r/z_i) dr \int_{Z_{i-1}}^{Z_i} f_Z(z_i) dz_i \quad (11)$$

where the random variable Z is used to characterize the sets of modal parameters, i.e. $Z_i = \omega_i \zeta_i$.

Further, Z_{i-1} and Z_i represent the lowest and the highest bounds of modal properties (frequency and modal damping ratio) in the i^{th} set. Also, $f_R(r)$ and $f_{R/Z_i}(r/z_i)$ are the total probability density function and the conditional probability density function for the response corresponding

to the i^{th} set, respectively. If $f_Z(z_i)$ is constant, the probability of occurrence of a particular set of modal properties in each interval is equal to $1/n$ and Eq. (11) becomes

$$\int_{-\infty}^{R_{0.84}} f_R(r) dr = \frac{1}{n} \sum_{i=1}^n \int_{-\infty}^{R_{0.84}} f_{R/Z_i}(r/z_i) dr \quad (12)$$

6.1 Mean of Conditional Responses

In general, the nature of probability density functions for $f_R(r)$ and $f_{R/Z_i}(r/z_i)$ is not known. On the other hand, availability of these density functions does not solve the problem of determining a method for evaluating $R_{0.84}$ in a response spectrum method in which we calculate the conditional response $R_{0.84/Z_i}$ for the given i^{th} set of modal properties. To do so, we need to determine the probability density function for $R_{0.84/Z_i}$ and also the relationship between the properties of this distribution and the design response $R_{0.84}$, if any. While the former can be evaluated numerically, determining the latter is not so simple especially if a relationship is to be developed based purely on theoretical concepts. Aradhya and Gupta [3] conducted a numerical study to conclude that $R_{0.84}$ is close to 50 percent non-exceedence probability of $R_{0.84/Z_i}$, i.e. the median value. Theoretically, we find that this observation is true only if the coefficient of variation for the conditional distribution is same for each set of modal properties and approximately equal to the coefficient of variation for the response R , i.e.

$$R_{0.84} = E[R] + \sigma[R] = (1 + \nu_R)E[R] \quad (13)$$

$$\begin{aligned}
R_{0.84/Z_i} &= E[R / Z = z_i] + \sigma[R / Z = z_i] \quad i = 1, \dots, N \\
&= (1 + \nu_{R/Z_i}) E[R / Z = z_i]
\end{aligned}
\tag{14}$$

in which ν_R is the coefficient of variation for the response R and ν_{R/Z_i} that for the conditional response corresponding to $Z = z_i$. Further,

$$E[R] = E[E[R / Z = z_i]] \tag{15}$$

Using Eq. (14) we can write

$$E[R_{0.84/Z_i}] = E[(1 + \nu_{R/Z_i}) E[R / Z = z_i]] \tag{16}$$

ν_{R/Z_i} and $E[R]$ are independent. Therefore,

$$E[R_{0.84/Z_i}] = E[1 + \nu_{R/Z_i}] E[E[R / Z = z_i]] \tag{17}$$

Eqs (13), (15), and (17) give

$$\begin{aligned}
R_{0.84} &= E[R] + \sigma[R] = (1 + \nu_R)E[R] \\
&= (1 + \nu_R)E[E[R/Z = z_i]] \\
&= \frac{(1 + \nu_R)}{E[1 + \nu_{R/Z_i}]} E[R_{0.84/Z_i}] \\
&= \frac{1}{N} \sum_{i=1}^N R_{0.84/Z_i} \Theta_i \quad \Theta_i = \frac{1 + \nu_R}{1 + \nu_{R/Z_i}}
\end{aligned} \tag{18}$$

Further, Θ_i is a factor that is dependent upon the coefficient of variation ν_R for the response R and ν_{R/Z_i} for the conditional response $R_{0.84/Z_i}$. If coefficients of variation are assumed to be constant and $\nu_R \approx \nu_{R/Z_i}$, $\Theta_i = 1$. This further simplifies Eq. (18) as

$$R_{0.84} = \frac{1}{N} \sum_{i=1}^N R_{0.84/Z_i} \tag{19}$$

Fig. 7 shows the maximum possible error that can occur due to the assumption that $\Theta_i = 1$ for a wide range of values of the coefficients of variation ν_R and ν_{R/Z_i} . We use Eq. (19) to evaluate the design response for all the cases that are summarized in Table 3. Fig. 8 compares the secondary system response evaluated by this method with those evaluated earlier by Monte Carlo simulation using multiple time history and response spectrum analyses. As shown in Fig. 8, the design responses obtained from the mean of conditional response are close to those given by time history analyses.

6.2 Square-Root-of-Mean-of-Squares (SRMS)

The mean of conditional responses was derived based on the assumption that the coefficient of variation for the conditional distribution is same for each set of modal properties and equal to the coefficient of variation for the response R . In Appendix A, we develop an alternative method that gives mean plus one standard deviation of response without such an assumption. The method can be termed as square-root-of-mean-of-squares (SRMS). According to the formulation presented in the Appendix A, we can write

$$R_{0.84} = \left[\frac{1}{N} \sum_{i=1}^N R_{0.84/Z_i}^2 \Theta_i \right]^{\frac{1}{2}} \quad \Theta_i = \left(\frac{1 - \frac{2\nu_{R/Z_i}}{(1 + \nu_{R/Z_i})^2}}{1 - \frac{2\nu_R}{(1 + \nu_R)^2}} \right) \quad (20)$$

For $\Theta_i = 1$, Eq. (20) can be simplified as

$$R_{0.84} = \left[\frac{1}{N} \sum_{i=1}^N R_{0.84/Z_i}^2 \right]^{\frac{1}{2}} \quad (21)$$

The maximum possible error due to the assumption that $\Theta_i = 1$ is shown in Fig. 9. Once again, we evaluate the design responses for all the cases in Table 3 using SRMS. Fig.10 compares the secondary system response evaluated by all methods mentioned in this paper. As shown in Fig. 10, the design responses obtained from SRMS are almost identical to those evaluated by time history analyses.

8. Summary and Conclusions

This study investigates the effect of uncertainties in modal properties of uncoupled primary and secondary systems in the seismic analysis of non-classically damped coupled system. It is shown that uncertainties in the frequencies and modal damping ratios of the two uncoupled systems can result in a significant variation of secondary system response when the frequencies of the two uncoupled systems are tuned or nearly tuned. On the contrary, the effect of uncertainties in the input excitation dominates the variation in secondary system response when the two uncoupled systems are detuned. Therefore, uncertainties in the frequencies and modal damping ratios do not have any meaningful influence on the secondary system response in detuned systems. While multiple time history analyses may be conducted in a Monte Carlo simulation to evaluate the design response corresponding to 84% non-exceedence probability, it is shown that a consideration of 84% non-exceedence probability in a Monte Carlo simulation with multiple response spectrum analyses gives excessively high values for tuned primary – secondary systems. This is so because the earthquake input is characterized in terms of a single design spectrum that in itself corresponds to 84% NEP. Consequently, the multiple analyses performed by sampling only the modal properties give excessively high values when the design response is defined at 84% NEP. First-order reliability method is used to evaluate the non-exceedence probability values at which the individual modal responses should be defined in order to evaluate a combined response that is close to the true design response as obtained from a Monte Carlo simulation with multiple time history analyses. Results of this study show that the modal responses need to be evaluated at 84% NEP in systems that have perfectly correlated modes ($\varepsilon_{12} = 1$) and in systems where only a single mode contributes to the particular response quantity

of interest. In other cases, characterizing modal responses at 84% NEP would give design response that is excessively higher than its true value. While FORM gives relatively accurate response, the results of first-order reliability method cannot be directly included in a response spectrum method due to computational complexity and inefficiency. Therefore, simplified methods based on total probability theorem are presented for this purpose. It is shown that the numerical results obtained from the simplified methods are very close to the true values of design responses.

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Table 1. Modal properties of uncoupled primary and secondary systems

| System | Primary system | | Secondary system | | Mass ratio (M_s/M_p) | Coefficient of variation |
|---------|-----------------|---------------|------------------|---------------|-----------------------------|---|
| | ω_p (Hz) | ζ_p (%) | ω_s (Hz) | ζ_s (%) | | |
| Detuned | 0.5 | 6.0 | 2.5 | 2.0 | 0.001 | $v_{\omega_p} = v_{\omega_s}$ $= v_{\zeta_p} = v_{\zeta_s} = 0.15$ |
| Tuned | 2.5 | 6.0 | 2.5 | 2.0 | | |

Table 2. List of real earthquake records

| Record No. | Date | Earthquake and Site | Component | PGA(g) |
|------------|----------------|-----------------------------|-----------|--------|
| 1 | 1940 May 18 | Imperial Valley | S00E | 0.89 |
| 2 | | Elcentro | S90W | 0.55 |
| 3 | 1952 Jul 21 | Kern County, Pasadena, | S00E | 0.12 |
| 4 | | Caltech-Athenaeum | S90W | 0.14 |
| 5 | 1952 Jul 21 | Kern County, Taft, | N21E | 0.40 |
| 6 | | Lincoln School Tunnel | S69E | 0.46 |
| 7 | 1952 Jul 21 | Kern County, Santa Barbara, | N42E | 0.23 |
| 8 | | Court House | S48E | 0.33 |
| 9 | 1952 Jul 21 | Kern County, Hollywood | S00W | 0.14 |
| 10 | | Storage, Basement | N90E | 0.11 |
| 11 | 1957 Mar 22 | San Francisco | N10E | 0.21 |
| 12 | | Golden Gate Park | S80E | 0.27 |
| 13 | 1933 Mar 10 | Long Beach | S08W | 0.34 |
| 14 | | Vernon CMD Building | N82W | 0.39 |
| 15 | 1934 Dec 30 | Lower California, ElCentro, | S00W | 0.41 |
| 16 | | Imperial Valley | S90W | 0.47 |
| 17 | 1935 Oct 31 | Helena, Montana | S00W | 0.37 |
| 18 | | Carrol Collage | S90W | 0.37 |

Table 2. (continued)

| Record No. | Date | Earthquake and Site | Component | PGA(g) |
|------------|-------------|--|--------------|--------------|
| 19 20 | 1949 Apr 13 | Western Washington, Seattle, Distr. Engs. Office | S02W N88W | 0.17 0.17 |
| 21 22 | 1949 Apr 13 | Western Washington, Olympia, Hwy. Test Lab | S04W N86E | 0.42 0.71 |
| 23 24 | 1965 Apr 29 | Puget Sound, Olympia, Hwy. Test Lab | S04W N86E | 0.35 0.51 |
| 25 | 1966 Jun 27 | Parkfield, California, Cholame Shandon Array No. 2 | N65E | 1.25 |
| 26 27 | 1966 Jun 27 | Parkfield, California, Cholame Shandon Array No. 5 | N05W N85E | 0.90 1.11 |
| 28 29 | 1966 Jun 27 | Parkfield, California, Cholame Shandon Array No. 8 | N50E N40W | 0.60 0.70 |
| 30 31 | 1966 Jun 27 | Parkfield, California, Cholame Shandon Array No. 12 | N50E N40W | 0.14 0.16 |
| 32 33 | 1966 Jun 27 | Parkfield, California Temblor California No. 2 | N65W S25W | 0.69 0.89 |
| 34 35 | 1971 Feb 9 | San Fernando Pacoima Dam | S16E S74W | 2.98 2.74 |
| 36 37 | 1971 Feb 9 | San Fernando 8244 Orion Blvd., 1 st Floor | N00W S90W | 0.65 0.34 |

Table 2. (continued)

| Record No. | Date | Earthquake and Site | Component | PGA(g) |
|------------|-------------|--|--------------|--------------|
| 38 39 | 1971 Feb 9 | San Fernando 250 E. First St., Basement | N36E N54W | 0.26 0.32 |
| 40 41 | 1971 Feb 9 | San Fernando, 445 Figueroa St. Sub Basement | N52W S38W | 0.38 0.30 |
| 42 43 | 1971 Feb 9 | San Fernando Hollywood Storage Basement | S00W N90E | 0.27 0.39 |
| 44 45 | 1971 Feb 9 | San Fernando, Caltec Seismological Lab | S00W S90W | 0.23 0.49 |
| 46 47 | 1971 Feb 9 | San Fernando Caltech-Athenaeum | N00E N90W | 0.24 0.28 |
| 48 49 | 1971 Feb 9 | San Fernando Caltech Millikan Lib., Basement | N00E N90E | 0.52 0.47 |
| 50 51 | 1971 Feb 9 | San Fernando Jet Propulsion Lab, Basement | S82E S08W | 0.54 0.36 |
| 52 53 | 1971 Feb 9 | San Fernando, Fire Station Storage Room | S60E S30W | 0.29 0.35 |
| 54 55 | 1971 Feb 9 | San Fernando 15250 Ventura Blvd., Basement | N11E N79W | 0.57 0.38 |
| 56 57 | 1992 Jun 28 | Landers Lucerne Valley Station | N15W N80W | 1.88 1.64 |
| 58 59 | 1994 Jan 17 | Northridge Pardee Station | S00E N90E | 1.26 0.75 |

Table 2. (continued)

| Record No. | Date | Earthquake and Site | Component | PGA(g) |
|------------|-------------|---|-----------|--------|
| 60 | 1994 Jan 17 | Northridge, Rinaldi Receiving Station | N42W | 1.22 |
| 61 | | | S48W | 2.12 |
| 62 | 1994 Jan 17 | Northridge, Sylmar Converter Station | N52E | 1.52 |
| 63 | | | S38E | 1.91 |
| 64 | 1994 Jan 17 | Northridge, Sylmar Converter Station East | N18E | 2.10 |
| 65 | | | N72W | 1.23 |
| 66 | 1994 Jan 17 | Northridge Newhall-LA County Fire Station | N90E | 1.51 |
| 67 | | | N00E | 1.50 |
| 68 | 1994 Jan 17 | Northridge Arleta Fire Station | N90E | 0.87 |
| 69 | | | N00E | 0.75 |
| 70 | 1994 Jan 17 | Northridge Tarzana-Cedar Hill Nuresery | N90E | 4.47 |
| 71 | | | N00E | 2.45 |
| 72 | 1994 Jan 17 | Northridge Sylmar-county Hospital | N90E | 1.51 |
| 73 | | | N00E | 2.24 |
| 74 | 1994 Jan 17 | Northridge, Santa Monica City Hall | N90E | 2.22 |
| 75 | | | N00E | 1.00 |

Table 3. Properties of uncoupled primary and secondary systems

| Case | Primary system | | | | Secondary system | | | | Coefficient of variation |
|------|----------------|-------------------------------|--------------------|------------------|------------------|-------------------------------|--------------------|------------------|--|
| | K_p (N/m) | M_p (Ns ² /m) | ω_p (Hz) | ζ_p (%) | K_s (N/m) | M_s (Ns ² /m) | ω_s (Hz) | ζ_s (%) | |
| I | 888.3 | 10 | 1.5 | 6.0 | 2.4674 | 0.01 | 2.5 | 2.0 | v_{ω_p} $= v_{\omega_s}$ $= v_{\zeta_p}$ $= v_{\zeta_s}$ $= 0.15$ |
| II | 2088.4 | | 2.3 | | | | | | |
| III | 2467.4 | | 2.5 | | | | | | |
| IV | 2878.0 | | 2.7 | | | | | | |

Table 4. Properties of uncoupled primary and secondary systems

| Case | Primary system | | | Secondary system | | | Coefficient of variation |
|------|----------------|-------------------------------|---------------|------------------|-------------------------------|---------------|--|
| | K_p (N/m) | M_p (Ns ² /m) | ζ_p (%) | K_s (N/m) | M_s (Ns ² /m) | ζ_s (%) | |
| I | 394.8 | 10 | 6.0 | 2.4674 | 0.01 | 2.0 | v_{ω_p} $= v_{\omega_s}$ $= v_{\zeta_p}$ $= v_{\zeta_s}$ $= 0.15$ |
| II | 888.3 | | | | | | |
| III | 2467.4 | | | | | | |

Table 5. Percentile value of R_1 and R_2 corresponding to design response defined at 84% NEP

| Case | Modal correlation coefficient (ϵ_{12}) | Mean modal response ratio (\bar{r}) | FORM | | Monte Carlo simulation | |
|------|---|---|-----------|-----------|------------------------|-----------|
| | | | R_1 (%) | R_2 (%) | R_1 (%) | R_2 (%) |
| I | 0.00 | 0.20 | 84 | 52 | 87 | 53 |
| II | 0.04 | 0.80 | 81 | 72 | 80 | 69 |
| III | 0.28 | 0.73 | 83 | 74 | 82 | 76 |

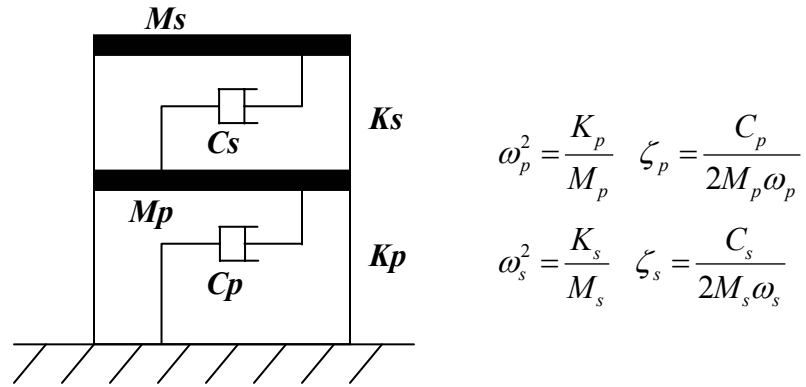


Fig. 1. SDOF primary - SDOF secondary system

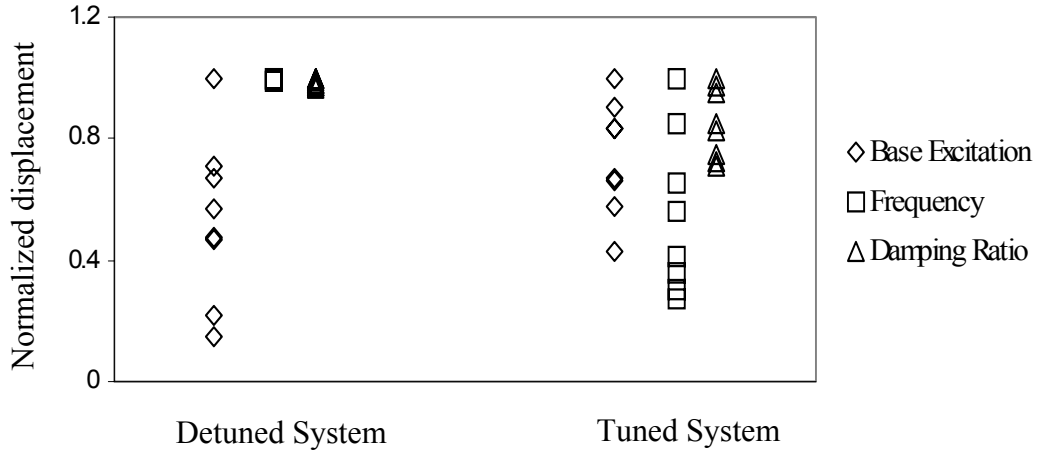


Fig. 2. Relative significance of uncertainties in base excitation and modal properties

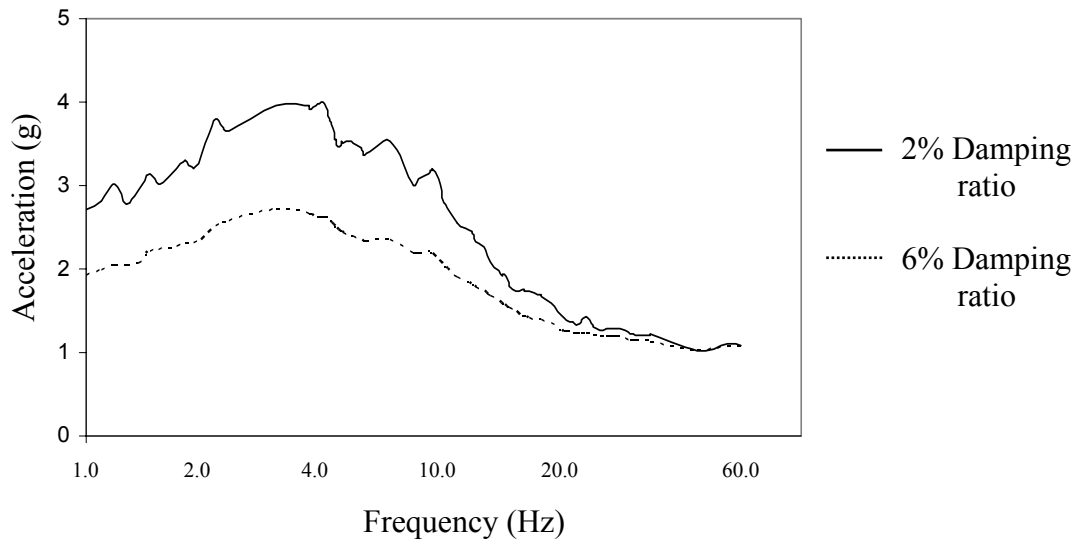


Fig. 3. Input response spectrum corresponding to 84% NEP (75 real earthquakes)



Fig. 4. Response corresponding to 84% NEP in Monte Carlo simulation

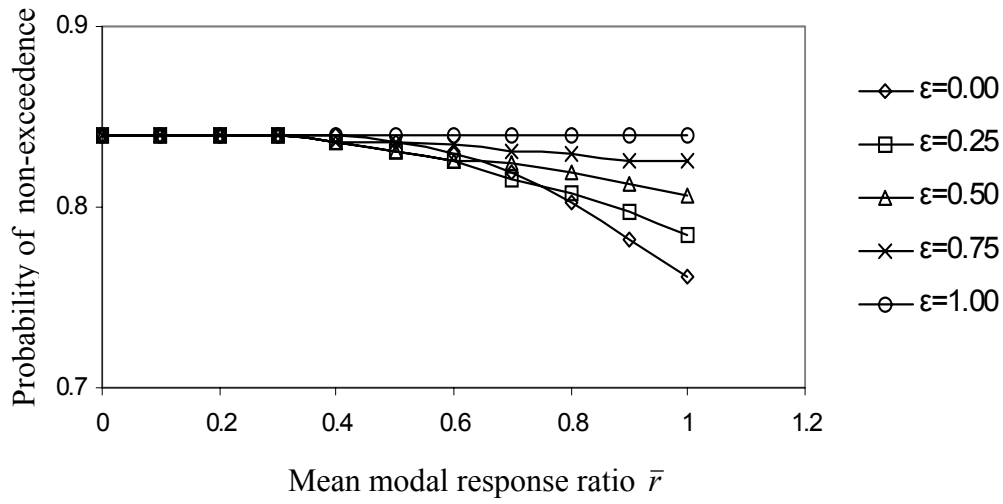


Fig. 5. Values of NEP for R_1 needed to evaluate design response defined at 84% NEP

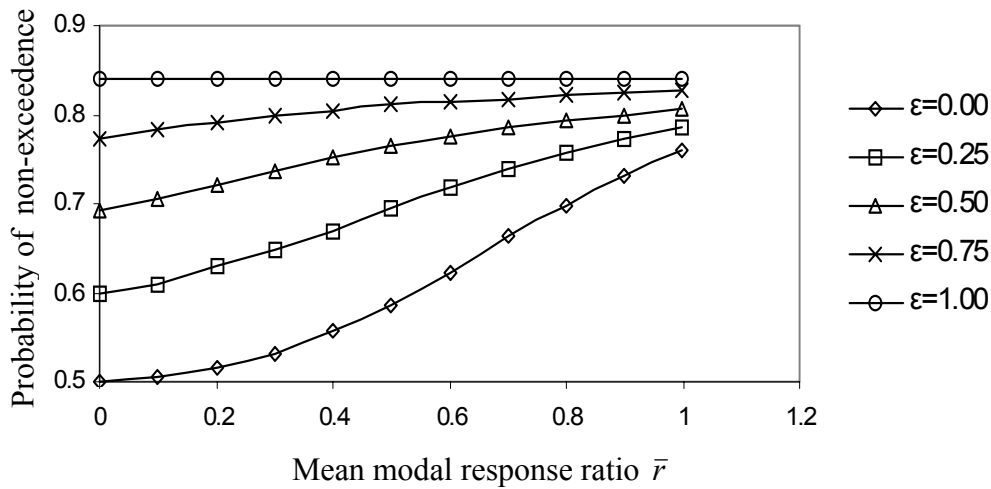


Fig. 6. Values of NEP for R_2 needed to evaluate design response defined at 84% NEP

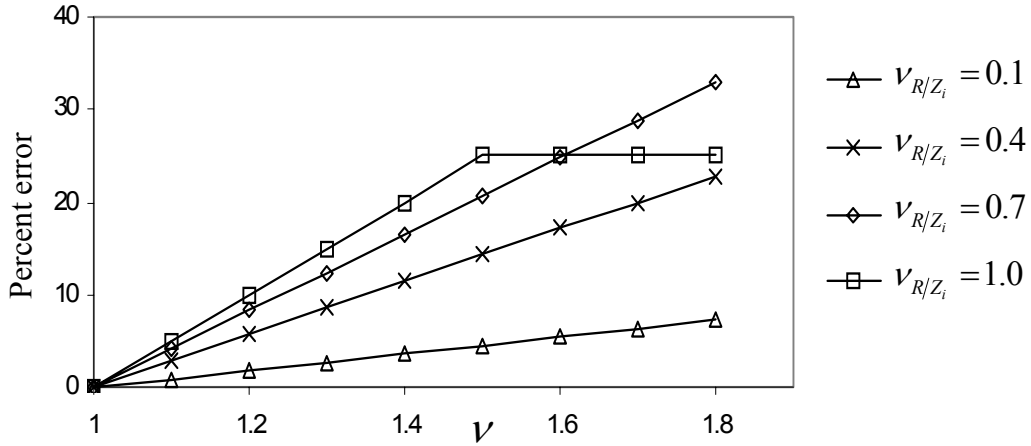


Fig.7. Maximum error in mean of conditional Response, $\nu = \frac{V_R}{V_{R/Z_i}}$

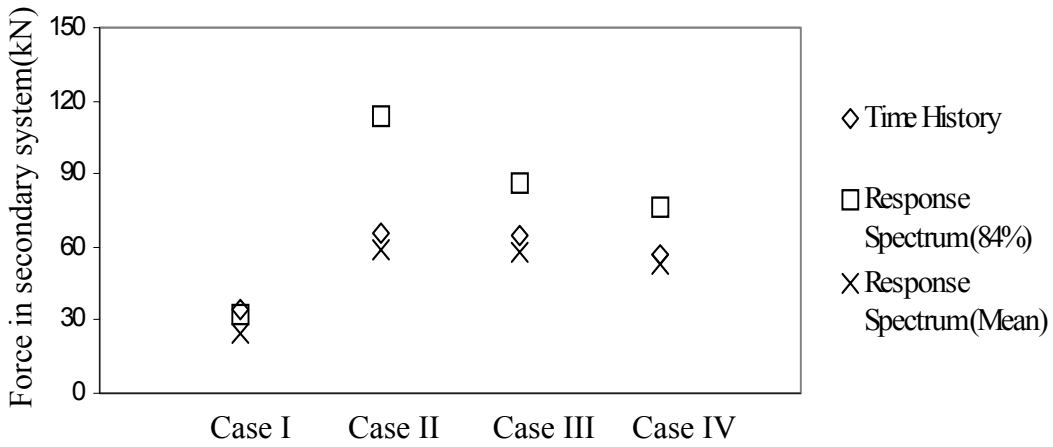


Fig. 8. Response corresponding to 84% NEP in Monte Carlo simulation

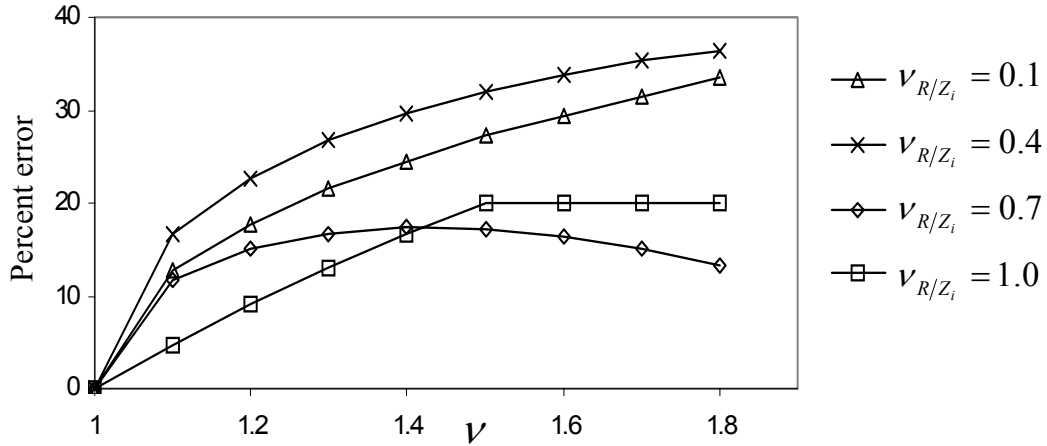


Fig.9. Maximum error in SRMS method, $\nu = \frac{\nu_R}{\nu_{R/Z_i}}$

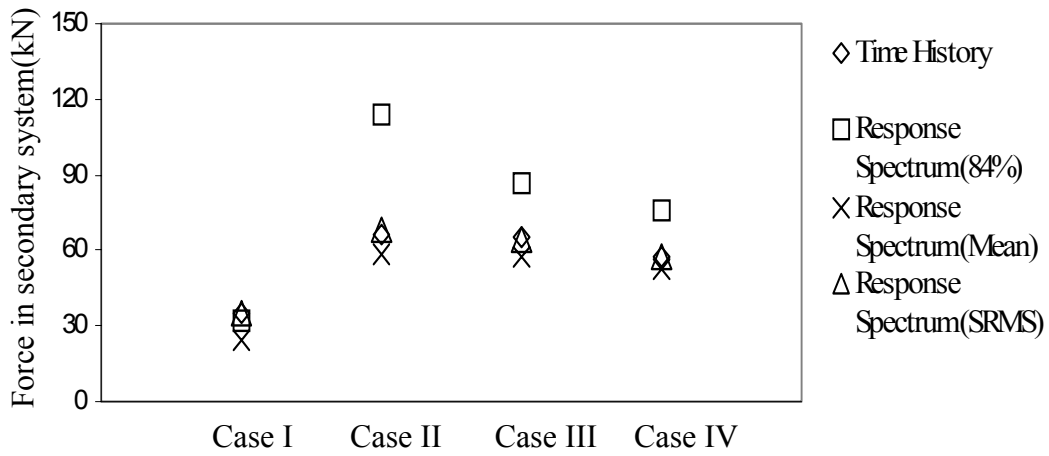


Fig. 10. Response corresponding to 84% NEP in Monte Carlo simulation

Appendix A

Square-Root-of-Mean-of-Squares

Let us begin with expressing the variance of response R

$$\sigma_R^2 = \frac{\sum_{i=1}^{NT} (R_i - \bar{R})^2}{NT} \quad (\text{A-1})$$

where σ_R^2 = variance of design response

\bar{R} = mean of design response

R_i = conditional response for i^{th} set of earthquake input and modal properties

(frequencies and modal damping ratios)

NT = the total number of responses over all earthquake inputs and modal property sets

According to Eq. (15)

$$E[R] = E[E[R/Z = z_i]] \quad (\text{A-2})$$

Eqs (A-1) and (A-2) give

$$\sigma_R^2 = \frac{1}{NT} \sum_{i=1}^{NT} \left[R_i - \frac{1}{NS} \sum_{j=1}^{NS} \bar{R}_{Z_j} \right]^2 \quad (\text{A-3})$$

where \bar{R}_{Z_j} = mean of conditional response using earthquake inputs for a given j^{th} set of

modal properties

$Z_j = j^{th}$ set of modal properties

NS = the total number of sets for modal properties

Next, we can simplify the following expression

$$\begin{aligned}
 \left[R_i - \frac{1}{NS} \sum_{j=1}^{NS} \bar{R}_{Z_j} \right]^2 &= R_i^2 - \frac{2}{NS} R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \frac{1}{NS^2} \left(\sum_{j=1}^{NS} \bar{R}_{Z_j} \right)^2 \\
 &= R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \\
 &\quad + 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} - \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 - \frac{2}{NS} R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \frac{1}{NS^2} \left(\sum_{j=1}^{NS} \bar{R}_{Z_j} \right)^2 \\
 &= A_i + B_i
 \end{aligned} \tag{A-4}$$

where,

$$\begin{aligned}
 A_i &= R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \\
 B_i &= 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} - \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 - \frac{2}{NS} R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \frac{1}{NS^2} \left(\sum_{j=1}^{NS} \bar{R}_{Z_j} \right)^2
 \end{aligned}$$

Let us subdivide NT into NS intervals such that $1 < Nk_1 < \dots < Nk_i < \dots < Nk_{NS}$. We can

then write

$$\begin{aligned}
\frac{1}{NT} \sum_{i=1}^{NT} A_i &= \frac{1}{NT} \sum_{i=1}^{NT} \left[R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right] \\
&= \frac{1}{NT} \left\{ \begin{aligned} &\left[\sum_{i=1}^{Nk_1} \left[R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right] + \sum_{i=1}^{Nk_2} \left[R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right] + \dots \right. \\ &\dots + \sum_{i=l}^{Nk_{l+1}} \left[R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right] + \dots + \\ &\left. \dots + \sum_{i=Nk_{NS-2}}^{Nk_{NS-1}} \left[R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right] + \sum_{i=Nk_{NS-1}}^{Nk_{NS}} \left[R_i^2 - 2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right] \right\}
\end{aligned} \right. \tag{A-5}
\end{aligned}$$

Note that $Nk_{NS} = NT$

Eq. (A-5) can be rearranged as

$$\begin{aligned}
\frac{1}{NT} \sum_{i=1}^{NT} A_i &= \frac{1}{NT} \left\{ \begin{aligned} &\sum_{i=1}^{Nk_1} \left[(R_i - \bar{R}_{Z_1})^2 - 2R_i \sum_{j=2}^{NS} \bar{R}_{Z_j} + \sum_{j=2}^{NS} \bar{R}_{Z_j}^2 \right] + \dots \\ &\dots + \sum_{i=Nk_{NS-1}}^{Nk_{NS}} \left[(R_i - \bar{R}_{Z_{NS}})^2 - 2R_i \sum_{j=1}^{NS-1} \bar{R}_{Z_j} + \sum_{j=1}^{NS-1} \bar{R}_{Z_j}^2 \right] \end{aligned} \right\} \\
&= \frac{1}{NT} \left\{ \begin{aligned} &\sum_{i=1}^{Nk_1} (R_i - \bar{R}_{Z_1})^2 - 2 \sum_{i=1}^{Nk_1} R_i \sum_{j=2}^{NS} \bar{R}_{Z_j} + \sum_{i=1}^{Nk_1} \sum_{j=2}^{NS} \bar{R}_{Z_j}^2 \dots \\ &\dots + \sum_{i=1}^{Nk_{NS}} (R_i - \bar{R}_{Z_1})^2 - 2 \sum_{i=1}^{Nk_{NS}} R_i \sum_{j=1}^{NS-1} \bar{R}_{Z_j} + \sum_{i=1}^{Nk_{NS}} \sum_{j=1}^{NS-1} \bar{R}_{Z_j}^2 \end{aligned} \right\} \tag{A-6}
\end{aligned}$$

For simplicity, let each interval be equal i.e. $Nk_1 = Nk_2 = \dots = Nk_{NS-1} = Nk_{NS} = K$. Note that $NT = (K)(NS)$. Eq. (A-6) can then be simplified as

$$\begin{aligned}
\frac{1}{NT} \sum_{i=1}^{NT} A_i &= \frac{1}{NT} \left\{ \begin{aligned} &K\sigma_{Z_1}^2 - 2K\bar{R}_{Z_1} \sum_{j=2}^{NS} \bar{R}_{Z_j} + K \sum_{j=2}^{NS} \bar{R}_{Z_j}^2 + \dots \\ &\dots + K\sigma_{Z_{NS}}^2 - 2K\bar{R}_{Z_{NS}} \sum_{j=1}^{NS-1} \bar{R}_{Z_j} + K \sum_{j=1}^{NS-1} \bar{R}_{Z_j}^2 \end{aligned} \right\} \\
&= \frac{1}{NS} \left\{ \sum_{i=1}^{NS} \sigma_{Z_i}^2 - 2 \sum_{i=1}^{NS} \left[\bar{R}_{Z_i} \sum_{\substack{j=1 \\ j \neq i}}^{NS} \bar{R}_{Z_j} \right] + (NS-1) \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 \right\}
\end{aligned} \tag{A-7}$$

Next, we simplify B_i in Eq. (A-4) following the same procedure as above.

$$\begin{aligned}
\frac{1}{NT} \sum_{i=1}^{NT} B_i &= \frac{1}{NT} \sum_{i=1}^{NT} \left[2R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} - \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 - \frac{2}{NS} R_i \sum_{j=1}^{NS} \bar{R}_{Z_j} + \frac{1}{NS^2} \left(\sum_{j=1}^{NS} \bar{R}_{Z_j} \right)^2 \right] \\
&= \frac{1}{NS} \left\{ \left(2 - \frac{1}{NS} \right) \left(\sum_{j=1}^{NS} \bar{R}_{Z_j} \right)^2 - NS \sum_{j=1}^{NS} \bar{R}_{Z_j}^2 \right\}
\end{aligned} \tag{A-8}$$

Substituting Eqs. (A-7) and (A-8), variance σ_R^2 can be expressed as

$$\begin{aligned}
\sigma_R^2 &= \frac{1}{NS} \left\{ \sum_{i=1}^{NS} \sigma_{Z_i}^2 - 2 \sum_{i=1}^{NS} \left[\bar{R}_{Z_i} \sum_{\substack{j=1 \\ j \neq i}}^{NS} \bar{R}_{Z_j} \right] + (NS-1) \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 \right\} \\
&\quad + \frac{1}{NS} \left\{ \left(2 - \frac{1}{NS} \right) \left(\sum_{i=1}^{NS} \bar{R}_{Z_i} \right)^2 - NS \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 \right\} \\
&= \frac{1}{NS} \left\{ \sum_{i=1}^{NS} \sigma_{Z_i}^2 - 2 \sum_{i=1}^{NS} \left[\bar{R}_{Z_i} \sum_{\substack{j=1 \\ j \neq i}}^{NS} \bar{R}_{Z_j} \right] - \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 + \left(2 - \frac{1}{NS} \right) \left(\sum_{i=1}^{NS} \bar{R}_{Z_i} \right)^2 \right\} \\
&= \frac{1}{NS} \left\{ \sum_{i=1}^{NS} \sigma_{Z_i}^2 - 2 \sum_{i=1}^{NS} \left[\bar{R}_{Z_i} \sum_{j=1}^{NS} \bar{R}_{Z_j} \right] + \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 + \left(2 - \frac{1}{NS} \right) \left(\sum_{i=1}^{NS} \bar{R}_{Z_i} \right)^2 \right\} \\
&= \frac{1}{NS} \left\{ \sum_{i=1}^{NS} \sigma_{Z_i}^2 + \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 - \frac{1}{NS} \left(\sum_{i=1}^{NS} \bar{R}_{Z_i} \right)^2 \right\}
\end{aligned} \tag{A-9}$$

Or,

$$\begin{aligned}
NS\sigma_R^2 &= \sum_{i=1}^{NS} \sigma_{Z_i}^2 + \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 - \frac{1}{NS} \left(\sum_{i=1}^{NS} \bar{R}_{Z_i} \right)^2 \\
&= \sum_{i=1}^{NS} \sigma_{Z_i}^2 + \sum_{i=1}^{NS} \bar{R}_{Z_i}^2 - NS\bar{R}^2 \\
&= \sum_{i=1}^{NS} \left(\sigma_{Z_i}^2 + 2\sigma_{Z_i} \bar{R}_{Z_i} + \bar{R}_{Z_i}^2 \right) - 2 \sum_{i=1}^{NS} \sigma_{Z_i} \bar{R}_{Z_i} - NS\bar{R}^2
\end{aligned} \tag{A-10}$$

$$NS(\sigma_R^2 + \bar{R}^2) = \sum_{i=1}^{NS} S_{Z_i}^2 - 2 \sum_{i=1}^{NS} \sigma_{Z_i} \bar{R}_{Z_i} \tag{A-11}$$

where, S_{Z_i} = mean plus one standard deviation for i^{th} set of modal properties.

Let us add $NS2\sigma_R\bar{R}$ to both sides of Eq. (A-11) and rearrange it.

$$NS(\sigma_R^2 + \bar{R}^2 + 2\sigma_R\bar{R}) = \sum_{i=1}^{NS} S_{Z_i}^2 - 2\sum_{i=1}^{NS} \sigma_{Z_i} \bar{R}_{Z_i} + 2NS\sigma_R\bar{R} \quad (\text{A-12})$$

$$S^2 = (\sigma_R + \bar{R})^2 = \frac{1}{NS} \left(\sum_{i=1}^{NS} S_{Z_i}^2 - 2\sum_{i=1}^{NS} \sigma_{Z_i} \bar{R}_{Z_i} + 2NS\sigma_R\bar{R} \right) \quad (\text{A-13})$$

Further if ν_R is the coefficient of variation for R , $\sigma_R\bar{R}$ can be expressed by

$$\sigma_R\bar{R} = \left[\frac{1}{\beta} (\sigma_R + \bar{R}) \right]^2 \quad \nu_R = \frac{\sigma_R}{\bar{R}} \quad (\text{A-14})$$

$$\nu_R\bar{R}^2 = \frac{\bar{R}^2}{\beta^2} (1 + \nu_R)^2 \quad (\text{A-15})$$

$$\frac{1}{\beta^2} = \frac{\nu_R}{(1 + \nu_R)^2} \quad (\text{A-16})$$

Substituting Eq. (A-16) into Eq. (A-14), we can write

$$\begin{aligned} \sigma_R\bar{R} &= \left[\frac{\sqrt{\nu_R}}{(1 + \nu_R)} (\sigma_R + \bar{R}) \right]^2 \\ &= \frac{\nu_R}{(1 + \nu_R)^2} (\sigma_R + \bar{R})^2 \\ &= \frac{\nu_R}{(1 + \nu_R)^2} S^2 \end{aligned} \quad (\text{A-17})$$

Similarly, $\sigma_{Z_i}\bar{R}_{Z_i}$ can be rewritten as

$$\sigma_{Z_i} \bar{R}_{Z_i} = \frac{\nu_{Z_i}}{(1 + \nu_{Z_i})^2} S_{Z_i}^2 \quad \nu_{Z_i} = \frac{\sigma_{Z_i}}{\bar{R}_{Z_i}} \quad (\text{A-18})$$

Eq. (A-13), Eq. (A-17) and Eq. (A-18) give

$$S^2 = \frac{1}{NS} \left(\sum_{i=1}^{NS} S_{Z_i}^2 - 2 \sum_{i=1}^{NS} \frac{\nu_{Z_i}}{(1 + \nu_{Z_i})^2} S_{Z_i}^2 + 2NS \frac{\nu_R}{(1 + \nu_R)^2} S^2 \right) \quad (\text{A-19})$$

$$NS \left(S^2 - 2 \frac{\nu_R}{(1 + \nu_R)^2} S^2 \right) = \sum_{i=1}^{NS} S_{Z_i}^2 - 2 \sum_{i=1}^{NS} \frac{\nu_{Z_i}}{(1 + \nu_{Z_i})^2} S_{Z_i}^2 \quad (\text{A-20})$$

$$NS \left(1 - \frac{2\nu_R}{(1 + \nu_R)^2} \right) S^2 = \sum_{i=1}^{NS} \left(1 - \frac{2\nu_{Z_i}}{(1 + \nu_{Z_i})^2} \right) S_{Z_i}^2 \quad (\text{A-21})$$

$$S^2 = \frac{1}{NS} \sum_{i=1}^{NS} S_{Z_i}^2 \left(\frac{1 - \frac{2\nu_{Z_i}}{(1 + \nu_{Z_i})^2}}{1 - \frac{2\nu_R}{(1 + \nu_R)^2}} \right) \quad (\text{A-22})$$

Finally, the response corresponding to mean plus one standard deviation (84% NEP if Gaussian distribution) can be obtained by

$$S = \left[\frac{1}{NS} \sum_{i=1}^{NS} S_{Z_i}^2 \Theta_{Z_i} \right]^{\frac{1}{2}} \quad \Theta_{Z_i} = \begin{pmatrix} 1 - \frac{2\nu_{Z_i}}{(1 + \nu_{Z_i})^2} \\ 1 - \frac{2\nu_R}{(1 + \nu_R)^2} \end{pmatrix} \quad (\text{A-23})$$

PART IV

PROBABILISTIC FRAMEWORK FOR CONDITION ASSESSMENT

Byounghoan Choi and Abhinav Gupta

PROBABILISTIC FRAMEWORK FOR CONDITION ASSESSMENT

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Abstract: A modified probabilistic framework for condition assessment based on Monte Carlo simulation is proposed in this study. Frequency and mode shapes data before and after degradation are used to evaluate the responses of a structure subjected to a sine sweep type of harmonic excitation. A degradation likelihood index is introduced to evaluate likelihood of degradation by considering the correlation between probability density functions of rigidities before and after degradation. Validity of the proposed framework is evaluated using two simple structures. A sensitivity analysis is also performed to study the consistency of the proposed framework due to not only changes in the mean of mode shapes and frequency but also the number of modes considered. Results for two simple structures show considerable promise, but they are not overwhelmingly positive. Additional work is needed to improve the consistency of the proposed approach.

1. Introduction

Nearly all in-service structures and equipment require some form of inspection to maintain their integrity and avoid catastrophic failure. Over the past decade and more, extensive research has been conducted on employing vibration data to investigate integrity, detect degradation, minimize downtime, and develop efficient inspection strategies. Applications of vibration based

methods for detecting structural degradation are mostly encountered in bridges, buildings and to some extent in aerospace structures. Potential benefits in industrial facilities such as power plants can be quite significant. Power plant structures (buildings together with mechanical and electrical equipment) are subjected to significant in-plant vibrations and any undetected degradation can lead to a malfunction or accident. The premise for using vibration data in detecting locations of structural degradation is based primarily on the fact that material and geometrical degradation causes a decrease in structural stiffness which is reflected by the corresponding changes in the vibration characteristics such as the natural frequencies and the mode shapes. Aktan et al. (1997), Chang (1997,1999), ASCE (1997), and Doebling et al. (1996) provide a comprehensive review of existing developments in structural condition assessment and their application to infrastructure systems. Application of these techniques to bridges has been widely studied at Sandia and Los Alamos National Laboratories (LANL 1995).

Different researchers have used a wide variety of vibration data for structural condition assessment. These include natural frequencies, mode shapes, mode shape curvatures, frequency response functions, transmissibility ratios, modal strain energy, etc (Messina et al 1997, Ren 2002, Pandey et al. 1991, Shi et al. 1998). While the accuracy and acceptability of a particular data type is highly dependent upon the nature of application and quality of data being considered, the natural frequencies and the mode shapes are the two most widely used parameters. Several studies have now indicated that methods based on employing the mode shape data are superior than the ones employing only the natural frequencies due to the spatial distribution of mode shape data. Irrespective of the parameters used, almost all the methods employ a finite element model which is updated to minimize the errors between the recorded data and the analytical results. The updated finite element model is then used to identify locations at which stiffness

degradation has occurred. A key limitation of these approaches lies in the sensitivity of the results to uncertainties associated with finite element modeling along with errors associated with recorded data (Xia et al. 2002). According to Farrar and Doebling (1998), measurement related uncertainties may cause these methods to detect degradation in structural elements that are otherwise intact and vice versa. Another limitation lies in the availability of only a few lower order mode shapes. Evaluation of high frequency modes from a typical set of recorded data is highly impractical if not impossible.

Abdelghani (1997) indicates that the concept of estimating the likelihood of degradation is more achievable than detecting degradation in an absolute sense due to unavoidable uncertainties and omissions as well as due to the difficulty in defining degradation in an absolute sense. Some studies have considered the effect of uncertainties in finite element modeling as well as recorded data for evaluating the sensitivity of modal properties, updating the finite element models, and conducting structural condition assessment (Collins et al. 1974, Papadopoulos and Garcia 1998, Xia et al. 2002, Shon and Law 1997). In almost all these methods, a perturbation analysis is used to relate the changes in frequency and mode shape data with the corresponding changes in the global stiffness matrix. The reductions in the elements of global stiffness matrix are then expressed in terms of the corresponding reductions in the element stiffness matrices. The set of algebraic equations obtained in this manner are then used to evaluate the statistical properties of the element stiffness matrices namely the mean and covariance matrices (Papadopoulos and Garcia 1998). To do so, a Taylor series approximation is employed to establish the set of algebraic equations together with a Monte Carlo simulation to evaluate the covariance matrices. For simplicity, Papadopoulos and Garcia (1998) considered all the random variables to be normally distributed. Structural degradation is then characterized in terms of a

probability damage quotient that indicates a confidence level on the existence of degradation by comparing the probability density functions for stiffness terms before and after degradation.

While the study conducted by Papadopoulos and Garcia (1998) was based primarily on changes in frequency, Xia et al. (2002) extended this concept by considering the frequencies as well as the mode shapes before and after the degradation. To account for differences in the sensitivity of frequencies and mode shapes to structural degradation, different weights are considered for the two sets of parameters in their study. However, their method also characterizes all the random variables to be Gaussian.

In this paper we propose a modified probabilistic framework for structural condition assessment that utilizes some of the concepts proposed by Papadopoulos and Garcia (1998) and those by Xia et al. (2002). To begin with, we illustrate that the definition of degradation in terms of a confidence level may not be sufficient in identifying elements with significant degradation especially if the probability density functions are not Gaussian. Unlike existing studies, our work is based purely on Monte Carlo simulation within the finite element analysis and not on perturbation-based algebraic equations. The proposed technique employs both the frequency and mode shape data of the structure before and after degradation. The premise lies in simulation-based decision making wherein multiple analyses can be conducted starting with updated finite element models that correlate with recorded data before and after degradation. The modal properties are used to analytically evaluate the structural responses by considering sine sweep type of harmonic excitation. Changes in structural response are then used to evaluate an indicator for structural degradation based on the statistical correlation between the random variables characterizing element stiffness before and after degradation. The proposed technique evaluates the necessary probability density functions numerically and does not make any assumption

regarding the nature of their distribution. The results presented in this paper are based purely on simulated data and not on any experimental results. Only simple systems are used to evaluate the validity of the proposed framework. While this paper represents a complete set of work, additional work is needed before considering it for submission to any technical journal.

2. Formulation of the Inverse Problem

The framework proposed in this paper is based on useful concepts represented by Papadopoulos and Garcia (1998) and Xia et al. (2002). As in these existing studies, we consider that the frequencies and mode shapes of the structure are available from the updated finite element models before and after degradation. Further, it is assumed that only a few and not all the mode shapes are available as is the case in real-life structural applications. For an N-DOF, the equations of motion before and after degradation can be written as

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{U_f\}f(t) = \{F(t)\} \quad (1)$$

$$[M - \Delta M]\{\ddot{U}_d\} + [C_d - \Delta C]\{\dot{U}_d\} + [K - \Delta K]\{U_d\} = \{F(t)\} \quad (2)$$

where \ddot{U} , \dot{U} and U are the acceleration, velocity, and displacement vectors of the structure before degradation, respectively. Subscript d denotes the corresponding quantities after degradation. Further, Δ indicates the degradation in structural properties. The vector $\{U_f\}$ represents the influence vector with respect to the location of the force $f(t)$. In general, most degradations are categorized by a change in only the stiffness of elements i.e. the degradations in mass and damping terms are neglected. While the modal damping ratios before and after

degradation are considered to be same, the damping matrices $[C]$ and $[C_d]$ are different because of changes in frequencies and mode shape matrices. Therefore, Eq. (2) can be rewritten as

$$[M]\{\ddot{U}_d\} + [C_d]\{\dot{U}_d\} + [K - \Delta K]\{U_d\} = \{F(t)\} \quad (3)$$

Substituting Eq. (3) into Eq. (1),

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = [M]\{\ddot{U}_d\} + [C_d]\{\dot{U}_d\} + [K - \Delta K]\{U_d\} \quad (4)$$

$$[M][\{\ddot{U}\} - \{\ddot{U}_d\}] + [C]\{\dot{U}\} - [C_d]\{\dot{U}_d\} + [K][\{U\} - \{U_d\}] = -[\Delta K]\{U_d\} \quad (5)$$

Let $[\Phi]$ and $[\Phi_d]$ represent the mode shape matrices before and after degradation; vectors $\{q\}$ and $\{q_d\}$ represent the corresponding normal coordinates; and ω_j and ω_{jd} represent the corresponding circular frequencies in j^{th} mode. Since $\{U\} = [\Phi]\{q\}$ and $\{U_d\} = [\Phi_d]\{q_d\}$, Eq. (5) can be rewritten as

$$[M][[\Phi]\{\ddot{q}\} - [\Phi_d]\{\ddot{q}_d\}] + [C][\Phi]\{\dot{q}\} - [C_d][\Phi_d]\{\dot{q}_d\} + [K][[\Phi]\{q\} - [\Phi_d]\{q_d\}] = -[\Delta K][[\Phi_d]\{q_d\}] \quad (6)$$

$$\begin{aligned} \ddot{q}_j + 2\omega_j\zeta_j\dot{q}_j + \omega_j^2q_j &= f_j(t) & f_j(t) &= \{\Phi\}_j^T \{U_f\} f(t) = \gamma_j f(t) \\ \ddot{q}_{jd} + 2\omega_{jd}\zeta_j\dot{q}_{jd} + \omega_{jd}^2q_{jd} &= f_{jd}(t) & f_{jd}(t) &= \{\Phi\}_{jd}^T \{U_f\} f(t) = \gamma_{jd} f(t) \end{aligned} \quad (7)$$

For harmonic excitation of the type $f(t)=f_0 \sin\Omega t$, the j^{th} modal response in Eq. (7) can be formulated by transient response q_{tj} and steady state response q_{sj} , respectively. For initial conditions $q_j(0)$ and $\dot{q}_j(0)$, we can write

$$\begin{aligned} q_j(t) &= q_{tj}(t) + q_{sj}(t) \\ &= e^{-\zeta_j \omega_j t} (A_j \cos \omega_{Dj} t + B_j \sin \omega_{Dj} t) + X_j \sin \Omega t + Y_j \cos \Omega t \end{aligned} \quad (8)$$

where, $\omega_{Dj} = \omega_j \sqrt{1 - \zeta_j^2}$

$$\begin{aligned} A_j &= q_j(0) - \frac{\gamma_j (2\omega_j \zeta_j \Omega) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2} \\ B_j &= \frac{\dot{q}_j(0) + \omega_j \zeta_j \left[q_j(0) - \frac{\gamma_j (2\omega_j \zeta_j \Omega) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2} \right] - \frac{\Omega \gamma_j (\omega_j^2 - \Omega^2) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2}}{\omega_{Dj}} \\ X_j &= \frac{\gamma_j (\omega_j^2 - \Omega^2) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2} \\ Y_j &= \frac{-\gamma_j (2\omega_j \zeta_j \Omega) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2} \end{aligned}$$

Similarly, modal responses after degradation can be formulated by replacing ω_j and γ_j with ω_{jd} and γ_{jd} .

In this study, it is assumed that the elemental stiffness matrix $[K]_e$ is uniformly degraded and its degradation results from a change in rigidity (EA for bar element or EI for beam element).

The element stiffness matrix can be separated into two parts, one corresponding to rigidity $\{R\}_e$ and the other corresponding to geometric properties $[G]_e$.

$$[K]_e = [G]_e \{R\}_e \quad (9)$$

For one-dimensional bar element, change in element stiffness before and after degradation can be expressed as

$$[\Delta K]_e = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} - \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}_d = \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \{EA - E_d A_d\} = [G]_e \{\Delta R\}_e \quad (10)$$

where, E , A , and L are elastic modulus, area, and length associated with a particular element, respectively. $\{\Delta R\}_e$ is the change in rigidity of an element before and after degradation. For a flexural element, the reduced rigidity $\{\Delta R\}_e$ is $\{EI - E_d I_d\}$. The change in global stiffness matrix of a structure can be expressed in terms of change in element stiffness matrices.

$$[\Delta K] = \sum_{i=1}^{NE} [\Delta K]_{e_i} = \sum_{i=1}^{NE} [G]_{e_i} \{\Delta R\}_{e_i} \quad (11)$$

where, NE is the total number of elements in the finite element model. We can also write,

$$\begin{aligned}
[\Phi_d]\{q_d\} &= \{\Phi_d q_d\} \\
&= \sum_{i=1}^{NE} \{\Phi_d q_d\}_i
\end{aligned} \tag{12}$$

The right side of Eq. (6) can then be expressed as

$$[\Delta K][\Phi_d]\{q_d\} = \sum_{i=1}^{NE} [G]_{e_i} \{\Delta R\}_{e_i} \{\Phi_d q_d\}_{e_i} = \sum_{i=1}^{NE} [G]_{e_i} \{\Phi_d q_d\}_{e_i} \{\Delta R\}_{e_i} = [T]\{\Delta R\} \tag{13}$$

$$[T] = \sum_{i=1}^{NE} [G]_{e_i} \{\Phi_d q_d\}_{e_i} \quad \{\Delta R\}^T = \{\Delta R_{e_1}, \Delta R_{e_2}, \dots, \Delta R_{e_{NE}}\} \tag{14}$$

Therefore, Eq. (6) becomes

$$[M][\Phi]\{\ddot{q}\} - [\Phi_d]\{\ddot{q}_d\} + [C][\Phi]\{\dot{q}\} - [C_d][\Phi_d]\{\dot{q}_d\} + [K][\Phi]\{q\} - [\Phi_d]\{q_d\} = [T]\{\Delta R\} \tag{15}$$

It should be noted that while reduced rigidity vector $\{\Delta R\}$ is constant, the transformation matrix $[T]$ is time variant. By taking the inverse of $[T]$ in Eq. (15), reduced rigidity vector can be evaluated as

$$\{\Delta R\} = [T]^{-1}\{S\} \tag{16}$$

$$\{S\} = [M][\Phi]\{\ddot{q}\} - [\Phi_d]\{\ddot{q}_d\} + [C][\Phi]\{\dot{q}\} - [C_d][\Phi_d]\{\dot{q}_d\} + [K][\Phi]\{q\} - [\Phi_d]\{q_d\}$$

As is well known, the total number of degrees of freedom is typically much larger than the number of modes. Therefore, the matrix $[T]$ may not be inverted directly. For such non-square matrices, the optimal solution of inverting the matrix $[T]$ can be obtained by using the pseudoinverse technique explained in Appendix A.

In this study, degradation is evaluated by using only the steady state part of the response to harmonic excitation. Therefore, time variation of the matrix can be eliminated. Let us write the steady state responses q_j in mode j before degradation and q_{jd} after degradation as follows

$$q_j \approx q_{sj} = X_j \sin \Omega t + Y_j \cos \Omega t ; \quad q_{jd} \approx q_{sjd} = X_{jd} \sin \Omega t + Y_{jd} \cos \Omega t \quad (17)$$

$$\text{where, } X_j = \frac{\gamma_j (\omega_j^2 - \Omega^2) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2} \quad \text{and} \quad Y_j = \frac{-\gamma_j (2\omega_j \zeta_j \Omega) f_0}{(\omega_j^2 - \Omega^2)^2 + (2\omega_j \zeta_j \Omega)^2}$$

$$X_{jd} = \frac{\gamma_{jd} (\omega_{jd}^2 - \Omega^2) f_0}{(\omega_{jd}^2 - \Omega^2)^2 + (2\omega_{jd} \zeta_{jd} \Omega)^2} \quad \text{and} \quad Y_{jd} = \frac{-\gamma_{jd} (2\omega_{jd} \zeta_{jd} \Omega) f_0}{(\omega_{jd}^2 - \Omega^2)^2 + (2\omega_{jd} \zeta_{jd} \Omega)^2}$$

All the $X_j, Y_j, X_{jd},$ and Y_{jd} terms together can be expressed as vectors $\{X\}, \{Y\}, \{X_d\},$ and $\{Y_d\},$ respectively. Substituting Eq. (17) into Eq. (15) and separating sine and cosine terms, we get

$$\{S\}_s \sin \Omega t + \{S\}_c \cos \Omega t = [T]_s \{\Delta R\}_s \sin \Omega t + [T]_c \{\Delta R\}_c \cos \Omega t \quad (18)$$

where,

$$\{S\}_s = -\Omega^2 [M][\Phi]\{X\} - [\Phi_d]\{X_d\} - \Omega [C][\Phi]\{\dot{q}\} - [C_d][\Phi_d]\{\dot{q}_d\} + [K][\Phi]\{X\} - [\Phi_d]\{X_d\}$$

$$\{S\}_c = -\Omega^2 [M][\Phi]\{Y\} - [\Phi_d]\{Y_d\} + \Omega [C][\Phi]\{\dot{q}\} - [C_d][\Phi_d]\{\dot{q}_d\} + [K][\Phi]\{Y\} - [\Phi_d]\{Y_d\}$$

$$[T]_s = \sum_{i=1}^{NE} [G]_{e_i} \{\Phi_d X_d\}_{e_i} \quad \{\Delta R\}_s^T = \{\Delta R_{e_1}, \Delta R_{e_2}, \dots, \Delta R_{e_{NE}}\}_s$$

$$[T]_c = \sum_{i=1}^{NE} [G]_{e_i} \{\Phi_d Y_d\}_{e_i} \quad \{\Delta R\}_c^T = \{\Delta R_{e_1}, \Delta R_{e_2}, \dots, \Delta R_{e_{NE}}\}_c$$

Therefore,

$$\{\Delta R\}_s = [T]_s^{-1} \{S\}_s ; \quad \{\Delta R\}_c = [T]_c^{-1} \{S\}_c \quad (19)$$

Results obtained from each of two equations should be same if no uncertainty is considered. When uncertainty is considered, the combined vectors can be obtained as the mean of individual vectors.

$$\{\Delta R\} = \frac{\{\Delta R\}_s + \{\Delta R\}_c}{2} \quad (20)$$

3. Proposed Probabilistic Approach

The proposed approach is based on consideration of uncertainties in both sets of data obtained before and after degradation. To account for these uncertainties, the frequencies and mode shapes (both before and after degradation) are characterized by uniformly distributed

independent random variables, Q_ϕ and Q_ω , respectively. One way to conduct the Monte Carlo simulation is to generate the values of each variable randomly by using the following equations

$$\begin{aligned}
 Q_\phi &= \mu_\phi (1 + \nu_\phi \kappa_\phi) \\
 Q_\omega &= \mu_\omega (1 + \nu_\omega \kappa_\omega) \\
 -1 &\leq \kappa \leq 1
 \end{aligned} \tag{21}$$

where, μ_ϕ and μ_ω are means of frequencies and mode shapes, respectively. Further, κ represents random number that is used to simulate values of random variables Q_ϕ and Q_ω about their respective mean values. Typically, Q_ϕ and Q_ω are varied between certain bounds for each variable that are governed by the values of coefficients of variation ν_ϕ and ν_ω for each random variable. These simulated values then give the corresponding values and probability distribution for the elements of reduced rigidity vector $\{\Delta R\}$ in accordance with Eqs. (16) and (21). In most of the existing studies, the frequency and mode shape values evaluated from updated finite element models are considered as means. This may not necessarily be true. In this study, we also characterize the means of frequency and mode shape μ_ϕ and μ_ω as random variables with specified coefficients of variations.

Before proceeding ahead, let us examine the nature of probability density functions for changes in the elements of rigidity vectors obtained using the above procedure. For simplicity of notation, let us represent ΔR by Z so that its probability density function is expressed as $f_Z(z)$. The simulated values of these quantities can be very sensitive to the variations in the mode shape

values. One factor that contributes to these sensitivities corresponds to the non-unique nature of pseudo-inverse problem given by Eq. (16). We consider a 5-DOF simple system shown in Fig. 1 to evaluate $f_z(z)$. Simulated values of the frequencies and mode shapes before and after degradation are evaluated by introducing a 30% degradation in 2nd storey and a 40% degradation in 3rd storey. While the simulated values are considered as means, the coefficients of variation for means are taken as 0.15 for the mean of mode shape and 0.01 for the mean of frequencies. Table 1 gives the frequencies of this system before and after degradation. To conduct the Monte Carlo simulation in accordance with Eq. (21), the coefficient of variation for Q_ϕ is taken as 0.15. Since the existing studies have shown that the frequencies are much less sensitive to degradation (Ren et al. 2002), the coefficient of variation for Q_ω is ignored, i.e. $\nu_\omega = 0$. The $f_z(z)$ curves are evaluated numerically by considering 5000 samples in the Monte Carlo simulation using steady-state response for a harmonic load applied at the top as shown in Fig. 1. The frequency of excitation is taken as 6 Hz. Figs. 2 to 6 show the $f_z(z)$ curves for each spring before and after degradation. Several key observations can be made about the usefulness as well as the limitations of the information provided by these probability density functions.

The most important observation relates to the fact that in general the $f_z(z)$ curve for a particular element undergoes an offset if the element experiences degradation as shown in Figs. 3 and 4 for the 2nd and 3rd storeys, respectively. It is important to note that a negative value of $\Delta R/R$ in these figures represents that the rigidity of the particular sample in the Monte Carlo simulation is higher than the value considered in the finite element analysis, i.e. $\Delta R/R = -1$ represents that rigidity of the particular element is twice the value considered in finite element analysis. A value of $\Delta R/R = +1$, on the other hand, represents that the element is completely

degraded with no rigidity. Even though it may be possible for $\Delta R/R$ to be close to positive unity, its values in the vicinity of negative unity appear impractical. Such large values exist due to sensitivity of results to the amount of uncertainty considered in mode shapes and the non-unique nature of the inverse-problem. Consideration of such large values in the Monte Carlo simulation would mean that the engineer/ decision maker has no confidence in the finite element model. A practical approach would be to constrain the Monte Carlo simulation by limiting the largest negative value that can be attained by $\Delta R/R$. In this study, we consider that $-0.2 \leq \Delta R/R \leq +1.0$. The observations made in the example considered above are similar to those made by Papadopoulos and Garcia (1998) as well as Xia et al. (2002). In both these studies, the observation regarding an offset in the probability density function is used to evaluate the likelihood of degradation in element j by evaluating the probability of ΔR_j^d after degradation being outside a specified confidence interval of $f_Z(z)$ before degradation. If P_j^d represents the likelihood of degradation in the j^{th} element, this criteria can be expressed as

$$P_j^d = 1 - P(\Delta R_{j,\alpha L} \leq \Delta R_j^d \leq \Delta R_{j,\alpha H}) = 1 - \int_{\Delta R_{j,\alpha L}}^{\Delta R_{j,\alpha H}} f_{Z_j^d}(z_j^d) dz_j^d \quad (22)$$

where $\Delta R_{j,\alpha L}$ and $\Delta R_{j,\alpha H}$ are the lower and upper bounds of reduction in ΔR defined at αL and αH percent non-exceeding probability values, respectively. Next, let us examine the quality of results evaluated using this approach. For the 5-DOF simple system described above, Table 2 gives the P_j^d values corresponding to the 95 percent non-exceedence probability before and after degradation. As seen in this Table 2, equal likelihood exists for degradation in elements 2 and 5.

Since no degradation occurred in element 5, it is possible that such a characterization of degradation with respect to a confidence interval may not be able to identify elements with significant degradation or it may identify degradation in elements that do not experience significant degradation. To avoid such scenarios, we propose a new index that is based on the correlation between the probability density functions for $f_z(z)$ before and after degradation. The premise for this approach is based on the work of Messina et al. (1997) who expressed the degradation in terms of correlation between the change of frequencies. They referred to this index as “damage location assurance criteria (DLAC)” which is defined as follows:

$$DLAC_j = \frac{|\{\Delta f\}^T \{\delta f_j\}|^2}{(\{\Delta f\}^T \{\Delta f\})(\{\delta f_j\}^T \{\delta f_j\})} \quad (23)$$

where, Δf is difference in measured frequency and δf_j is difference in frequency obtained from analytical model when the j^{th} element is assumed to be degraded. Similar to $DLAC$, it is possible to define a degradation likelihood index (DLI) at the j^{th} element by evaluating correlation between ΔR_j and ΔR_j^d , the changes in rigidity before and after degradation.

$$DLI_j = \left(1 - \frac{\left[\int_{-0.2R_j}^{R_j} f_{Z_j}(z_j) f_{Z_j^d}(z_j) dz_j \right]^2}{\int_{-0.2R_j}^{R_j} f_{Z_j}^2(z_j) dz_j \int_{-0.2R_j}^{R_j} f_{Z_j^d}^2(z_j) dz_j} \right) \times 100 \quad (24)$$

where, $f_{Z_j}(z_j)$ and $f_{Z_j^d}(z_j)$ are the probability density functions for the j^{th} element of reduced rigidity vector before and after degradation, respectively. DLI lies in the range of 0 to 100 with 0 indicating the lowest likelihood of degradation and 100 indicating the highest likelihood of degradation. For a discrete random variables ΔR_j and ΔR_j^d , the integrals in Eq. (24) can be replaced by corresponding summation as follow.

$$DLI_j = \left(1 - \frac{\left[\sum_{i=1}^{NT} \left(f_{Z_{ji}^d}(z_{ji}) f_{Z_{ji}}(z_{ji}) \right) \right]^2}{\sum_{i=1}^{NT} \left(f_{Z_{ji}^d}(z_{ji}) \right)^2 \sum_{i=1}^{NT} \left(f_{Z_{ji}}(z_{ji}) \right)^2} \right) \times 100 \quad (25)$$

where, NT is the total number of uniform intervals within the specified range $(-0.2\{R\} \leq \{\Delta R\} \leq \{R\})$.

At this point in this discussion, we would like to mention the significance of input excitation in the quality of results obtained from such an approach. Clearly, the response of a system and the nature of probability density function are highly dependent upon the frequency of harmonic excitation. It is even more so when only a few lower order modes are considered. Consequently, degradation likelihood index is a function of excitation frequency, i.e. $DLI(\Omega)$. Figs. 7 to 11 show the variation of DLI with excitation frequency for the 5-DOF example described earlier in this paper. While these curves show that DLI gives good results in identifying elements with degradation for several different excitation frequencies, there can be certain excitation frequencies which the DLI may not give good results. Therefore, defining DLI as mean over all the DLI variation would eliminate such ambiguity i.e.

$$DLI_j \approx E[DLI_j(\Omega)] = \sum_{i=1}^{NF} DLI_j(\Omega_i) \quad (26)$$

Where NF is the number of excitation frequencies considered. Table 3 gives the mean values of DLI for each element in the example considered. It is evident from these values that the likelihood of degradation in the 2nd and 3rd storeys is significantly much larger than that in other storeys.

4. Additional Example

Next we consider an additional example to evaluate the validity of this approach. For this purpose we consider a fixed-fixed flexural beam shown in Fig. 12. This example is representative of a straight pipe in the power plant that may experience corrosion/ erosion related degradation resulting in reduced rigidities EI . A total of 8 elements are considered in the 8m long pipe with 2-DOF at each node as shown in Fig 12. Three different elements are considered to be degraded as shown in Table 4. The corresponding frequencies of the structure are also given in Table 5. For all cases, 5% and 1% uncertainties are considered in the means of mode shapes and natural frequencies. Only five lower most modes are considered in the analysis. The harmonic excitation is considered to act at the mid-span. The mean values of DLI are given in Table 4. As seen in Table 4, DLI at the degraded elements are relatively much higher than those for the other elements.

5. Sensitivity Analysis

In this section, we present the results of a parametric study conducted to explore the sensitivity of DLI not only to the degree of uncertainty in means of mode shapes and frequencies but also to the number of modes considered. To do so, we consider a 5-DOF system similar to that shown in Fig. 1 with the exception that $k=3000 \text{ lb/in}$. Table 6 gives the frequencies of this system before and after degradation. It is assumed that 30% degradation occurs in 2nd and 4th storeys.

(i) *Degree of uncertainty in the mean of mode shapes*: We consider four cases with different uncertainties in the mean of mode shapes that vary from 0 to 15% with 5% increment. DLIs for all the four cases are summarized in Table 7. DLIs for the first three cases represent relatively higher values at the actual degraded elements. But, for 15% uncertainty, DLI at 5th storey is similar to that for the 2nd storey. It is observed that the proposed probabilistic approach give reasonably good results when uncertainties are relatively small. For large uncertainties, it may also indicate degradation in elements that are otherwise intact.

(ii) *Degree of uncertainty in the mean of frequency*: Effect of uncertainty in the mean of natural frequencies is studied by considering no uncertainty in the mean of mode shapes and four cases corresponding to 1%, 3%, 5%, and 7% uncertainties in the means of frequencies. As seen in Table 8, the proposed probabilistic approach is insensitive to the degree of uncertainty in the mean of frequency.

(iii) *Number of modes considered* : To study the effect of the number of modes considered, we consider 5% uncertainty in the mean of mode shapes and 1% uncertainty in the mean of the frequency. Four different cases corresponding to 1, 2,3, and 5 modes are considered. As seen in

Table 9, the quality of results evaluated using the proposed approach improves as the number of modes is increased.

6. Summary and Conclusions

This study presents a modified probabilistic framework for structural condition assessment by using the frequency and mode shape data before and after degradation. Unlike existing studies, our work is based purely on Monte Carlo simulation. The modal properties are used to analytically evaluate the structural responses by considering a sine sweep type of harmonic excitation. In an existing framework proposed by Papadopoulos and Garcia (1998) and later modified by Xia et al (2002) the degradation likelihood is expressed in terms of the probability that the reduction in rigidity lies within a specified confidence interval. It is shown that this characterization may or may not be sufficient in identifying elements with significant degradation especially if the probability density functions are not Gaussian. We propose a new degradation likelihood index, DLI, that characterizes the likelihood of degradation by evaluating correlation between the probability density functions of rigidities before and after degradation. We use two simple systems to evaluate the consistency and validity of the proposed framework. We also use these systems to conduct a sensitivity analysis. While the results for these simple systems show considerable promise, they are not overwhelmingly attractive. Additional work is needed to evaluate the consistency of the proposed approach. It may also be possible to define the DLI based on a derived response quality such as element strain energy rather than directly using the rigidity values. Such exploration may be necessary to improve the quality and consistency of the proposed approach.

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Table. 1 Natural frequencies (Hz) for 5-DOF system

| | 1 st mode | 2 nd mode | 3 rd mode | 4 th mode | 5 th mode |
|--------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Before degradation | 2.026 | 5.914 | 9.322 | 11.975 | 13.659 |
| After degradation | 1.815 | 5.708 | 8.459 | 10.707 | 12.67 |

Table. 2 P_j^d values corresponding to 95% non-exceedence probability for 5-DOF system

| Storey | P_j^d |
|--------|---------|
| 1 | 0.01 |
| 2 | 0.34 |
| 3 | 1.00 |
| 4 | 0.12 |
| 5 | 0.34 |

Table. 3 Mean values of DLI for 5-DOF system

| Storey | DLI |
|--------|-------|
| 1 | 24.04 |
| 2 | 71.35 |
| 3 | 98.82 |
| 4 | 39.84 |
| 5 | 44.73 |

Table. 4 Representative piping with both ends fixed

| Element | Degradation (%) | DLI |
|---------|-----------------|-------|
| 1 | 0 | 23.54 |
| 2 | 0 | 17.82 |
| 3 | 40 | 40.38 |
| 4 | 30 | 35.31 |
| 5 | 0 | 20.12 |
| 6 | 0 | 18.03 |
| 7 | 0 | 16.26 |
| 8 | 30 | 56.66 |

Table. 5 Natural frequencies (Hz) for representative piping

| | 1 | 2 | 3 | 4 | 5 |
|--------------------|------|------|------|-------|-------|
| Before degradation | 1.75 | 4.85 | 9.53 | 15.81 | 23.78 |
| After degradation | 1.61 | 4.42 | 8.86 | 14.78 | 21.82 |

Table. 6 Natural frequencies (Hz) for 5-DOF system in sensitivity analysis

| | 1 st mode | 2 nd mode | 3 rd mode | 4 th mode | 5 th mode |
|--------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Before degradation | 4.53 | 13.22 | 20.84 | 26.78 | 30.54 |
| After degradation | 4.47 | 12.99 | 19.12 | 24.05 | 27.45 |

Table. 7 Effect of uncertainty in the mean of mode shape

| Story | DLI | | | |
|-------|-------------------|-------------------|--------------------|--------------------|
| | 0% uncertainty | 5% uncertainty | 10% uncertainty | 15% uncertainty |
| 1 | 25.04 | 21.93 | 27.15 | 42.41 |
| 2 | 63.31 | 69.06 | 71.86 | 67.31 |
| 3 | 22.13 | 22.38 | 27.35 | 40.98 |
| 4 | 74.42 | 73.06 | 81.13 | 94.22 |
| 5 | 21.40 | 23.53 | 42.37 | 64.98 |

Table. 8 Effect of uncertainty in the mean of frequency

| Story | DLI | | | |
|-------|-------------------|-------------------|-------------------|-------------------|
| | 1% uncertainty | 3% uncertainty | 5% uncertainty | 7% uncertainty |
| 1 | 25.04 | 18.59 | 21.64 | 23.12 |
| 2 | 63.31 | 62.03 | 58.67 | 58.55 |
| 3 | 22.13 | 23.85 | 27.00 | 33.46 |
| 4 | 74.42 | 79.92 | 83.20 | 79.13 |
| 5 | 21.40 | 26.31 | 31.55 | 20.58 |

Table. 9 Effect of number of modes considered

| Story | DLI | | | |
|-------|-------|---------|---------|---------|
| | 1mode | 2 modes | 3 modes | 5 modes |
| 1 | 55.06 | 12.43 | 16.74 | 21.93 |
| 2 | 99.85 | 92.60 | 74.42 | 69.06 |
| 3 | 16.69 | 24.76 | 21.86 | 22.38 |
| 4 | 22.00 | 64.76 | 75.33 | 73.06 |
| 5 | 22.00 | 56.41 | 65.65 | 23.53 |

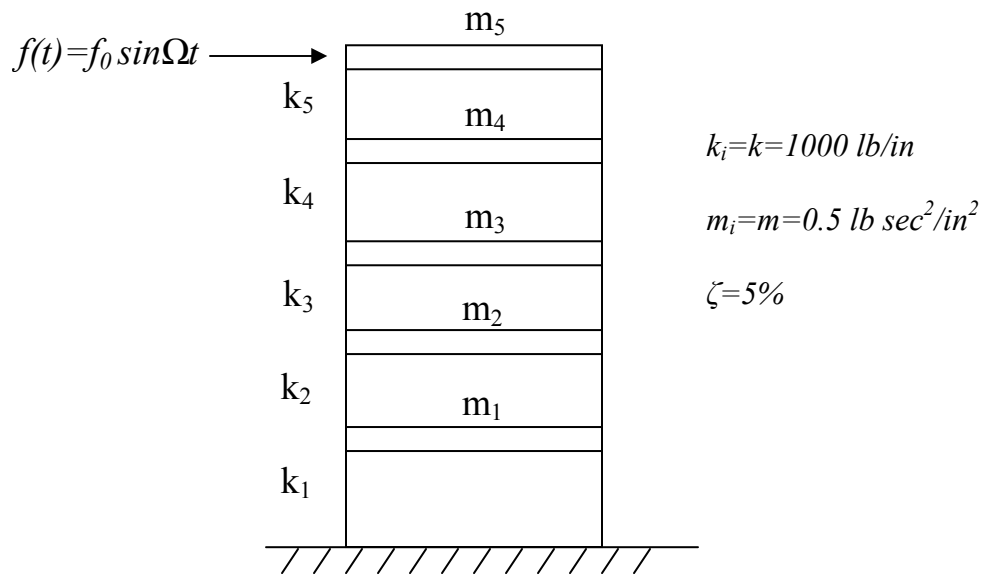


Fig. 1. Simple representative 5-DOF system

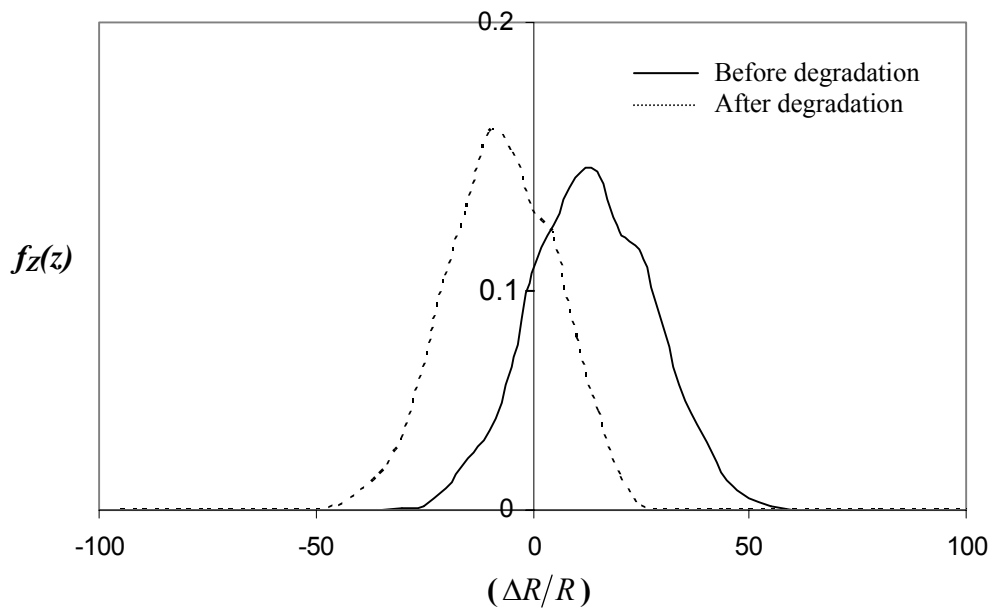


Fig. 2. Probability density functions $f_z(z)$ for 1st storey rigidity

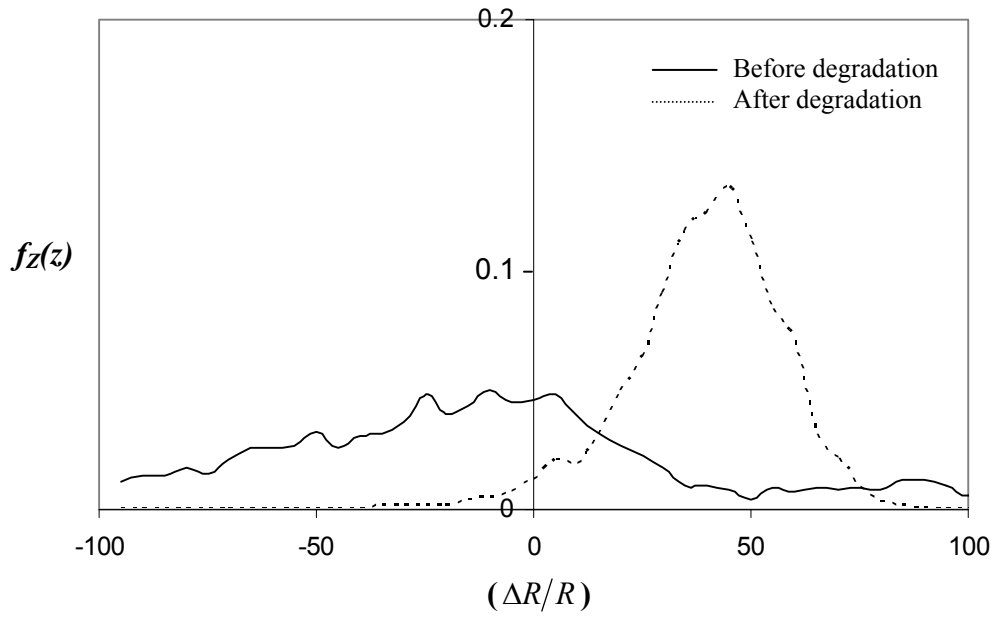


Fig. 3. Probability density functions $f_z(z)$ for 2nd storey rigidity

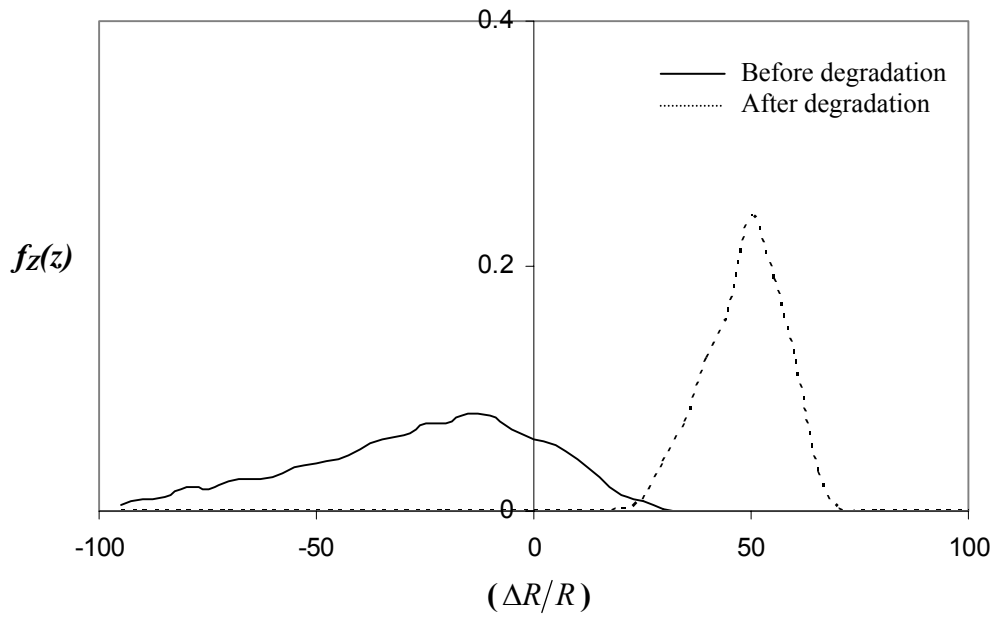


Fig. 4. Probability density functions $f_z(z)$ for 3rd storey rigidity

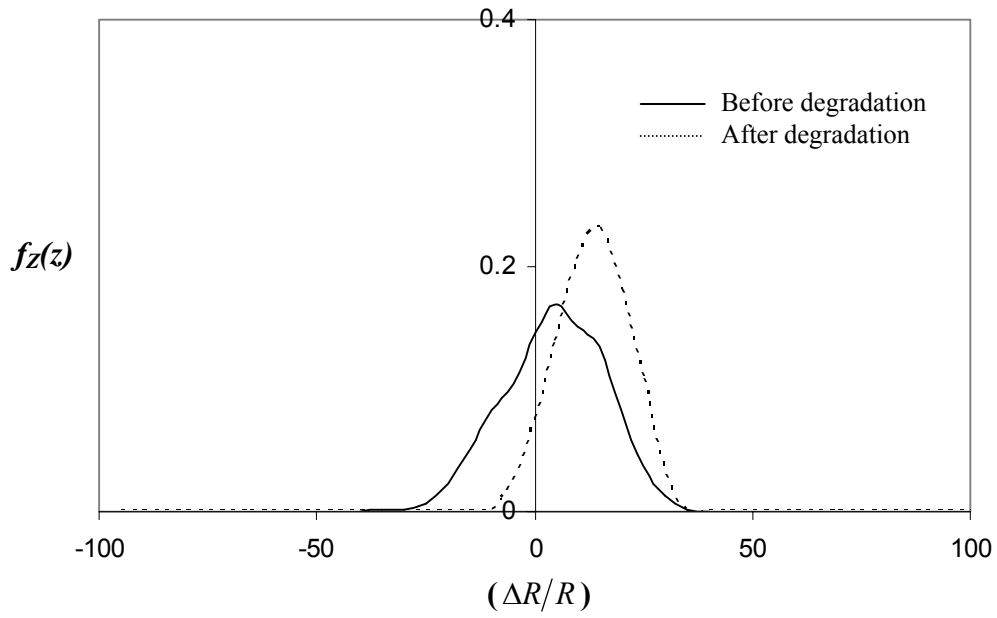


Fig. 5. Probability density functions $f_z(z)$ for 4th storey rigidity

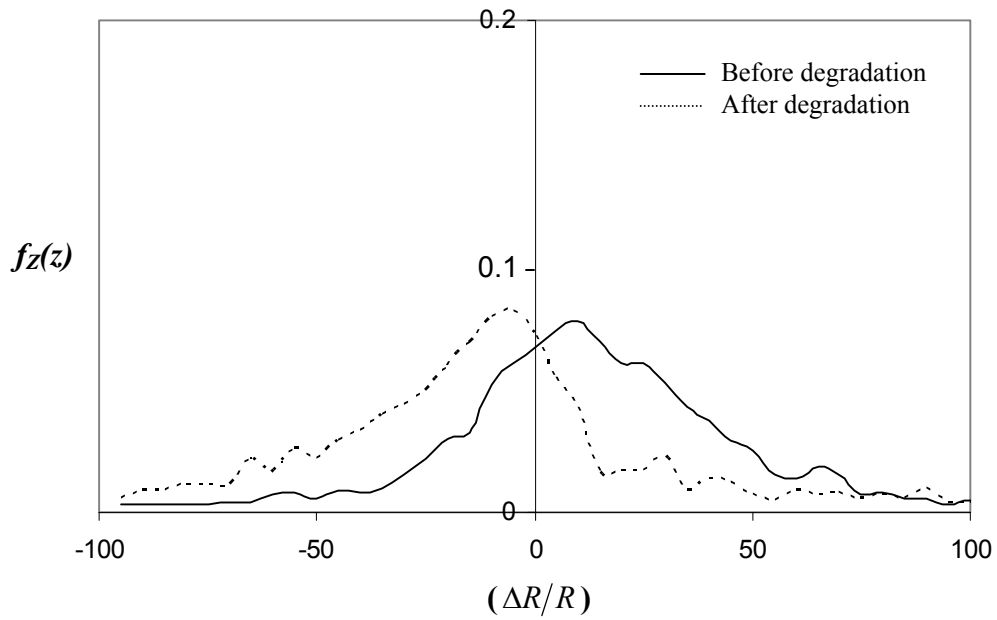


Fig. 6. Probability density functions $f_z(z)$ for 5th storey rigidity

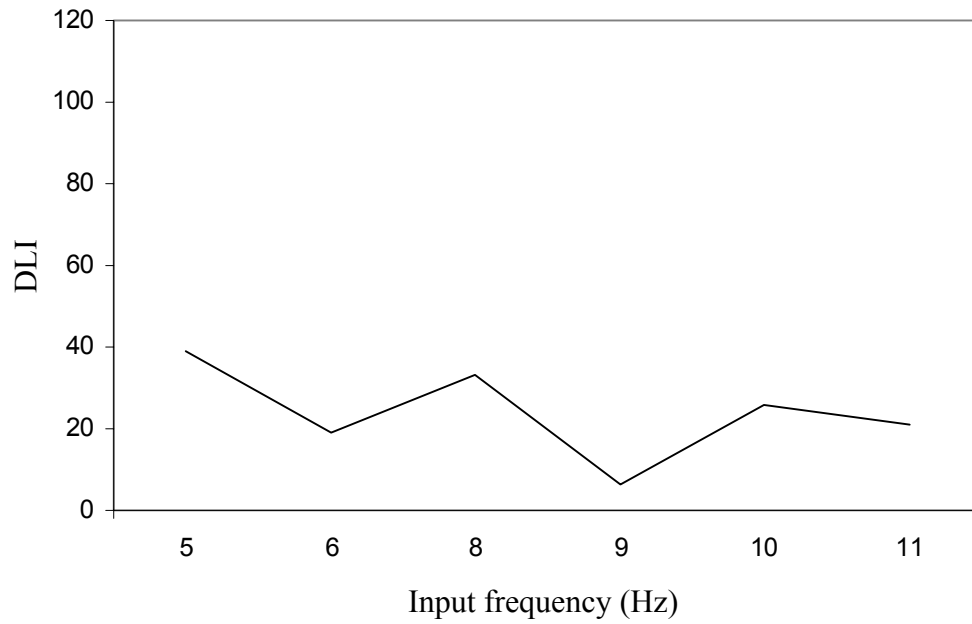


Fig. 7. Degradation likelihood index for 1st storey

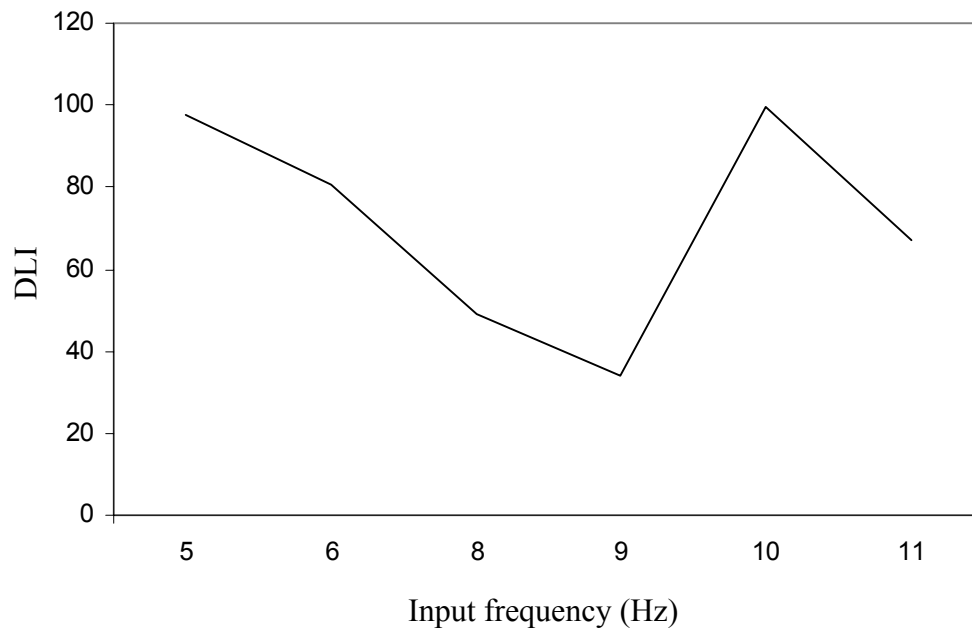


Fig. 8. Degradation Likelihood Index for 2nd Storey

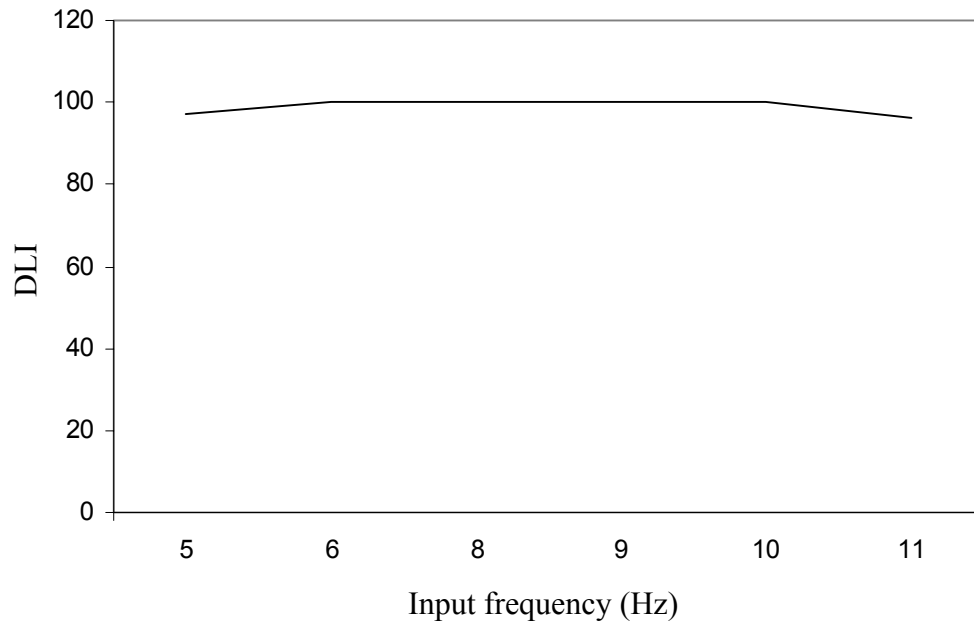


Fig. 9. Degradation Likelihood Index for 3rd Storey

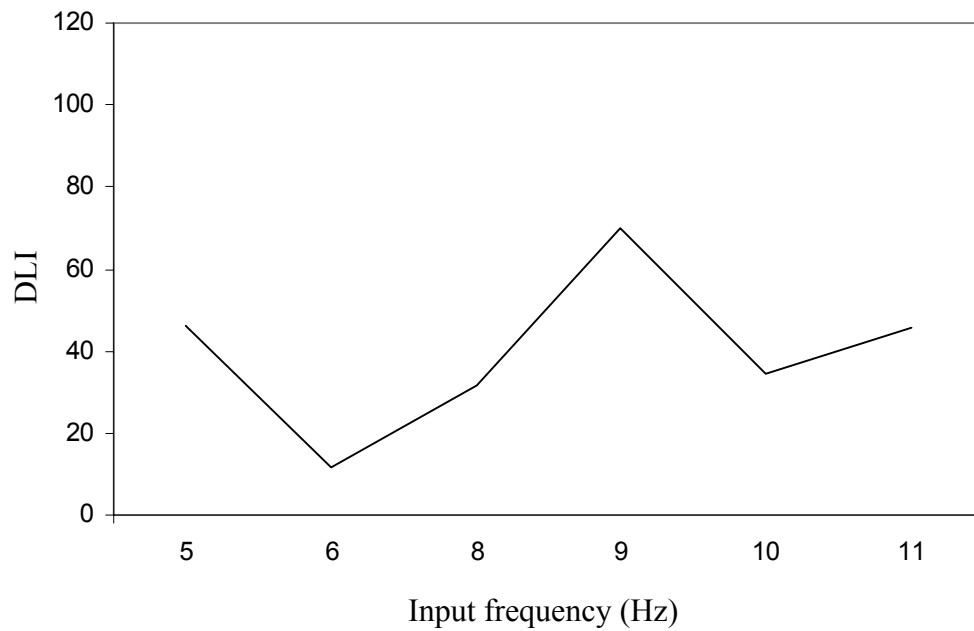


Fig. 10. Degradation Likelihood Index for 4th Storey

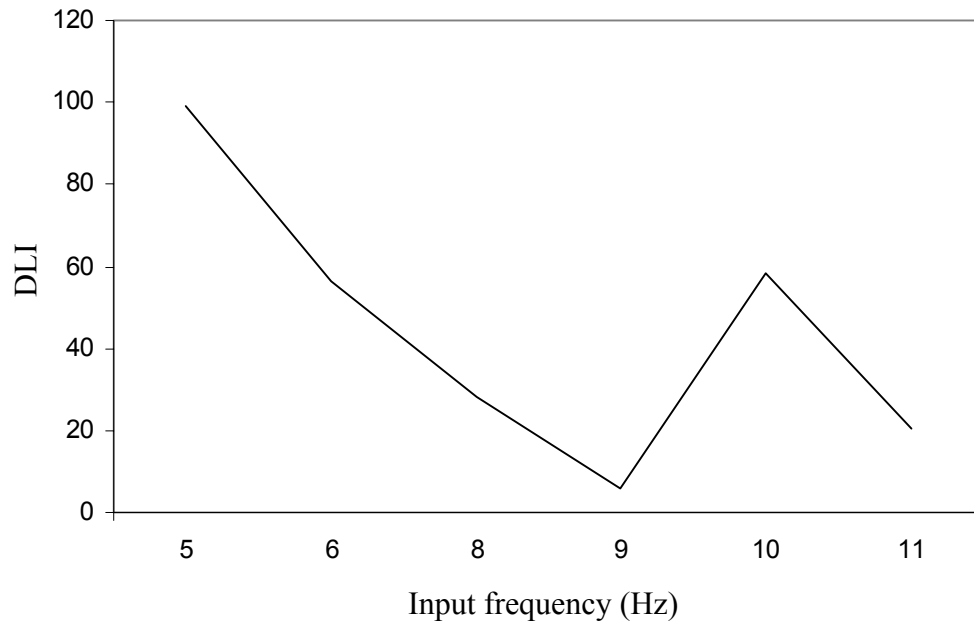


Fig. 11. Degradation Likelihood Index for 5th Storey

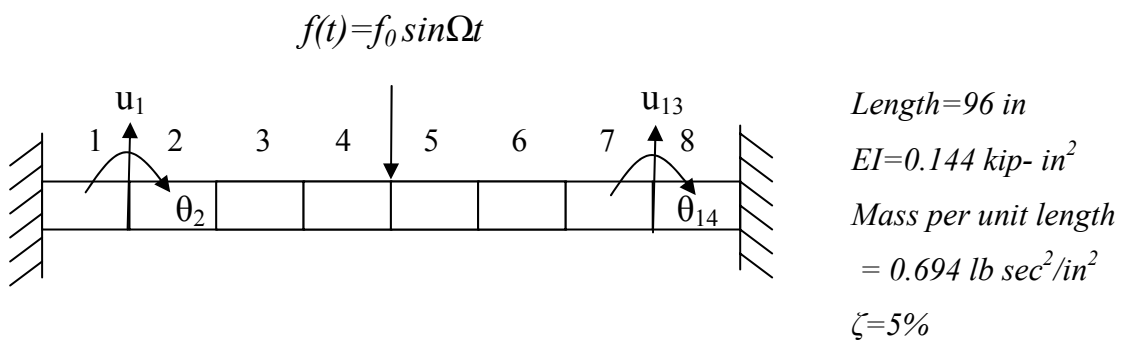


Fig. 12. Fixed-Fixed Beam Considered for Representative Piping System

Appendix A

Pseudoinverse technique using singular value decomposition (SVD)

For non-square matrices such as underdetermined and overdetermined matrix, the unique solution to their inverses does not exist due to their rank-deficiencies. In such cases, the pseudoinverse technique can be used to obtain the optimal solution to non-square matrices. The pseudoinverse technique using SVD is widely used to map the dependent vector spaces into the independent vector spaces by using eigenvalues and eigenvector of the original matrix. Let us consider $m \times n$ non-square matrix $[T]$ and the matrix $[T]$ can be decomposed into three matrices.

$$[T] = [U][\Lambda][V]^T \quad (\text{A-1})$$

where, $[U]$ and $[V]$ are $m \times m$ matrix of eigenvectors of $[T][T]^T$ and $n \times n$ matrix of $[T]^T[T]$, respectively and $[\Lambda]$ is $m \times n$ matrix of eigenvalues obtained by combining each matrix of eigenvalues for $[T][T]^T$ and $[T]^T[T]$. Eigenvalue matrix $[\Lambda]$ can be written as

$$[\Lambda] = \begin{bmatrix} \Lambda_{11} & & \dots & & \Lambda_{1m} \\ & \ddots & & & \\ \vdots & & & \Lambda_{ij} & \vdots \\ \vdots & & & & \vdots \\ \Lambda_{n1} & & \dots & & \Lambda_{nm} \end{bmatrix}_{m \times n}$$

$$\Lambda_{ij} = \begin{cases} \sigma_{ii} & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$\sigma_{11} \geq \sigma_{22} \geq \dots \geq \sigma_{mm} \geq 0 \quad \text{if } m < n$$

(A-2)

where, eigenvalue matrix $[\Lambda]$ contains r non-zero eigenvalues corresponding to the minimum rank of $[T][T]^T$ and $[T]^T[T]$ ($r \leq m$, if $m < n$). Since each of $[U]$ and $[V]$ is orthogonal, their orthogonal properties can be expressed as

$$\begin{aligned}
 [U]^T &= [U]^{-1} & \text{and} & & [U][U]^T &= [U]^T [U] = [I] \\
 [V]^T &= [V]^{-1} & \text{and} & & [V][V]^T &= [V]^T [V] = [I]
 \end{aligned}
 \tag{A-3}$$

By taking inverse of the decomposed matrices in Eq. (A-1), the pseudoinverse of $[T]$ can be evaluated by

$$[T^+] = [V]^{-1} [\Lambda]^{-1} [U]^{-T}
 \tag{A-4}$$

Further, using orthogonal properties of eigenvectors in Eq. (A-3), Eq. (A-4) can be simplified as

$$[T^+] = [V][\Lambda]^{-1} [U]^T
 \tag{A-5}$$

PART V

SUMMARY AND CONCLUSIONS

1. Summary and Conclusions

This research addresses the following three problems associated with the design, operation and inspection of power plant structural systems including equipment and piping : (i) Reliability-based load and resistance factor design of piping components, (ii) Consideration of the effects of uncertainties on piping response in a coupled building-piping system analysis, and (iii) Probabilistic framework for condition assessment of piping systems using recorded vibration data. Summary and conclusions for each of these three topics are given below.

(a) Reliability-based load and resistance factor design of piping components

An exploratory case study on the application of the LRFD approach to the Section III of ASME Boiler and Pressure Vessel Code for piping design is presented in this part. The objective is to provide useful input to the similar but more comprehensive study being undertaken by the ASME working group on piping design. While a complete piping system consists of several components such as straight pipes, elbows, branch connections, etc., only a cold straight pipe section is considered in this study. The performance function is defined with respect to a failure mode that is defined by plastic instability. For simplicity, only service level D is considered and the effects of pressure and seismic moment are considered. Since the effect of dead weight is insignificant with respect to the DBE loading in service level D, it is neglected in the present study. As a first step in this process, the presently used design equation that is based on the working stress method of design is calibrated by calculating the minimum reliability levels associated with it for various values of design pressure and the diameter to thickness D/t ratio. It is observed that the minimum reliability index varies between a narrow range of 1.86 and 2.21

when mean design pressure is less than equal to $2P_a$. The D/t ratio has no influence on the minimum reliability levels. It is also shown that the D/t ratio has no influence on the partial safety factors calculated using LRFD approach. Monte Carlo simulation is used to verify the computation of partial safety factors using the First Order Reliability Method. It is illustrated that the total safety factor for the presently used design equation is same as that for a design equation based on the LRFD format in which the target reliability is equal to the minimum reliability of the presently used design equation.

The partial safety factors are calculated for a range of target reliability values. The purpose is to provide consistency with a systems-based approach. In such an approach, a plant wide PRA may be used to identify critical structural systems and components and to allocate the target reliabilities for different piping systems based on the outcome from a Master Logic Diagram (MLD) that is generated using the top-down hierarchical approach. Identification of critical systems and components from such an approach also assists in the appropriate allocation of redundancies for defense in depth. Finally, the sensitivity of the partial safety factors is studied by (i) considering a variability in the values of the coefficient of variation for various random variables, (ii) varying the ratio of mean design pressure to mean design moment, (iii) considering different distribution types for each random variables, and (iv) considering the statistical correlation between specific random variables. The observations made from the sensitivity studies show that corresponding to a specified target reliability level β_0 the safety factors are insensitive to the values of mean stress ratio when the mean design pressure is less than equal to $2P_a$. The load factor γ_M is quite sensitive to the value of coefficient of variation for the moment especially for larger values of β_0 . This is so because a large uncertainty in moment characterized by a higher value of its coefficient of variation governs the design. The effect of

changes in the coefficient of variation for the pressure and the strength is relatively negligible on γ_P and ϕ_S . It is also observed that the load factors can vary significantly for different types of distributions considered. For reliability index values corresponding to those encountered in practical applications, consideration of a lognormal distribution for the moment gives almost similar values of γ_M as those given by the consideration of the Type II extreme value distribution. Significant differences exist for target β_0 values greater than 2.25. Characterization of variables by normal distribution gives higher values for γ_P and ϕ_S but much lower values for γ_M . Consideration of lognormal distribution for all the variables appear to be a reasonable choice. Statistical correlation between the ultimate stress and the section modulus appears to have no influence on loads and resistance factors.

(b) Consideration of uncertainties in seismic analysis of coupled building-piping systems

This study investigates the effect of uncertainties in modal properties of uncoupled primary and secondary systems in the seismic analysis of non-classically damped coupled system. It is shown that uncertainties in the frequencies and modal damping ratios of the two uncoupled systems can result in a significant variation of secondary system response when the frequencies of the two uncoupled systems are tuned or nearly tuned. On the contrary, the effect of uncertainties in the input excitation dominates the variation in secondary system response when the two uncoupled systems are detuned. Therefore, uncertainties in the frequencies and modal damping ratios do not have any meaningful influence on the secondary system response in detuned systems. While multiple time history analyses may be conducted in a Monte Carlo simulation to evaluate the design response corresponding to 84% non-exceedence probability, it is shown that a consideration of 84% non-exceedence probability in a Monte Carlo simulation

with multiple response spectrum analyses gives excessively high values for tuned primary – secondary systems. This is so because the earthquake input is characterized in terms of a single design spectrum that in itself corresponds to 84% NEP. Consequently, the multiple analyses performed by sampling only the modal properties give excessively high values when the design response is defined at 84% NEP. First-order reliability method is used to evaluate the non-exceedence probability values at which the individual modal responses should be defined in order to evaluate a combined response that is close to the true design response as obtained from a Monte Carlo simulation with multiple time history analyses. Results of this study show that the modal responses need to be evaluated at 84% NEP in systems that have perfectly correlated modes ($\varepsilon_{12} = 1$) and in systems where only a single mode contributes to the particular response quantity of interest. In other cases, characterizing modal responses at 84% NEP would give design response that is excessively higher than its true value. While FORM gives relatively accurate response, the results of first-order reliability method cannot be directly included in a response spectrum method due to computational complexity and inefficiency. Therefore, simplified methods based on total probability theorem are presented for this purpose. It is shown that the numerical results obtained from the simplified methods are very close to the true values of design responses.

(c) Probabilistic framework for condition assessment of piping

This study presents a modified probabilistic framework for structural condition assessment by using the frequency and mode shape data before and after degradation. Unlike existing studies, our work is based purely on Monte Carlo simulation. The modal properties are used to analytically evaluate the structural responses by considering a sine sweep type of

harmonic excitation. In an existing framework proposed by Papadopoulos and Garcia (1998) and later modified by Xia et al (2002) the degradation likelihood is expressed in terms of the probability that the reduction in rigidity lies within a specified confidence interval. It is shown that this characterization may or may not be sufficient in identifying elements with significant degradation especially if the probability density functions are not Gaussian. We propose a new degradation likelihood index, DLI, that characterizes the likelihood of degradation by evaluating correlation between the probability density functions of rigidities before and after degradation. We use two simple systems to evaluate the consistency and validity of the proposed framework. We also use these systems to conduct a sensitivity analysis. While the results for these simple systems show considerable promise, they are not overwhelmingly attractive. Additional work is needed to evaluate the consistency of the proposed approach. It may also be possible to define the DLI based on a derived response quality such as element strain energy rather than directly using the rigidity values. Such exploration may be necessary to improve the quality and consistency of the proposed approach.

2. Recommendations for Future Research

Based on the conclusions reached in this study, the following recommendations are made for future research:

- It is desirable to study the application to LRFD approach to piping design for additional failure modes and loading conditions.
- In a coupled primary-secondary system analysis, the proposed simplified methods for evaluating the response corresponding to 84% NEP employ Monte Carlo simulation within

the response spectrum method. It is recommended to further simplify the proposed formulations by using closed-form solutions.

- The probabilistic approach for evaluating likelihood of degradation is applied to two simple systems. It is recommended that additional cases be considered to evaluate its validity and consistency.
- It is recommended that additional parameters that are based on derived response quantities such as strain energies be considered to improve the quality of results in the proposed probabilistic framework for structural condition assessment.
- It is also recommended that the proposed framework for structural condition assessment be evaluated by considering experimental data.

References

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