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RELIABILITY ANALYSIS OF PRESTRESSED CONCRETE CONTAINMENT STRUCTURES

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ABSTRACT

The reliability analysis of prestressed concrete containment structures subjected to combinations of static and dynamic loads with consideration of uncertainties of structural and load parameters is presented. Limit state probalities for given parameters are calculated using the procedure developed at BNL, while that with consideration of parameter uncertainties are calculated by a fast intergration for time variant structural reliability. The limit state surface of the prestressed concrete containment is constructed directly incorporating the prestress. The sensitivities of the Choleskey decomposition matrix and the natural vibration character are calculated by simplified procedures.

1. INTRODUCTION

The containment structure is an important nuclear safety–related structure and its reliability evaluation is of high interest. Recently a reliability analysis methodology of structures especially of containment structures based on probability is developed at Brookhaven National Labortory(BNL)[1]. However in BNL's procedure only the randomnees of time invariant loadings and the uncertainties due to random time histories of time variant loadings are considered. In this study a fast intergration for time variant structural reliability[3][4] is used for including uncertainties of structural and time variant loading's parameters. The limit state analysis and limit state probability calculation under given parameters are conducted following the BNL's procedure. But the structural analysis is conducted using the semi–analytical finite ring element method and the stochastic seismic response is analysed by stochastic modal decomposition. The earthquake loadings are specified by response spectra and the design response spectra of NGR No.1.60, USNRC, are used. The limit state surface of the prestressed concrete containment is constructed[2].

The containment consists of a circular cylindrical wall with a hemispherical dome on the top, as shown in Fig.1. The prestressing tendon quantity is 34.41 cm^2per meter in meridional direction and 59.86 cm^2in hoop direction. The effective prestress factor is taken to be 0.60. The finite ring element model for the containment is shown in Fig.2. Four loads are considered in this study. They are: the weight load (D / L), the accidental internal pressure (P), the tornado load (W) and the earthquake acceleration (E). The limit state probabilities for three load combinations: (D / L+P), (D / L+W) and (D / L+E) are calculated. In this paper only the limit state probability for (D / L+E) is presented.

2. LIMIT STATE

In prestressed concrete containment the concrete cross–section has subjected a initial stress induced by the unbonded prestressing tendons, which brings difficulties in construction of limit state surface. BNL's procedure uses the limit state equations of the RC containment and treats the prestress as an equivalent variation in temperature in the truss elements modeling the prestressing tendons. In this study the limit state surface is constructed directly incorporating the prestress.

The limit state is defined as the prestressing tendons initial to yield (in tension or compression) and / or the crushing strength of the concrete at the cross–section's extreme fibers is reached. The limit state surface in term of the menberance stress component τ and the corresponding bending moment component m may be shown as in Fig.3. Let a_k be the effective prestressd factor, then the tensile stress in prestressing tendons is $a_k f_y$, and the compressive stress in concrete is $-2a_k a_s f_y / b$, here a_s is the prestressing tendon area in width b of the cross–section. The coordinates of points in the limit state surface can be calculated from the stain state of the corresponding points[2]. For example, for point f, the cross–section becomes fully cracked. The tendons in one side yield and the stress at another side is zero. For point e, i.e. the balance point, the cross–section partially cracks. The tendons in tension yield and the compressive stress of concrete at another side reaches crushing strength f_{cm}. The strain state for these cases is shown as in Fig.4.

For point f, from the strain state we obtain

$$\tau_f = \frac{a_s}{b}(x_s + f_s) \tag{1}$$

$$m_f = \frac{a_s}{b}(x_s - f_s)(\frac{t}{2} - t_0) \tag{2}$$

in which

$$f_s = \frac{x_c n}{x}(1 - k_m + x)$$

$$x_s = (1 - a_k) f_y$$

$$x = \frac{n x_c k_m}{x_s - n x_c}$$

$$x_c = \frac{2 a_k a_s f_y}{bt} \quad , \quad k_m = \frac{t - t_0}{t} \quad , \quad n = \frac{E_s}{E_c}$$

Specially when $a_k = 0$, eqs(1) and (2) reduce to that of RC containment.

3. FAST INTERGRATION FOR TIME VARIANT STRUCTURAL RELIABILITY

For time variant structural reliability, the probability of failure can be expressed as

$$p_f = \int_{G(\mathbf{X},\mathbf{R}) \leq 0} f_{\mathbf{X},\mathbf{R}}(\mathbf{x},\mathbf{r}) d\mathbf{x} d\mathbf{r} \tag{3}$$

in which, $G(\mathbf{X},\mathbf{R})$is a performance function, $\mathbf{X}$ denote all parameters in structure and loadings, R is a vector of random response variables. In order to carry out the intergration in eq.(3) by first–order method, Wen[3] constructs a performance function directly in transformed standard nomal space:

$$G_u(\mathbf{U}, U_{n+1}) = U_{n+1} - \Phi^{-1}\{p_f[T^{-1}(\mathbf{U})]\} \tag{4}$$

in which, U_{n+1} is an auxiliary standard variate independent of $\mathbf{U}$, $T^{-1}(\mathbf{U})$are inverse Rosenblatt transformation.

If we start from a general reliability formaula:

$$p_f = \int_{\mathbf{x}} P_f(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} \tag{5}$$

in which, $p_f(\mathbf{x})$is the conditional probability of failure for given $\mathbf{X}=\mathbf{x}$, f(x) is the joint pdf of X, then it can be shown that one can use a general performance function[4]:

$$G(\mathbf{X}, X_{n+1}) = X_{n+1} \tag{6}$$

to calculate the intergration in eq.(5). Here X_{n+1} is an auxiliary variable, which may be of arbitrary distribution as long as it satisfies

$$F_{x_{n+1}}(0|\mathbf{x}) = p_f(\mathbf{x}) \tag{7}$$

The gradient of G at the point on failure surface can be obtained with the aid of Rosenblatt transformation:

$$\{\frac{\partial X_{n+1}}{\partial u_i}\}_{i \leqslant n} = -[J_n^{-1}]^T \{\frac{\partial p_f(\mathbf{x})}{\partial x_i}\}_{i \leqslant n} \tag{8}$$

$$\frac{\partial X_{n+1}}{\partial u_{n+1}} = \Phi(u_{n+1}) \tag{9}$$

where

$$u_{n+1} = \Phi^{-1}[p_f(\mathbf{x})] \tag{10}$$

J^{-1} is the inverse of Jacobian matrix.

This method can also be used for a general reliability problem including uncertainties of parameters.

4. SENSITIVITY ANALYSIS

In the reliability analysis the sensitivity analysis involves complicated operations of arithmetic. In this study the calculation of the sensitivities of Choleskey decomposition matrix and natural vibratrion character are conducted by simplified procedures:

In general the Choleskey decomposition matrix can be expressed as

$$[A] = [L] \ [L]^T \tag{11}$$

in which, $[A]$ is a symmestrical positive definite matrix, $[L]$ is a triangular matrix. The problem is knowing $\partial[A]/\partial p$, how to obtain $\partial[L]/\partial p$, here p is an arbitrary parameter. From eq(11), we have

$$\frac{\partial[A]}{\partial p} = [B] + [B]^T \tag{12}$$

where

$$[B] = \frac{\partial[L]}{\partial p} [L]^T \tag{13}$$

Denoting $\partial[L]/\partial p$ by $[L_p]$, $\partial[A]/\partial p$ by $[a_p]$, then it can be shown that

$$L_p(i,j) = \frac{a_p(j,i) - b(j,i) - \sum_{r=1}^{j-1} L_p(i,r) L(j,r)}{L(i,j)} \qquad j \leqslant i \tag{14}$$

$$L_p(i,i)=\frac{\frac{1}{2}a_p(i,i)-\sum_{r=1}^{i-1}L_p(i,r)L(i,r)}{L(i,i)}\qquad j=i \tag{15}$$

$$L_p(i,j)=0\qquad j>i \tag{16}$$

$$b(i,j)=\sum_{r=1}^{i}L_p(i,r)l(j,r)\qquad j\geqslant i \tag{17}$$

Eqs.(14)–(17) are a series of recurrence formulas and can be used to calculate all the elements of $\partial[L]/\partial p$.

It is well known that the sensitivity of engenvalue is easy to calculate, but that of engenvector is not so easy. According to modal decomposition, the response {u} can be expressed as

$$\{u\}=\sum_i F_i\{x_i\}Y_i \tag{18}$$

in which, F_i is the modal participation factor, Y_i is the normal coordinate, X_i is the modal shape. From eq.(18) we obtain

$$\frac{\partial\{u\}}{\partial p}=[A]+\sum_i F_i\{x_i\}\frac{\partial Y_i}{\partial\omega_i}\frac{1}{2\omega_i}\frac{\partial\lambda_i}{\partial p} \tag{19}$$

in which

$$\{A\}=\sum_i\{\frac{\partial F_i}{\partial p}\{x_i\}Y_i+F_i\frac{\partial\{x_i\}}{\partial p}Y_i\} \tag{20}$$

λ_i and ω_i are the engenvalue and natural freqency respectively.

It is shown that for the the shear building and containment structures term {A} in eq.(19) can be neglected in comparison with the second term. That is the sensitivity of seismic response can be approximated by neglecting the sensitivities of the mode shape and modal participation factor.

5. LIMIT STATE PROBABILITY

Using the fast intergration described above, the limit state probability of the prestressed concrete containment with consideration of parameter's uncertainties was analyzed. The unceratinties of seven structural parameters are included as shown in Table 1. These parameters are the Young's modulus E_c, Poisson's tatio ν, mass density ρ and the bending compressive strength f_{cm} of concrete, the thickness of the shell t, the damping ratio ξ and the yielding strength of the prestressing tendon f_y. As for the exitation characteristics the peak acceleration a_g, duration D and standard response spectra Z_a of earthquake are considered. The pdf of duration D is taken as[2]

$$f_D(d)=0.139d^{-0.222}exp(-0.0989d) \tag{21}$$

The spectra Z_a fit the lognormal distribution and the variability coefficient may be taken as 0.26 for the natural frequency f less than 12.5Hz, 0.0127(33−f) for 12.5<f<33Hz and 0 when f is larger than 33 Hz[2].

The initial parameter values for iteration are shown in Table 2.

It is found that the critical element is located at the lowest place and the limit state is reached as the meridional prestressing tendons begin to yield with the cross–section partially cracked. The overall limit state probability of the containment in 40 year lifetime under the combination of weight load and earthquake acceleration with consideration of parameter's uncertainties is $4.60 \times 10^{-5} \sim 1.517 \times 10^{-4}$, while that without consideration of parameter's uncertainties is $9.65 \times 10^{-6} \sim 1.83 \times 10^{-5}$. It shows that the parameter's unceratinties play a major role in the reliability of the containments.

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Table 1 Uncertainties of Parameters

Parameter	Distribution	Mean value	Variability
E_c	Normal	33000 N / mm^2	0.1
ν	Normal	0.2	0.2
ρ	Normal	2.5g / cm^2	0.02
t	Normal	100cm	0.01
f_{cm}	Normal	29.8 N / cm^2	0.15
f_y	Lognormal	1200 N / mm^2	0.03
ξ	Normal	0.05	0.60

Table 2 Design Point

Parameter	Initial Value	Design Point	Sensitivity
E_c	32340	31982	-0.565×10^{-4}
ν	0.2	0.20051	0.893761
ρ	2.5	2.5018	2.01717
t	100	100.0104	0.029188
f_{cm}	29.8	29.774	−0.00218
f_y	1200	1189.84	−0.002435
ξ	0.05	0.04169	−25.7889
a_g	500	707.384	0.008624
Z_a	2.07	2.2079	2.10486
D	7.87	4.9571	0.003721

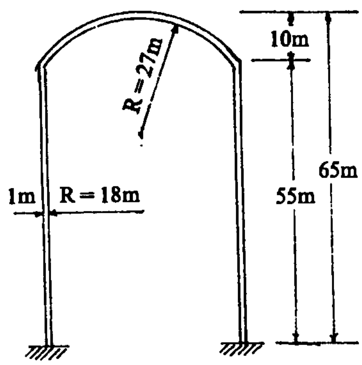

Fig.1 Containment Structure

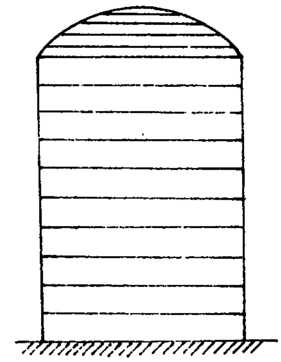

Fig.2 Finite Ring Element Model

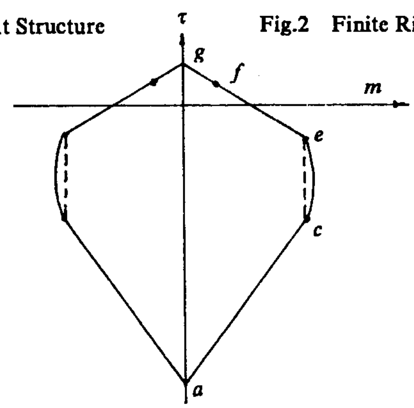

Fig.3 Limit State Surface

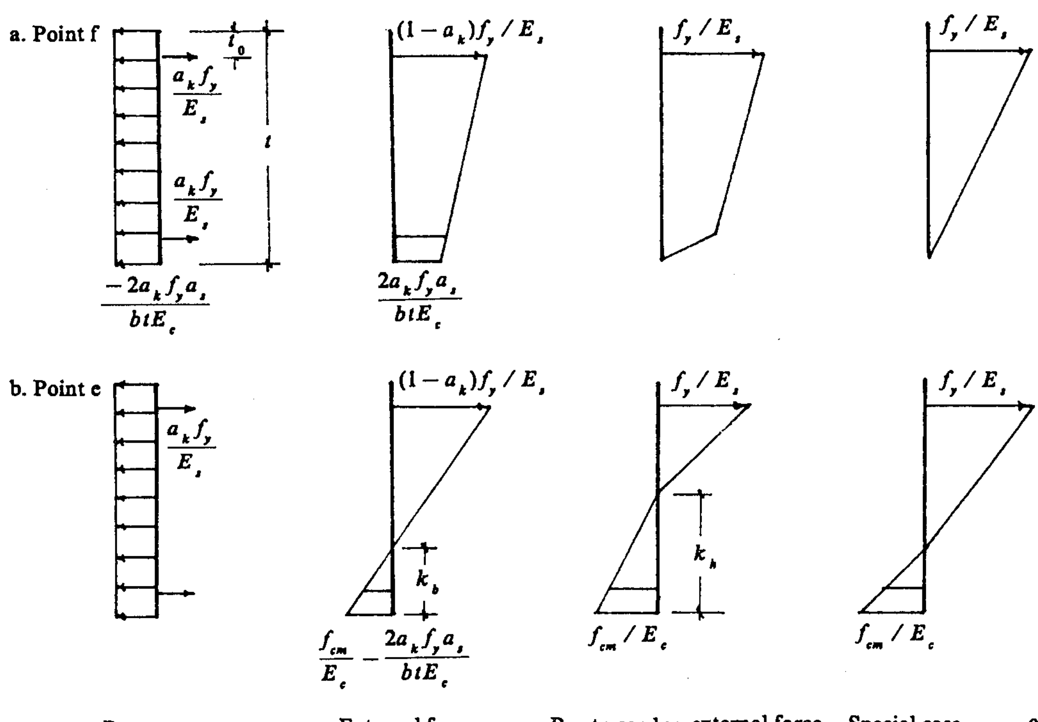

Fig.4 Strain state for the points on limit state surface