INELASTIC THERMODYNAMICALLY COUPLED RESPONSE AND EVALUATION OF TEMPORAL OPERATORS

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SUMMARY

This paper examines the numerical analysis of inelastic thermodynamically coupled structures as related to its solution stability and accuracy. The formulation is based on finite elements via first and second laws of thermodynamics. For clarity of the analysis, two-dimensional example problems are shown.

If an equation of motion is coupled with that of heat conduction, then the increase in temperature contributes to deformation whereas mechanical deformation gives rise to heat generation. Numerical stability involved in the equation of motion has been the subject of study for many years. However, if heat conduction is coupled with mechanical deformations, the stability criteria alter significantly, with the mechanical and thermal waves propagating at different speeds. These phenomena become more complicated if the material is inelastic as a function of time. Viscoelastic or damping properties contribute to additional difficulties in stability and convergence.

In the previous studies, the finite element analysis for inelastic thermodynamically-coupled structures has been demonstrated. Applying the 3rd order Taylor series expansion in explicit or implicit scheme, it can be shown that the coupled global finite element equations take the form

$$\begin{bmatrix} A_{ij} & -J_{i\beta} \\ H_{\alpha j} & N_{\alpha\beta} \end{bmatrix} \begin{bmatrix} u^{n+1} \\ T_\beta^{n+1} \end{bmatrix} = \begin{bmatrix} F_i \\ P_\alpha \end{bmatrix}^{n,n-1,n-2} \tag{1}$$

where u_i^{n+1} and T_β^{n+1} denote the nodal displacements and temperatures at the $n+1$th time step and F_i and P_α refer to the inelastic properties and response histories, as well as the input information.

To ensure accuracy and stability of simultaneous solution of (1), we derive the stability criterion in the form

$$|g_{\alpha\beta}| = |W_{\alpha\gamma}^{-1} Q_{\gamma\beta}| \leq 1$$

where $W_{\alpha\gamma}$ represents the square matrix on the left-hand side of (1) and $Q_{\gamma\beta}$ denotes the thermodynamic response histories.

From these results, we note that the temporal operator of the initially explicit or implicit scheme applied to the coupled equation of motion and heat conduction causes the resulting form to be always implicit. Various material constants such as specific heat and thermal conductivity coefficient, viscosity constant, etc., influence significantly the amplification factor $g_{\alpha\beta}$.

Our study includes other forms of temporal operators and establishes relationships between the limiting Δt and various thermomechanical constants in order to achieve accurate and stable solutions.

In summary, direct simultaneous solutions of equations of motion and heat conduction are more efficient than solving them separately. This is made possible via selecting suitable temporal operators and the limiting integration time increments. The present study shows that the 3rd order Taylor series expansion is most effective in comparison with other temporal operators.