UNSTEADY THERMAL STRESSES IN A FINITE SHORT CIRCULAR CYLINDER WITH AN ARBITRARY HEAT SUPPLY

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SUMMARY

A number of axisymmetrical thermoelastic problems in an infinite circular cylinder have been considered and also several investigations in a finite cylinder as to the end-effect problems have been published. In most works, however, the consideration of heating or heat supply is limited to a particular case. An attempt has here been made to the problems of a finite short cylinder due to an arbitrary heat supply in axial direction, that is a case of fairly general boundary condition. An exact solution is given for the thermal stress distribution in radial and axial direction of solid circular cylinder. In case of asymmetrical heating, it is obviously seen that the boundary value problem becomes pretty complex because of the necessity of considering the more general form of the Love's function which has not been used so far. The analysis is developed by the integral transforms with the method of thermoelastic potential and Love's function. Numerical results in the form of curves are given for typical cases. It will be seen that asymmetrical heat supply can be divided into a symmetrical part and an anti-symmetrical one. Then we consider both problems separately. Hence the required solutions of temperature and stress distribution can evidently be obtained as the sum of both effects. By the method of separation of variables, the general solution of Love's function can be written as

$$L^s/2GK = A_0 z^3/6 + C_0 zr^2/2 + \sum \{A_n I_0(k_n r) + B_n k_n r\, I_1(k_n r)\} \sin k_n z/k_n^3 +$$
$$+ \sum \{C_s \sinh \lambda_s z + D_s \lambda_s z \cosh \lambda_s z\} J_0(\lambda_s r)/\lambda_s^3 +$$
$$+ \sum \{E_m I_0(\beta_m r) + F_m \beta_m r\, I_1(\beta_m r)\} \sin \beta_m z/\beta_m^3 .$$

Above expression has to be used for the case of symmetrical problem only. Anti-symmetrical problem needs to have another form. For simplicity we show only a symmetrical solution.

The numerical results are presented for various ratios of length per radius (=0.5, 1.0, 2.0 and 3.0).

From our results the end-effect evidently appears in the finite cylinder with large influence for smaller ratios of length/radius.