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THE APPLICATION OF LARGE USER-ORIENTATED SYSTEMS OF PROGRAMS TO THE SOLUTION OF NON-LINEAR AND DYNAMIC PROBLEMS

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SUMMARY

The application of the user-orientated and modular Structural Analysis Systems offers many advantages when solving complex and costly non-linear and dynamics problems and could also have a significant impact on development of new models in the system.

For the solution of creep and plasticity problems the CEGB system of structural programs contains at present six different flow rules covering the creep behaviour of metals, viscoelastic materials and plastic deformation in two dimensions. Two alternative finite element solution techniques, the direct-band approach and an iterative procedure, are available. In the solution of dynamic problems a modified front solution combined with nodal condensation is used and various finite elements for two and three dimensional solid bodies, plates, beams and pipe structures are available. In general various models and solution techniques are available in various programs in the system, but all programs use a standardized input/output and a common library, to store data and results, on 3330 magnetic disks.

The elastic and creep analysis of the stainless steel guide tube, a part of 2 × 660 MW(e) Advanced Gas-cooled Reactor Hartlepool Power Station has been performed. The guide tube with inside diameter 10.25″ and thickness 0.16″ is assumed to be under external pressure 40 p.s.i.a. and heated to a uniform temperature of 600°C. Material properties for Type 321 steel have been used in the analysis. Time hardening flow rule is assumed and two alternative finite element solution techniques have been applied, direct approach using TESS program and iterative procedure using STAG program. The dynamic behaviour of the same guide tube is examined using programs DYSAFE and BERDYNE for natural and forced vibrations. The results are presented and the interaction of design criteria and maximum stresses from both analyses examined. The non-linear and dynamic behaviour of a mild steel guide tube with inside diameter 3.75″ and thickness 0.18″, a part of 585 MW(e) Trawsfynydd Nuclear Power Station under different loading conditions is examined in the same way as the Hartlepool guide tube. Some representative results are shown. In both cases the possibility of failure of the tube during the working life of the power station due to creep collapse, dynamic loading or combined effects is discussed.

The solution of some non-linear stress problems including the effects of creep and plasticity of fast reactor components is examined using the improved TESS program.

In the presented analyses various models and solution techniques and different finite element types have been used in running a number of programs available in the system. The advantages of the system approach to an engineer-designer solving complex non-linear and dynamics problems are presented from the point of the selection of the optimum model and solution technique, data management, the combined running of various programs and computer economy.

The impact of the system approach on further non-linear and dynamic research work is examined and some recent advances in these fields at the Central Electricity Generating Board are discussed.

1. Introduction

The structural evaluation of nuclear power plant components requires increasingly more complex analysis considering combined thermal and mechanical loading conditions. Typical examples are the reactor components of AGR, HTR and CFR at high temperatures which under thermal and mechanical loading conditions have to be examined for creep, plasticity and dynamic effects in steady state and transient conditions.

The paper presents examples of combined elastic-creep and dynamic analysis of AGR Power Station components using various programs linked into a system network described more fully by Jezernik and Miller [1]. The elastic-creep and dynamic analysis of the stainless steel guide tube, a part of 2 x 660 MW(e) AGR Hartlepool Power Station and dynamic analysis of a mild steel guide tube a part of 585 MW(e) Trawsfynydd Nuclear Power Station have been performed. Recent advances at CEGB in developing programs for dynamic analysis of linear elastic structures are described (see Leech [2], [3]. Some results of analyses are presented and conclusions summarized.

A more detailed review of results and of the advantages of using system facilities to an engineer-designer solving complex non-linear and dynamics problems from the point of the selection of the optimum model and solution technique, data management, the combined running of various programs and computer economy, will be given in the final paper. Some recent developments of non-linear models including the combined effects of creep and plasticity in the improved version of TESS program to be used e.g. for analyses of fast reactor components will be examined.

2. The Basic Theory and Computer Programs

2.1 Nonlinear Analysis

In the elastic and non-elastic stress analysis of structures by the finite element displacement method a set of simultaneous algebraic equations

$$\{R\} - \{\{R^t\} + \{R^c\} + - \ldots\} = [K].\{\partial\} \qquad \text{eq. (1a)}$$

or

$$\{\partial\} = [K]^{-1} \{\{R\} - \{R^t\} - \{R^c\} - \ldots\} \qquad \text{eq. (1b)}$$

is obtained and have to be solved for the displacements in terms of the nodal forces for each elastic case or at each time step for creep and/or plastic calculation. In the equations above $[K]$ is the structural stiffness matrix, $\{\partial\}$ is a column vector of unknown generalised displacements at nodal points and $\{R\}$, $\{R^t\}$, $\{R^c\}$etc. are the corresponding column vectors of applied loads and non-linear thermal, creep or other effects. For non-linear problems the system of equations (1b) is solved stepwise in time and different flow rules considering creep, plastic or creep-plastic effects for different materials are normally used. One of the approaches widely used in the solution of non-linear problems is the so called initial strain approach. This approach is based on the idea of a repeated use of equation (1b) and proceeding with step-wise solutions in time. The TESS suite of programs and program STAG21 have been used for elastic, creep, plastic and viscoelastic analyses to solve a number of engineering problems in recent years (see for example Jezernik and Leech 4).

2.2 Dynamic Analysis

In dynamic calculations $\{R\}$ and $\{\partial\}$ in eq. (1b) are functions of time and inertia and damping effects occur. For the special case of undamped systems the equation of motion is as follows:-

$$\{R\} = [K]\,\{\partial\} + [M]\,\{\ddot{\partial}\} \qquad \text{eq. (2)}$$

where M is the mass matrix of the structure. For the special case of a sinusoidal applied force in the steady state

$$\{R\} = \{R_o\}\ \sin \omega t \qquad \text{eq. (3)}$$

Equation (2) can be transformed into a form similar to equation (1):

$$\{R_o\} = ([K] - \omega^2\,[M])\,\{\partial\} \qquad \text{eq. (4)}$$

The equation (4) is used to modify the static analysis program BERSAFE (see Hellen 5) and the undamped dynamic response can be calculated. The modified program is called DYSAFE. All forces, stresses and displacements in the program are the amplitudes of sinusoidally varying quantities. In the BERDYNE program the equation (2) is again solved but $\{\partial\}$ is no longer sinusoidal.

We first solve

$$[K]\,\{\partial\} = \omega^2\,[M]\,\{\partial\} \qquad \text{eq. (5)}$$

using an eigenvalue routine to give an estimate of the natural frequencies w_m and corresponding mode shapes $\{\partial_n\}$ where n is the number of natural frequencies. Since the eigenvalue routine can deal with at most 200 values the matrices $[K]$ and $[M]$ must be reduced in size. In BERDYNE we use the distributed kinetic energy but retain only a small proportion of the unknown degrees of freedom known as "masters". The remaining 'slave' degrees of freedom take values giving least strain energy.

It is possible to set up a matrix $[Q]$ consisting of eigenvector columns obtained from eq. (6) and assume that the response $\{\partial\}$ is represented by the addition of proportions of mode shapes:-

$$\{\partial\} = [Q]\,\{\phi\} \qquad \text{eq. (6)}$$

where $\{\phi\}$ is a vector of generalised displacements.

Equation of motion (2) can be transformed into a set of uncoupled second order ordinary differential equations:

$$\{\ddot{\phi}\} + [\Omega^2]\,\{\phi\} = [\bar{M}]^{-1}\,\{R\} \qquad \text{eq. (7)}$$

where $[\bar{M}] = [Q^T]\,[M]\,[Q]$

$\{\bar{R}\} = [Q^T]\,\{R\}$

$[\bar{K}] = [Q]^T\,[K]\,[Q]$

and $[\Omega^2] = [M]^{-1}\,[K]$

The solution of eq. (7) is of the form

$$\{\partial\} = [Q]\,[C_1]\,[\sin \omega t] + [Q]\,[C_2]\,[\mathrm{Cos}\ \omega t] + [Q]\,[\Omega]^{-1}\,[\bar{M}]^{-1} \int_{t_o}^{t} [\sin(\omega(t-\tau))]\,\{\bar{R}\}\,d\tau \qquad \text{eq. (8)}$$

By differentiating eq. (8) with respect to t we can eliminate the matrices of arbitrary constants $[C_1]$ and $[C_2]$. The eq. (8) is then solved and values of $\{\partial\}$ and $\{\dot{\partial}\}$ calculated at time $t=t_1$. These values are then used to calculate $\{\partial\}$ and $\{\dot{\partial}\}$ at time $t=t_2$ etc. The complementary solution is given by the first two terms of eq. (8) and the particular solution is given by the third term where the integral represent the Duhamel integral. BERDYNE contains a library of these integrals for various types of forcing function.

The equation of motion for a forced damped system corresponding to eq. (2) is as follows:-

$$\{R\} = [K]\{\partial\} + [C]\{\dot{\partial}\} + [M]\{\ddot{\partial}\} \qquad \text{eq. (9)}$$

where $[C]$ is the damping matrix

The eq. (9) is again solved for displacement vector $\{\partial\}$. In BERDYNE three types of damping are considered: damping proportional to mass, to stiffness or expressed as percentage of critical damping.

3. Results and Summary

The elastic-creep and dynamic analysis of the stainless steel guide tube a part of 2X 660MW(e) Advanced Gas-Cooled Reactor Hartlepool Power Station has been performed. The guide tube with inside diameter 10.25" and thickness 0.16" is assumed to be under external pressure 40 psia and heated to a uniform temperature of 600°C. Material properties for Type 321 steel have been used in the analyses. In the non-linear analysis time hardening flow rule is assumed and two alternative finite element solution techniques have been applied, direct approach using TESS program and iterative procedure using STAG program. The dynamic behaviour of the same guide tube is examined applying a transient dynamic load and using the BERDYNE program. Geometrical, material, and loading date and some results of analyses are presented in Fig. 1, 2 and TABLE I. The results of dynamic analysis in TABLE I are for pressure 40 psia.

The dynamic behaviour of a mild steel guide tube with inside diameter 3.75" and thickness 0.18", a part of 585 MW(e) Trawsfynydd Nuclear Power Station subjected to an exponential type transient load which may arise as a result of a sudden depressurization in a gas-cooled nuclear reactor, is examined using the BERDYNE program the data and some results are presented in Fig. 2 and TABLE II.

Fig. 3 shows the computing time requirements (one time interval) for non-linear and dynamic analysis based on a number of engineering problems solved using CEGB programs TESS, BERSAFE and BERDYNE. It is apparent that both types of analyses require a substantial amount of computing time and a search for efficient solution techniques appears to be feasible. A qualitative comparison of iterative and direct solution techniques for the solution of non-linear problems has already been performed (see Jezernik and Leech 9).

In linear, non-linear and dynamic analysis using CEGB finite element programs BERSAFE, TESS, STAG and DYSAFE-BERDYNE a large amount of data can be used by all programs in a standardized form within a system network which enables any use and choice of the most appropriate module(s). A detailed qualitative examination of the combined analyses and results will be given in the final paper.

4. References

[1] JEZERNIK, A., MILLER, M.C., " Large User-Orientated Systems of Programs for Structural Analysis and Design", Paper M1/2* to be presented at 2nd SMiRT Conference, Berlin, Sept. 10-14, 1973.

[2] LEECH, A., "BERDYNE (Phase 1) a Computer System for the Dynamic Analysis of Structures, CEGB Report MS/C/P295, May 1973.

[3] LEECH, A., "DYSAFE a Computer System for the Simple Dynamic Response of Structures", CEGB Report MS/C/P223, May 1973.

[4] JEZERNIK, A., LEECH, A., "The Comparison of Iterative and Direct Solution Techniques in the Analysis of Time-Dependent Stress Problems, Including Creep, by the Finite Element Method", Paper presented at the Conference on the Mathematics of Finite Elements and Applications, Brussels University, April 18-20, 1972.

[5] HELLEN, T.K., "The Application of the BERSAFE Finite Element System to Nuclear Design Problems", 1st SMiRT Conference, Berlin, Sept. 20-24, 1971.

TABLE I.

Creep and Dynamic Analysis of the Stainless Steel Guide Tube (Fig. 1)

	TESS	STAG21
Average hoop stress (N/cm^2)	≈-900	≈-800

Time t (Int. 1)

	TESS		STAG21	
	Displacements (cm)			
NODAL POINT	x-	y-	x-	y-
1	0.	0.000588	0.0	0.000604
361	-0.000834	0.0	-0.000592	0.0

Time 4000 hours

Differnce between the inward radial displacement of nodal points 361 and 1 during first 4000 hours (cm).

	TESS (automatic adjustment) of time-step)	STAG21 (constant time step)
($R\text{-}DISPL_{361}-R\text{-}DISPL_1$)	≈-0.000010	≈-0.000012

DYSAFE

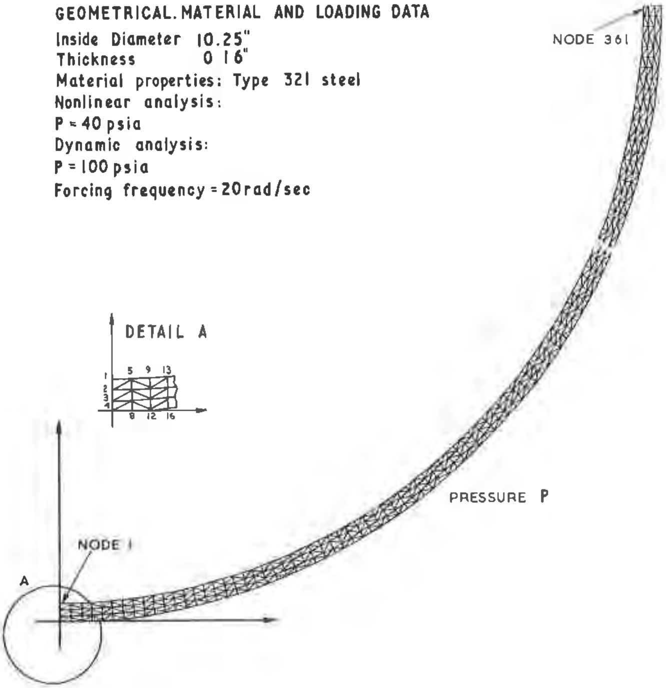
	Displacements (cm)	Stresses (N/cm^2)	
Nodal point	r	r	θ
5	-0.05689	451.732	-65556.5
6	-0.05671	-946.993	-65249.6
7	-0.05655	-1392.28	-64543.7
8	-0.05639	-1162.0	-63924.8

TABLE II

The maximum and minimum values of displacements and stresses is a mild steel guide tube subjected to a harmonic point load. The tube is represented by 46 nodes and 45 beam elements of length 6" with 6 degrees of freedom per node three translations Ux, Uy, Uz and three corresponding rotations θx, θy, θz. The table gives results using Euler beam theory and Timoshenko beam theory with rotary inertia and shear effects included

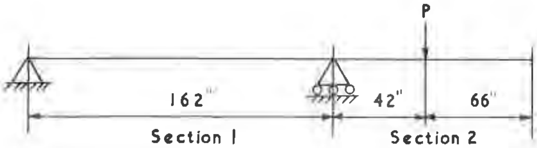
DISPLACEMENTS IN INS. ROTATIONS IN RADIANS STRESSES IN P.S.I.

	MAXIMUM Z DISPLACEMENT AT NODE 46	MINIMUM Z DISPLACEMENT AT NODE 34	MAXIMUM Y ROTATION AT NODE 45	MINIMUM Y ROTATION AT NODE 31
EULER	6.1367×10^{-3}	-2.9850×10^{-2}	7.2015×10^{-4}	-2.2000×10^{-4}
TIMOSHENKO	7.1022×10^{-3}	-3.0860×10^{-2}	7.4833×10^{-4}	-2.2366×10^{-4}
	MAXIMUM Z STRESS AT NODE 45	MINIMUM Z STRESS AT NODE 1	MAXIMUM BENDING MOMENT ABOUT Y AT NODE 35	MINIMUM BENDING MOMENT ABOUT Y AT NODE 28
EULER	9.7000×10	-1.6500×10^{2}	1.8000×10^{4}	-1.1900×10^{4}
TIMOSHENKO	9.9900×10	-1.6300×10^{2}	1.9300×10^{4}	-1.1900×10^{4}



GEOMETRICAL. MATERIAL AND LOADING DATA
Inside Diameter 10.25"
Thickness 0 16"
Material properties: Type 321 steel
Nonlinear analysis:
P = 40 psia
Dynamic analysis:
P = 100 psia
Forcing frequency = 20 rad/sec

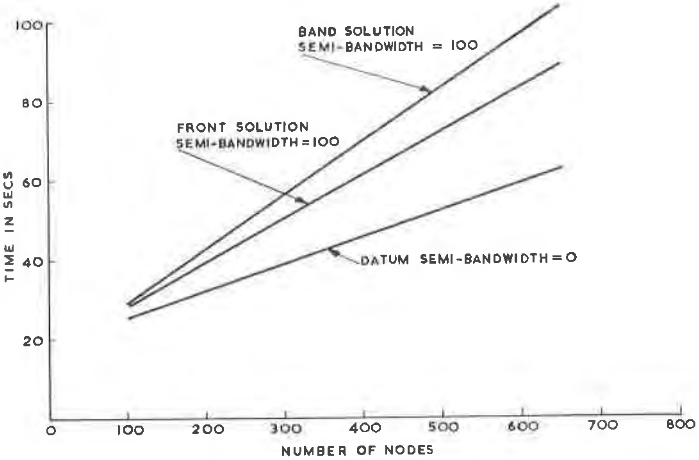
FIG I NONLINEAR AND DYNAMIC ANALYSIS OF GUIDE TUBE IN PLANE STRAIN CONDITIONS



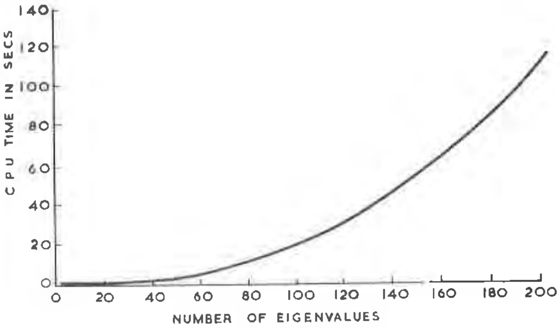
GEOMETRICAL, MATERIAL AND LOAD DATA

Inside diameter Section 1 = 6.68"
Thickness " 1 = 0.36"
Inside diameter Section 2 = 6.25"
Thickness " 2 = 0.25"
Second moment of inertia section 1 = 51.58939
Second moment of inertia section 2 = 27.0
Force P = 1000.0 lbs.
Forcing frequency = 20 rads/sec
Young's modulus = 30×10^6 lbs/in^2.
Density = 0 283 lbs/in^3.
Poissons ratio = 0.3
Material = mild steel.

FIG 2. DYNAMIC ANALYSIS OF SIMPLY SUPPORTED GUIDE TUBE



3a. TYPICAL COMPUTING TIME REQUIREMENTS FOR BAND AND FRONT SOLUTIONS (IBM 360/85)



3b. VARIATION OF CPU TIME (IBM 370/165) WITH NUMBER OF EIGENVALUES CALCULATED

FIG. 3 COMPUTING TIME REQUIREMENTS FOR NONLINEAR AND DYNAMIC ANALYSIS