APPLICATION OF THE SPLIT-RIGIDITY CONCEPT TO CONCRETE CRACKING IN REACTOR CONTAINMENT DESIGN

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Summary

The state of stress in a reactor containment shell with large openings (e.g. equipment hatch, personnel lock, etc.) can be readily determined using available analytical-numerical techniques if the usual assumptions of elastic, isotropic material could be justified. However, a conventionally reinforced concrete containment structure will crack when subjected to design thermal and mechanical loads and will consequently fail to satisfy this assumption. Cracking in concrete results in different membrane and bending stiffnesses (hence split-rigidity) which vary with the depth of cracking. Based on these stiffnesses the shell is considered to have an equivalent reduced membrane thickness t_m and a bending stiffness t_B. Using a similar approach to that for analyzing stiffened shells these effective thicknesses are incorporated in the governing differential equations to yield the following shell characteristic parameters for the cracked section case.

$$\beta = \left[1/2 \quad \frac{3\ (1-\nu^2)}{R^2\ t_B^2}\right]^{\frac{1}{4}} \left(\frac{t_m}{t_B}\right)^{\frac{1}{4}}$$

$$KB = \frac{1}{\sqrt{12\ (1-\nu^2)}} \qquad t_B \left(\frac{t_B}{t_m}\right)^{\frac{1}{2}}$$

These parameters when introduced in the formulation of Eringen et.al. results in a simplified procedure for determining the state of stress around circular openings in the presence of concrete cracking.

The values of the above parameters, dependent as they are on t_B amd t_m, vary with the depth of cracking. With the actual t_m and t_B being somewhere between an uncracked and a fully cracked concrete section, an iterative procedure can be adopted to determine the equilibrium state of cracking. In practical applications however, (e.g. equipment hatch and personnel lock) the parameters β and KB exhibited limited variability which reduced the effort of obtaining an iterative solution.

1. Introduction

The State of Stress in a reactor containment shell around large openings (e.g. equipment hatch, personnel locks, etc.) can be readily determined using available analytical-numerical techniques (e.g. finite element, finite difference) if the usual assumptions of elastic, isotropic material could be justified. However, a conventionally reinforced concrete containment structure will crack when subjected to design thermal and mechanical loads and will consequently fail to satisfy the foregoing assumption. Cracking in concrete results in different membrane and bending stiffnesses (hence the split rigidity) which vary with the depth of cracking. In a typical finite element formulation of the problem a thru the thickness layered shell idealization can be adopted wherein cracking is accomplished by modifying the material constitutive properties of the appropriate layer and recycling the solution until a desirable degree of convergence is achieved. Use of such nonlinear FEM solution will however be complex and expensive,

It has been shown that the closed form solutions of isotropic shells can be resolved for structually orthotropic shells by considering an effective membrane thickness, t_m, and an effective bending thickness, t_B (1). A similar approach can also be taken for concrete shells where based on the depth of cracking equivalent reduced membrane and bending thicknesses can be derived (2). The purpose of the present paper is to show how this split rigidity concept can be used in conjunction with available analytical-numerical techniques to analyze the state of stress surrounding large openings in cylindrical containments such as an equipment hatch.

2. Theoretical Development

In Reference 3 a quasi analytical solution has been presented for the determination of the state of stress in a circular cylindrical thin elastic shell with a lateral circular cutout. The governing partial differential equation

$$\Delta^2\psi + 8i\beta^2 \frac{\partial^2\psi}{\partial x^2} = 0 \qquad (1)$$

has been solved in Reference 3 in the presence of different force boundary conditions around the edge of the opening and the resulting state of stress has been determined by superposition. In Eq. 1

$$\left.\begin{aligned} \psi &= \frac{\omega}{\rho_0} + iK\phi, \quad i = \sqrt{-1} \\ \beta &= 1/2\,[3(1-\nu^2)]^{1/4}(Rt)^{-1/2} \\ K &= \frac{2}{Et^2\rho_0}\,[3(1-\nu^2)]^{1/2} \\ \Delta &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{aligned}\right\} \quad (2)$$

and ω (x,y) and ϕ (x,y) are the radial displacement and the stress function defining the complex displacement function ψ in the right handed coordinate system xyz (See Fig. 1). The constants E,ν , t are respectively, the Young's moduls, the Poisson's ratio and the thickness of the shell.

For a reinforced concrete shell where cracking would result in different membrane and bending stiffnesses the parameter β and K can be modified as follows

$$\left.\begin{aligned} \beta &= 1/2\,\left[\frac{3(1-\nu^2)}{R^2 t_B^2}\right]^{1/4} \left(\frac{t_m}{t_B}\right)^{1/4} \\ K &= \frac{[12(1-\ ^2)]^{1/2}}{E\rho_o t_B^{3/2}\ t_m^{1/2}} \end{aligned}\right\} \quad (3)$$

Where t_m and t_B are the effective membrane and bending thicknesses of the cracked section (2). Similarly another flexural rigidity parameter B used in Ref. 3 can be expressed as

$$\left.\begin{aligned} B &= \frac{Et_B^3\rho_o}{12(1-\nu^2)} \\ \text{and}\quad KB &= \frac{1}{12(1-\nu^2)}\quad t_B\left(\frac{t_B}{t_m}\right)^{1/2} \end{aligned}\right\} \quad (4)$$

The tables given in Ref. 4 can be used to find moments, shears and normal forces in the shell.

3. Analysis

A typical equipment hatch opening, 19 ft. in diameter, in a reinforced concrete containment shell has been analyzed using the methodology presented before. The containment shell, 3'-6" thick, was assumed to be subjected to an accident internal pressure of 30 psig. The loading conditions considered in the analysis comprise of dead (D), live (L), hydrostatic (B), pressure (P), temperature (T_a), and seismic ($E_B = E_{OBE}$ and $E^1 = E_{SSE}$) loads. Some of the critical load combinations for the analysis were:

$$\left.\begin{aligned} U &= 1.0D + 1.0L + 1.0\,T_a + 1.5P \\ U &= 1.0D + 1.0L + 1.0\,T_a + 1.0P + 1.0E^1 \\ U &= 1.0D + 1.0L + 1.0B + 1.0E_B \end{aligned}\right\} \quad (5)$$

$$S = 1.0D + 1.0L + 1.15P$$

The step by step procedure followed in the analysis is outlined below:

A. Determine the state of stress in the shell without any opening under the applicable loading combinations. These stresses will be designated as <u>nominal stresses</u>. Normally a linear elastic analysis will be performed to obtain the nominal stresses. However, for the thermal loading since cracking will reduce thermal moments, the elastic moments were modified using the approach given in Ref. 5.

B. Determine the response of the shell subject to such loading conditions which will give edge stresses around the opening equal and opposite to the appropriate nominal stresses determined above. This analysis will be carried out primarily by using the formulation in Ref. 3 wherein the elastic geometric material parameters β, K and B are replaced by the appropriate cracked section properties (see Equations 3, 4).

C. Superpose A and B to obtain the final stresses.

The specific edge loadings around the openings that have been considered are:

(i) Uniform edge moment

(ii) Axial load in the cylinder

(iii) Internal pressure in the cylinder and on the equipment hatch cover producing hoop tension and radial shear

The stress distribution in the vicinity of the opening due to the aforementioned edge loadings were obtained from Ref. 4 where solutions have been tabulated for different values $\beta\rho_0$ and a fixed poisson ratio $\nu = 0.3$. Since it has been noted in Ref. 3 that the effect of ν on the stresses is insignificant, the tabulated values could be used for concrete shells.

It has been indicated in Ref. 3 that the analysis presented therein will yield reasonably accurate solution as long as (i) $R/t \geq 10$ and (ii) $\frac{\rho_0}{R} \leq = .35$. For the case under investigation ($R/t = 18$, $\frac{\rho_0}{R} = .15$) as well as in general these restrictions are easily met for containment shells.

4. Variation in the Geometric-Material Parameters

In determining the stress amplification around the opening, the formulation in Ref. 3 uses the geometric material parameters $\beta\rho_0$ and KB. These parameters for a cracked shell are functions of the effective thicknesses t_m and t_B (See Equations 3, 4) which vary with the depth of cracking. Since the latter varies with the resulting stresses in the shell around the opening, it appears that an iterative procedure would have had to be used to solve the problem. Moreover the spatial extent of the cracking is not known apriori and had to be determined iteratively. However if the parameters β and KB do not exhibit significant variations with different state of cracking, it would then be practicable to bound the solution with representative values of the parameters.

Consider a 42" thick concrete cylindrical containment shell which is reinforced with #18 at 12" each way each face. The calculated geometric material properties for different state of cracking (i.e. from fully uncracked to fully cracked) are given in Table 1. It is easily seen that the cardinal parameters β and KB do not show wide variation and practical stress analysis can be carried out by assuming a representative set of values for β and KB. Any such selection of the parameters should take into consideration the fact concrete will not remain uncracked especially during an accident condition; large tensile forces in the shell generated by the internal pressure and temperature during an accident condition would conceivably crack the shell through the entire thickness. On the other hand during the operating condition, it is probable that the concrete will only be partially cracked.

5. Results of the Analysis

Based on a selected value of $\beta\rho_o$ and KB and using the tabulated values in Ref. 4, the results of the three edge loading cases mentioned before were combined with the nominal stresses resultants in the shell for all applicable load combinations. Such stresses resultants were evaluated at selected points surrounding the opening. At $\rho = \rho_o$, the force boundary conditions enforced apriori resulted in $M_\rho = o$, $N_\rho = o$ and Q_ρ = reaction from the hatch cover in the radial direction.

Generally it was observed that the effect of the opening on the stress distribution in the shell dimenished very significantly for $\rho \geq 3\rho_o$ (i.e. $\rho = 28.5$ ft. in the example problem. The radial and circumferential force resultants N_ρ and N_ϕ were primarily amplified in the vicinity of the opening due to the application of the self equilibrated internal pressure edge effect. The amplification in the circumferential moment was caused by the application of the corrective radial edge moment.

6. Conclusion

The methodology presented here for predicting the complex stress distribution surrounding a large opening in a reinforced concrete shell is realistic, strightforward and sufficiently accurate for this type of design work.

REFERENCES

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TABLE I

GEOMETRIC-MATERIAL CONSTANTS FOR A 42" THICK SHELL

	CONCRETE UNCRACKED	CONCRETE CRACKED TO HALF DEPTH	CONCRETE CRACKED TO 3/4 DEPTH	CONCRETE FULLY CRACKED (REBAR ONLY)
β	.00359	.00419	.00379	.00386
βρo	.414	.484	.438	.446
KB (in)	12.71	9.32	11.4	10.98

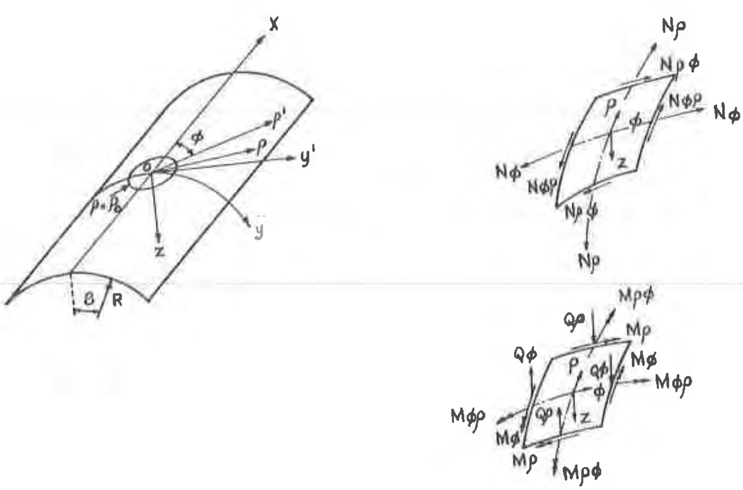

Figure 1. COORDINATE SYSTEM AND SHELL STRESS RESULTANTS

J 3/1