

# Seismic damage analysis of r/c nuclear structures

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## ABSTRACT

An approach for damage analysis of reinforced concrete structures in nuclear power plants is presented based on hysteretic random vibration. Reinforced concrete nuclear facilities may be classified into two types; namely, space frame structures and continuous wall-type structures. For frame-type structures, a maximum deformation-cumulative energy damage model developed earlier is summarized. For continuous wall-type structures, a recently developed nonlinear finite element approach is described for evaluating the stochastic response statistics necessary for damage assessment.

## 1 INTRODUCTION

The paper presents an approach for the damage analysis of reinforced concrete nuclear power plant structures subjected to earthquake ground excitations. Since the time history characteristics of future earthquake motions may be described in probabilistic terms, the random vibration technique is appropriate for estimating the response statistics necessary for the assessment of structural damage. Two types of structures are considered in the analysis; namely, discrete space frame structures and continuous wall-type systems. A damage model based on maximum deformation and cumulative hysteretic energy, developed earlier by Park, Ang and Wen (1985), is summarized for frame-type structures; whereas, for continuous wall-type structures, recent developments of a nonlinear finite element technique for random vibration analysis is described with an illustrative application to a four-story building.

## 2 FRAME-TYPE STRUCTURES

The seismic structural damage of frame-type structures can be evaluated by combining the damages of the constituent components. Structural damages of column and beam components under earthquake loadings are generally caused by the combination of repeated stress reversals and

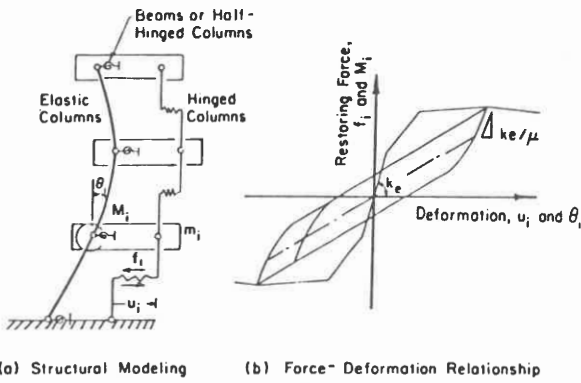


Figure 1. Modeling of Frame-Type Structures

high stress excursions; accordingly, structural damage may be expressed in terms of a damage index,

$$(1) \quad D = \frac{\delta_m}{\delta_u} + \frac{\beta}{Q_y} \frac{\delta_u}{\delta_u} \int dE$$

where:

- $\delta_m$  = maximum response deformation;
- $dE$  = incremental dissipated hysteretic energy;
- $\delta_u$  = ultimate deformation capacity under monotonic loading;
- $Q_y$  = yield strength; and
- $\beta$  = deterioration factor due to cyclic loading (non-negative constant).

The response statistics,  $\delta_m$  and  $dE$ , can be obtained through hysteretic random vibration analysis (Wen 1980), whereas the structural parameters,  $Q_y$ ,  $\delta_u$  and  $\beta$  have been determined as a function of component properties based on available laboratory test data (Park, Ang and Wen 1985).

For response analysis, idealization and simplifications are necessary; such idealizations, however, must be able to capture the main characteristics of the response statistics. In this regard, the model for reinforced concrete frames is schematically illustrated in Figure 1. Elastic bending columns (or shear walls) are connected to each floor by inelastic rotational springs in order to include the effect of story-coupling. Such a structural model may be properly combined to realistically describe the force-deformation relationship of a large class of frame structures. The overall damage sustained by an entire frame structure may be expressed as the sum of the damage indices of the constituent components, or of the stories,  $D_i$ , weighted by the corresponding energy contribution factors,  $\lambda_i$ ; namely,

$$(2) \quad D_T = \sum_i \lambda_i D_i$$

in which,  $\lambda_i = E_i / \sum E_i$ , and  $E_i$  is the dissipated energy in component  $i$  or story  $i$ .

Post-earthquake damage analysis of low-rise reinforced concrete buildings indicated that the values of the foregoing damage index,  $D$ , are such that  $D \geq 0.4$  represents damage beyond repair, and  $D \geq 1.0$  represents total collapse. The damage analysis method described herein may be applied to those structures in a nuclear power plant constructed with beam and column elements. In characterizing the significance of the damage index, however, the above damage criteria (which is based on calibration with buildings) may be re-examined to reflect the particular safety requirements for nuclear facilities.

### 3 CONTINUOUS WALL-TYPE STRUCTURES

Major parts of nuclear structures are often composed of continuous shear wall construction, including the reactor containment structure. The random vibration analysis of such a structure is currently limited largely to linearly elastic systems. Recent developments of a nonlinear finite element approach to hysteretic random vibration analysis is presented for two-dimensional reinforced concrete structures. The essential components of the approach are presented below.

#### 3.1 Orthotropic Constitutive Model

Under bi-axial or multi-axial loading conditions, a constitutive model is required; this must be consistent with the stochastic hysteretic behavior of inelastic materials. For wall-type elements, the proposed model is characterized by a two-dimensional yield criterion using the principal stress concept, i.e.,

$$(3) \quad \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi \pm \tau_{xy} \sin 2\phi$$

in which,  $\sigma_{1,2}$  are the principal stresses (assumed to be Gaussian random processes); and  $\phi$  is the principal stress angle. The value of  $\phi$  may be determined on the basis of least squares; that is

$$(4) \quad \frac{\partial}{\partial \phi} E[\sigma_{1,2}^2] = 0$$

The principal stresses are then expressed in terms of the corresponding strains,  $\epsilon_{1,2}$ , and the auxiliary inelastic strain components,  $\xi_{1,2}$ , as follows (given for plain stress condition):

$$(5) \quad \begin{aligned} \sigma_1 &= (1-H) \frac{E}{1-\nu^2} (\xi_1 + \nu \xi_2) + H \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) \\ \sigma_2 &= (1-H) \frac{E}{1-\nu^2} (\nu \xi_1 + \xi_2) + H \frac{E}{1-\nu^2} (\nu \epsilon_1 + \epsilon_2) \end{aligned}$$

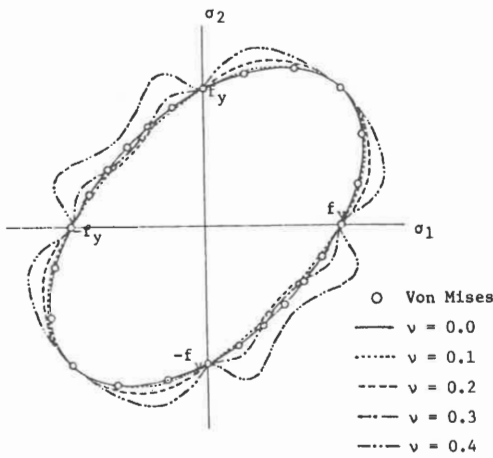


Figure 2. Yield Surfaces

in which,  $E$  is the modulus of elasticity;  $\nu$  is the Poisson's ratio; and  $H$  is the strain hardening ratio. The above strain components satisfy the following coupled nonlinear differential equations:

$$\dot{\xi}_1 = \dot{\epsilon}_1 - \alpha |\dot{\epsilon}_1 \xi_1| \xi_1 - \alpha \dot{\epsilon}_1 \xi_1^2 - \alpha |\dot{\epsilon}_2 \xi_2| \xi_1 - \alpha \dot{\epsilon}_2 \xi_1 \xi_2 - \beta \dot{\epsilon}_1 \xi_1 \xi_2 \quad (6)$$

$$\dot{\xi}_2 = \dot{\epsilon}_2 - \alpha |\dot{\epsilon}_2 \xi_2| \xi_2 - \alpha \dot{\epsilon}_2 \xi_2^2 - \alpha |\dot{\epsilon}_1 \xi_1| \xi_2 - \alpha \dot{\epsilon}_1 \xi_1 \xi_2 - \beta \dot{\epsilon}_2 \xi_1 \xi_2$$

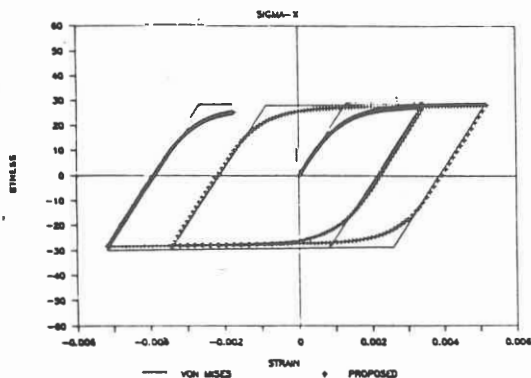
in which,  $\alpha$  and  $\beta$  are functions of the material parameters;

$$\alpha = \frac{1}{2} (E/f_y)^2 \quad (7a)$$

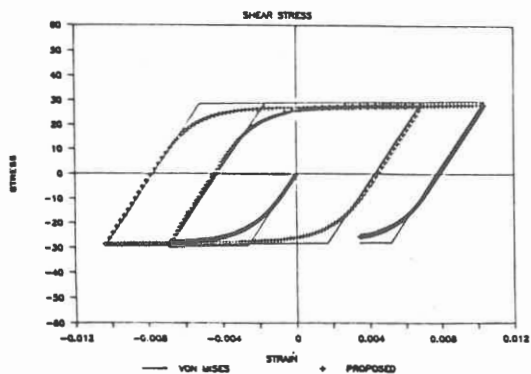
$$\beta = \alpha \{2/(1-\nu)^2 - 4\} \quad (7b)$$

in which,  $f_y$  is the uniaxial yield stress.

The above formulation is based on the basic principles of classical plasticity theory (namely, the yielding, hardening, and flow rules) by introducing the auxiliary inelastic strain components,  $\xi_{1,2}$ . The corresponding yield surfaces are illustrated in Figure 2 for various values of the Poisson's ratio. As indicated in Figure 2, the yield surface changes its shape for different values of the Poisson's ratio. However, when the Poisson's ratio is relatively low, the observed fluctuation is small and the corresponding yield surface agrees well with the von Mises' theory (e.g., for reinforced concrete,  $\nu$  is around 0.16 to 0.2). Some of the resulting hysteretic behavior is illustrated in Figure 3a for the nominal stress,  $\sigma_x$ , and in Figure 3b for the maximum shear stress,  $\tau$ , in which the element is subjected to stress reversals with  $\sigma_x = -\sigma_y$ . The corresponding hystereses associated with the von Mises' theory are also shown (in broken lines) in Figure 3.



(a) Normal Stress,  $\sigma_x$



(b) Shear Stress,  $\tau$

Figure 3. Hysteretic Behavior

### 3.2 FEM Analysis

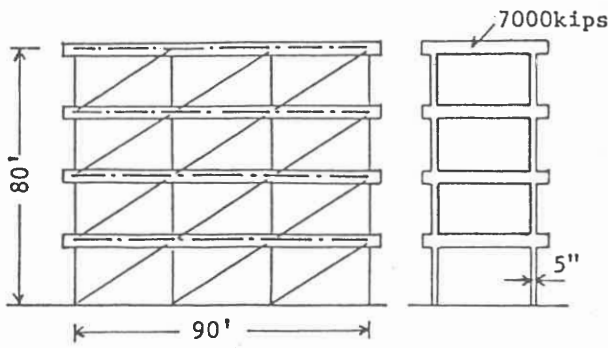
The foregoing nonlinear differential equations, equation 6, can be linearized using equivalent linearization technique (Atalik and Utku 1976). Moreover, by replacing the principal strain components,  $\epsilon_{1,2}$ , by the nodal displacements,  $\{u\}$ , through matrix operations, equation 6 may be rewritten as follows:

$$(8) \quad \{\dot{\xi}_1\} = [F]\{\dot{u}\} + [C_2]\{\xi_1\}$$

Similarly, the equivalent nodal forces for each finite element,  $\{q_e\}$ , may be expressed in the following form:

$$(9) \quad \{q_e\} = [K]\{u\} + [G]\{\xi_1\}$$

Therefore, the equations of motion can be expressed by a set of first order differential equations,



$E = 3000 \text{ ksi}$ ,  $\nu = 0.2$ ,  
 $H = 0.5$ ,  $f'_c = 4 \text{ ksi}$ ,  
 $\sigma_y = 50 \text{ ksi}$ ,  $\rho_w = 0.4\%$

Story	$\tau_u$ (ksi)	$f_y$ (ksi)
1	0.529	0.916
2	0.478	0.828
3	0.428	0.741
4	0.377	0.653

Figure 4. Continuous Wall Type Structure

$$(10) \quad \frac{d}{dt} \begin{Bmatrix} u \\ \dot{u} \\ \xi \end{Bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}K & -M^{-1}C & -M^{-1}G \\ 0 & F' & C_2 \end{bmatrix} \begin{Bmatrix} u \\ \dot{u} \\ \xi \end{Bmatrix} + \begin{Bmatrix} 0 \\ f(t) \\ 0 \end{Bmatrix}$$

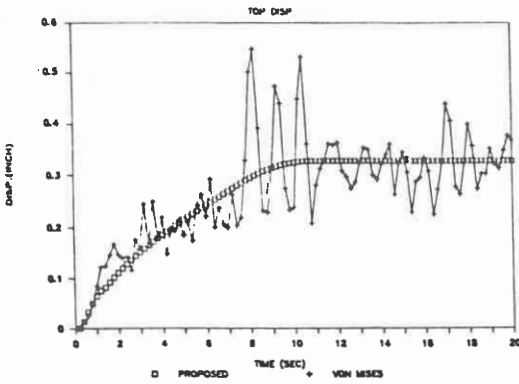
in which,  $M$  and  $C$  are mass and damping matrices;  $I$  is the identity matrix; and  $f(t)$  is the random excitation vector. The above set of stochastic differential equations can be solved for the covariance matrix (Wen 1980).

### 3.3 An Illustrative Application

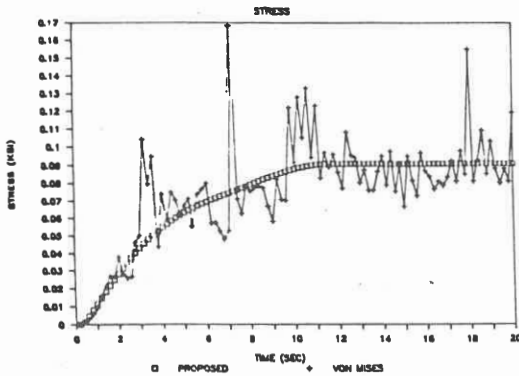
As an application, a four-story reinforced concrete building with a shear wall of uniform thickness,  $t = 10$  in, is considered. The building is subjected to horizontal earthquake motions modeled as a filtered shot-noise process. The shear wall is discretized into a total of 24 uniform-strain triangular elements. Since shear failure is the only significant failure mode, the yield criterion may be determined based on the overall shear strength,  $\tau_u$ , e.g.,

$$(11) \quad \tau_u = 2 \left( 1 + \frac{\sigma_0}{500} \right) \sqrt{f'_c} + \rho_w \sigma_y$$

in which,  $\sigma_0$ ,  $f'_c$  and  $\sigma_y$  are the axial stress, concrete strength, and steel yield stress in psi, respectively; and  $\rho_w$  is the wall reinforcement ratio. The calculated uniaxial yield stresses,  $f_y$ , for the elements in the different stories are indicated in Figure 4, together with other parameters. The responses of the four-story shear-wall building subjected to two rms excitation levels, 0.03g and 0.05g, are presented, respectively, in Figures 5 and 6. At the rms excitation of 0.03g, the responses shown in Figure 5 are essentially elastic; whereas at 0.05g the results shown in Figure 6 represent inelastic responses. Figures 5a and 6a show the rms lateral displacements of the



(a) Top Displacement



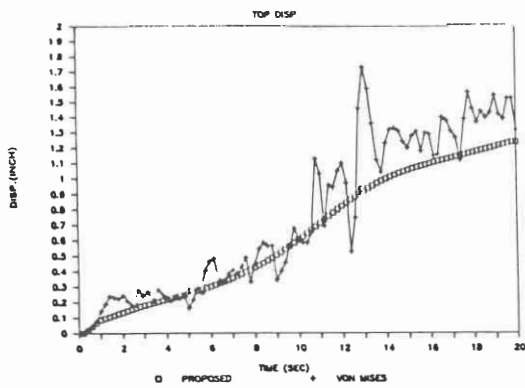
(b) Principal Stress

Figure 5. Near Elastic rms Responses ( $\sigma = 0.03g$ )

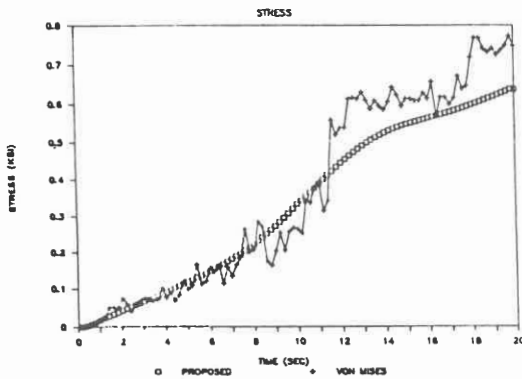
top of the building, whereas the principal stresses in the middle of the first story are illustrated in Figures 5b and 6b. For comparison, the results of Monte-Carlo simulation (with sample size of 20) obtained with the von Mises' criterion are also shown in Figures 5 and 6. When the rms excitation is  $0.05g$ , the shear wall in the first story would be subjected to a mean maximum shear deformation of  $\gamma = 0.6\%$ ; this would indicate significant shear cracks throughout the first story.

#### 4 SUMMARY AND CONCLUSIONS

Methods for seismic damage analysis applicable to reinforced concrete nuclear structures are presented for both frame-type and continuous wall-type structures. Results of recent developments in nonlinear finite element are used for the latter type of structures; these are described in some detail with an illustrative application to a four-story shear wall building. The damage analysis of continuous wall-type structures requires the development of suitable damage functions for general two-dimensional structural components. One such formulation is presented and discussed. Additional studies are obviously necessary.



(a) Top Displacement



(b) Principal Stress

Figure 6. Inelastic rms Responses ( $\sigma = 0.05g$ )

#### REFERENCES

- Atalik, T.S. & S. Utku. 1976. Stochastic linearization of multi-degree-of-freedom nonlinear systems. *J. Earthq. Engr. and Struct. Dyn.* 4,4:411-420.
- Hill, R. 1950. *The mathematical theory of plasticity*. Oxford.
- Park, Y.J. & A.H-S. Ang & Y.K. Wen. 1985. Seismic damage analysis of reinforced concrete buildings. *J. Struct. Engr., ASCE* 111,4:740-757.
- Park, Y.J. & Y.K. Wen & A. H-S. Ang. 1986. Random vibration of hysteretic systems under bi-directional ground motions. *J. Earthq. Engr. and Struct. Dyn.* 14:543-557.
- Wen, Y. K. 1980. Equivalent linearization for hysteretic systems under random excitations. *J. Applied Mech., ASME* 47,1:150-154.