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Passive TMDS for seismic response reduction of multi-story torsionally-coupled buildings

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ABSTRACT

This paper illustrates the practical considerations and vibration control effectiveness of passive tuned mass dampers (PTMDs) for multi-story torsionally-coupled buildings under incidental horizontal earthquake excitations. The PTMD is designed to control the mode which makes most contribution to the largest response of the building. Its optimum installation location and moving direction are determined from the mode shape values of controlled mode. The optimal system parameters of PTMD are then calculated by minimizing the mean-square modal displacement response ratio of controlled mode between the building with and without PTMD under earthquake excitation from critical direction. As two PTMDs are used to reduce both translational responses, this study arranges the two mass dampers to achieve the largest vibration reduction. Numerical results from a long and a square five-story torsionally-coupled buildings subjected to 1995 Kobe earthquake verify that the proposed optimal PTMDs can significantly reduce the dynamic responses of primary structure.

INTRODUCTION

Due to recent intensive research and development, vibration control of structures by means of passive tuned mass dampers (PTMDs) is gaining more acceptance not only in the design of new structures but also in the retrofit of existing structures to enhance their reliability against winds, earthquakes, and human activities[1-4]. PTMDs can be incorporated into an existing structure with less interference compared with other passive energy dissipation devices. Since 1971, lots of PTMDs have been successfully installed in high-rise buildings and tower in USA, Japan, and other countries and reported to be able to reduce wind-induced vibrations significantly. Most previous studies assumed the primary structure such as a building as a single degree-of-freedom (SDOF) system with its fundamental modal properties[1-6], or as a torsionally uncoupled plane structure to design the PTMDs[7-9]. However, a real building usually possesses a large number of degrees of freedom and is actually asymmetric to some degree even with nominally symmetric plan. It will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. Thus, the simplified SDOF system neglects the structural torsion-coupled effect and the PTMD effect on different modes which could overestimate the control effectiveness of PTMD. This study is a pioneer work dealing with the optimal installed floor, planar position and moving direction of PTMDs for multiple DOF torsionally-coupled buildings.

It is generally known that the vibration reduction of primary structure using PTMD is mainly attributed to the suppression of controlled modal responses. The previous studies determined the system parameters of PTMD based on this general concept. However, we find, in this paper, that the vibration control effectiveness of PTMD varies not only with the controlled modal parameters of the primary structure, but also with the installed location and moving direction of PTMD as well as the incident angle of earthquake excitation. Thus, for a torsionally-coupled real structure, the previous simplified model may lead to incorrect design of PTMD and overestimation of vibration control effectiveness.

This paper illustrates the above practical considerations in designing PTMDs for multistory torsionally-coupled buildings under incidental horizontal earthquake excitations. The general torsionally-coupled building is modeled by three DOFs for each floor with one rotation and two translations. The PTMD is designed to control the mode which makes most contribution to the largest response of the building. Its optimum installation location and moving direction are determined from the mode shape values of controlled mode. The optimal system parameters of PTMD are then calculated by minimizing the mean-square modal displacement response ratio of controlled mode between the building with and without PTMD under earthquake excitation from critical direction. As two PTMDs are used to reduce both translational responses, this study arranges the two mass dampers to achieve the largest vibration reduction. Numerical results from a long and a square five-story torsionally-coupled buildings subjected to 1995 Kobe earthquake verify that the proposed optimal PTMDs can significantly reduce the dynamic responses of primary structure.

DYNAMIC EQUATION OF BUILDING-TMD SYSTEMS

With reference to the building idealization consisting of rigid floors supported on massless axially inextensible columns and walls, the general torsionally-coupled multistory buildings as shown in Fig. 1 have the following features : (1) The principal axes of resistance for all the stories are identically oriented, along the x and y-axes shown; (2) the centers of mass of the floors do not lie on a vertical axis; (3) centers of resistance of the stories do not lie on a vertical axis, either, i.e. the static eccentricities at each story are not equal; (4) all floors do not have the same radius of gyration r about the vertical axis through the center of mass; and (5) ratios of the three stiffness quantities — translational stiffnesses in x and y directions and torsional stiffness — for any story are different.

For the above general torsionally-coupled N-story building as shown in Fig. 1, each floor has three degrees-of-freedom: x- and y-displacements, relative to the ground, of the center of mass and rotation about a vertical axis. For floor k, they are denoted by x_k, y_k and θ_k respectively. Assumed that a SDOF PTMD of mass, m_{sy}, damping coefficient, c_{sy} and stiffness, k_{sy}, is installed at the lth floor, and moving in y direction. The dynamic equation of motion of the combined building-TMD system under an incidental horizontal earthquake excitation (incident angle β from x direction) can be written as

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0}^T & m_{sy} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{u}_{sy} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0}^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}} \\ \dot{u}_{sy} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0}^T & 0 \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ u_{sy} \end{Bmatrix}$$

$$+\begin{Bmatrix} 0 \\ \vdots \\ 0 \\ k_{sy}y_l + k_{sy}\upsilon_y r_l\theta_l - k_{sy}u_{sy} \\ k_{sy}\upsilon_y y_l + k_{sy}\upsilon_y^2 r_l\theta_l - k_{sy}\upsilon_y u_{sy} \\ k_{sy}(u_{sy} - y_l - \upsilon_y r_l\theta_l) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ c_{sy}\dot{y}_l + c_{sy}\upsilon_y r_l\dot{\theta}_l - c_{sy}\dot{u}_{sy} \\ c_{sy}\upsilon_y\dot{y}_l + c_{sy}\upsilon_y^2 r_l\dot{\theta}_l - c_{sy}\upsilon_y\dot{u}_{sy} \\ c_{sy}\left(\dot{u}_{sy} - \dot{y}_l - \upsilon_y r_l\dot{\theta}_l\right) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} = \begin{Bmatrix} -\boldsymbol{M}\boldsymbol{r}\ddot{u}_g \\ -m_{sy}\sin\beta\ddot{u}_g \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F} \\ f_s \end{Bmatrix} \tag{1}$$

In Eq. (1), $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are $3N \times 3N$ mass, damping and stiffness matrices of building; $\boldsymbol{u}^T = [x_1 \quad y_1 \quad r_1\theta_1 \quad \cdots \quad x_N \quad y_N \quad r_N\theta_N]^T$ and u_{sy} denote the displacement vector of primary structure and PTMD displacement relative to base, respectively, T is the matrix transpose operator. $\boldsymbol{r}^T = [\cos\beta \quad \sin\beta \quad 0 \quad \cos\beta \quad \sin\beta \quad 0 \quad \cdots]^T$ is the ground influence coefficient vector; $\ddot{u}_g$ represents the earthquake ground acceleration; d_y is the distance in x direction between PTMD and mass center of lth floor, r_l is the radius of gyration of lth floor, $\upsilon_y = d_y / r_l$. Assumed that $\boldsymbol{C}$ is a classical damping matrix, the equation of motion of ith mode of controlled structure is expressed as

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \frac{1}{m_i^*}\sum_{k=1}^{3N}\phi_{k,i}F_k + \frac{\phi_{3l,i}}{m_i^*}\left[\upsilon_y m_{sy}\left(2\xi_{sy}\omega_{sy}\dot{v}_{sy} + \omega_{sy}^2 v_{sy}\right)\right] + \frac{\phi_{3l-1,i}}{m_i^*}\left[m_{sy}\left(2\xi_{sy}\omega_{sy}\dot{v}_{sy} + \omega_{sy}^2 v_{sy}\right)\right] \tag{2}$$

In Eq. (2), $v_{sy} = u_{sy} - \left(y_l + d_y\theta_l\right)$ is the displacement of PTMD relative to the lth floor, or say PTMD's stroke; m_i^* and η_i are the ith generalized modal mass and displacement; $\omega_{sy} = \sqrt{k_{sy}/m_{sy}}$ and $\xi_{sy} = c_{sy} / \left(2m_{sy}\omega_{sy}\right)$ represent the natural frequency and damping ratio of PTMD, respectively. $\phi_{3l-1,i}$ denotes the ($3l$-1)th element of ith mode shape ϕ_i. F_k is the kth element of vector $\boldsymbol{F}$. Define

$$\mu_{iy} = \left(\phi_{3l-1,i} + \upsilon_y\phi_{3l,i}\right)\frac{m_{sy}}{m_i^*} = \rho_{iy}\left(1 + \upsilon_y(\phi_{3l,i} / \phi_{3l-1,i})\right) \text{ and } F_i^* = \frac{1}{m_i^*}\sum_{k=1}^{3N}\phi_{k,i}F_k \tag{3}$$

where $\rho_{iy} = \phi_{3l-1,i}\left(m_{sy}/m_i^*\right)$ denotes the ith modal mass ratio of PTMD. Then, Eq. (2) can be rewritten as

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i - \mu_{iy}\left(2\xi_{sy}\omega_{sy}\dot{v}_{sy}\right) + \omega_i^2\eta_i - \mu_{iy}\left(\omega_{sy}^2 v_{sy}\right) = F_i^* \tag{4}$$

Similarly, the equation of motion of PTMD in Eq. (1) becomes

$$\left(\sum_{k=1}^{3N}\phi_{3l-1,k}\ddot{\eta}_k + \upsilon_y\sum_{k=1}^{3N}\phi_{3l,k}\ddot{\eta}_k + \ddot{v}_{sy}\right) + 2\xi_{sy}\omega_{sy}\dot{v}_{sy} + \omega_{sy}^2 v_{sy} = f_s^* \tag{5}$$

where $f_s^* = f_s / m_{sy}$. Provided that the PTMD is tuned to the ith mode of controlled structure and only the ith modal response is considered, from Eqs. (4) and (5), the equation of motion for ith mode and PTMD are expressed in matrix form as

$$\begin{bmatrix} 1 & 0 \\ \left(\phi_{3l-1,i} + \upsilon_y\phi_{3l,i}\right) & 1 \end{bmatrix}\begin{Bmatrix} \ddot{\eta}_i \\ \ddot{v}_{sy} \end{Bmatrix} + \begin{bmatrix} 2\xi_i\omega_i & -\mu_{iy}\left(2\xi_{sy}\omega_{sy}\right) \\ 0 & 2\xi_{sy}\omega_{sy} \end{bmatrix}\begin{Bmatrix} \dot{\eta}_i \\ \dot{v}_{sy} \end{Bmatrix} + \begin{bmatrix} \omega_i^2 & -\mu_{iy}\omega_{sy}^2 \\ 0 & \omega_{sy}^2 \end{bmatrix}\begin{Bmatrix} \eta_i \\ v_{sy} \end{Bmatrix} = -\begin{Bmatrix} \Gamma_i \\ \sin\beta \end{Bmatrix}\ddot{u}_g \tag{6}$$

where $\Gamma_i = (\phi_i^T\boldsymbol{M}\boldsymbol{r}) / (\phi_i^T\boldsymbol{M}\phi_i)$ is the ith modal participation factor. It has been proved that as the primary structure has no torsion-coupling, Eq. (6) is reduced to the same form as that of previous studies[10]. For the case of primary structure without PTMD, its ith modal equation is given as

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^{\,2}\eta_i = -\Gamma_i\ddot{u}_g \tag{7}$$

The comparison of modal responses in Eqs. (6) and (7) leads to the determination of optimal PTMD's parameters and the verification of vibration control effectiveness of PTMD.

OPTIMAL SYSTEM PARAMETERS OF PTMDS

According to Eqs. (6) and (7), the optimal PTMD's parameters are determined by minimizing the mean-square displacement response ratio of the ith tuned mode (or say controlled mode), $R_{dE,i}$, between the structure with and without installation of PTMD under an incidental horizontal earthquake excitation. As derived in reference [4], $R_{dE,i}$ takes the form as

$$R_{dE,i} = \frac{E\left[\eta_i^{\,2}\right]_{TMD}}{E\left[\eta_i^{\,2}\right]_{NOTMD}} = \frac{A}{B} \tag{8}$$

in which

$$\begin{aligned}
A = {} & 4\xi_i\xi_{sy}^{\,3}r_{fy}^{\,3}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right) \\
& +4\xi_i^{\,2}\xi_{sy}^{\,2}r_{fy}^{\,4}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right) \\
& +4\xi_i^{\,3}\xi_{sy}r_{fy}^{\,3}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2 + 4\xi_i^{\,2}\xi_{sy}^{\,2}r_{fy}^{\,2}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2 \\
& -\xi_i\xi_{sy}r_{fy}^{\,3}\left(\Gamma_i^2 - \rho_{iy}^{\,2}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right)\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right) \\
& -\xi_i\xi_{sy}r_{fy}^{\,3}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2 + \Gamma_i^{\,2}\xi_i\xi_{sy}r_{fy} + \xi_i^{\,2}r_{fy}^{\,2}\left(\rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2 \\
& +\xi_i\xi_{sy}r_{fy}^{\,5}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right)^2 \\
& +\xi_i^{\,2}r_{fy}^{\,4}\left(\Gamma_i + \rho_{iy}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))\right)^2\left(\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right)
\end{aligned}$$

$$\begin{aligned}
B = {} & 4\Gamma_i^2\xi_i\xi_{sy}^{\,3}r_{fy}^{\,3}\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right) + 4\Gamma_i^{\,2}\xi_i^{\,2}\xi_{sy}^{\,2}r_{fy}^{\,2} \\
& +\Gamma_i^{\,2}\xi_{sy}^{\,2}r_{fy}^{\,2}\left(\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right) + 4\Gamma_i^{\,2}\xi_i^{\,2}\xi_{sy}^{\,2}r_{fy}^{\,4}\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right) \\
& +\Gamma_i^{\,2}\xi_i\xi_{sy}r_{fy} + 4\Gamma_i^2\xi_i^{\,3}\xi_{sy}r_{fy}^{\,3} - 2\Gamma_i^{\,2}\xi_i\xi_{sy}r_{fy}^{\,3} + \Gamma_i^{\,2}\xi_i^{\,2}r_{fy}^{\,4}\left(\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right)^2 \\
& +\Gamma_i^{\,2}\xi_i\xi_{sy}r_{fy}^{\,5}\left(1+\rho_{iy}\phi_{3l-1,i}(1+\upsilon_y(\phi_{3l,i}/\phi_{3l-1,i}))^2\right)
\end{aligned}$$

where $r_{fy} = \omega_{sy}/\omega_i$ is defined as the frequency ratio of PTMD to the controlled mode. Owing to the magnitude of elements, $\phi_{k,i}$, in mode shape vector ϕ_i is relative, the value $R_{dE,i}$ depends on $\phi_{k,i}$. If ϕ_i is selected to normalize the ith effective modal mass $M_i = \left(\sum_{k=1}^{3N}\phi_{k,i}m_k r_k\right)^2 \Big/ \sum_{k=1}^{3N}\phi_{k,i}^2 m_k$. $R_{dE,i}$ does not vary with $\phi_{k,i}$, and $\Gamma_i = 1$. Under this definition, the modal mass ratio ρ_{iy} in Eq. (3) has a definite physical meaning and its expression becomes $\rho_{iy} = (\phi_{3l-1,i}m_{sy})/M_i$.

The value of $R_{dE,i}$ smaller than unity represents attenuation of the structural responses due to the presence of PTMDs. It is found from Eq. (8) that $R_{dE,i}$ equals to unity as $\upsilon_y = (-\phi_{3l-1,i}/\phi_{3l,i})$. That means, in this case, there is no vibration reduction. This finding indicates the importance of the considerations of torsion-coupling effect and the installation location of PTMDs.

It is seen from Eq. (8) that $R_{dE,i}$ is a function of the controlled modal parameters (ξ_i and

ϕ_i), the PTMD's system parameters (ρ_{iy}, ξ_{sy} and r_{fy}), the installed floor ($\phi_{3l-1,i}$, $\phi_{3l,i}$), moving direction and planar position (υ_y) of PTMD, as well as the seismic incident angle β. For an existing building, when the controlled modal parameters, the installed floor, planar position and moving direction of PTMD, and the seismic incident angle β are known or given (to be discussed later), the optimal PTMD's system parameters can be obtained by differentiating $R_{dE,i}$ with respect to ρ_{iy}, r_{fy} and ξ_{sy} and equating to zero, respectively, to minimize $R_{dE,i}$. Their values may be found by solving the following simultaneous equations

$$\frac{\partial R_{dE,i}}{\partial \rho_{iy}} = 0, \quad \frac{\partial R_{dE,i}}{\partial r_{fy}} = 0, \quad \frac{\partial R_{dE,i}}{\partial \xi_{sy}} = 0 \tag{9}$$

In practice, $\left(\rho_{iy}\right)_{opt}$ is rarely used due to economic considerations. Hence, in general, we find out $\left(r_{fy},\ \xi_{sy}\right)_{opt}$ for various values of ρ_{iy} and then search for $\left(\rho_{iy}\right)_{opt}$. As mentioned above, prior to the determination of the optimal PTMD's design parameters from Eq. (9), we must first determine (1) the controlled structural mode; (2) the installed floor, moving direction and planar position of PTMD, and (3) the critical seismic incident angle β_{cr}. These factors play very important roles in optimum design of PTMDs and their control efficacy.

Controlled modes. A PTMD is simply a SDOF system. As seen in the theoretical development, it reduces the primary structural responses by means of minimizing the mean-square displacement response ratio of certain mode of the primary structure. It is obvious that a PTMD is optimally designed to control the mode which makes the most contribution to a specified response of the primary structure. For a torsionally-coupled shear building, the first three modes are the most important to translational and torsional responses of each floor. However, the x and y translations have different dominant modes. It could be possible to reduce the dynamic responses of all degrees of freedom using one PTMD. But, this PTMD will not be the optimal one to every degree of freedom. In general, the conventional design of a PTMD is to reduce the largest response, which may cause damage, of the primary structure. Therefore, the dominant mode to the largest structural response is selected as the controlled mode of PTMD.

Installed floor, moving direction and planar position of PTMD. It has been shown by Lin et al [10] for planar buildings that the floor corresponding to the tip of controlled mode shape will be the optimum location for PTMD because more response reduction can be achieved. Similarly, for a torsionally-coupled building, the terms of $\phi_{3l-1,i}$ and υ_y in Eq.(6) clearly denote the installed floor l, moving direction y (or say DOF $3l-1$), and planar position d_y. Therefore, the optimum installation floor and planar position of PTMD can be determined by means of maximizing the absolute values of $(\phi_{3l-1,i} + \upsilon_y\phi_{3l,i})$ for moving in y-direction or $(\phi_{3l-2,i} + \upsilon_x\phi_{3l,i})$ for moving in x-direction. To achieve this, the following steps are suggested: (i). following reference [10], we may choose the floor corresponding to the tip of controlled mode shape as the installed floor. (ii). choose the degree of freedom of the largest response as the moving direction of PTMD. (iii). from Eq. (6), we know that the optimal planar position of PTMD is related with the sign of translation ($\phi_{3l-1,i}$ or $\phi_{3l-2,i}$) and rotation ($\phi_{3l,i}$) mode shape values. When both mode shape values have the same sign, we choose υ_y or υ_x having maximum positive value allowable in the installed floor. On the other hand, when $\phi_{3l-1,i}$ or $\phi_{3l-2,i}$ and $\phi_{3l,i}$ have opposite signs, we will choose υ_y or υ_x to be the maximum negative value. Through above steps, we can obtain the maximum absolute value of $(\phi_{3l-1,i} + \upsilon_y\phi_{3l,i})$ or

$(\phi_{3l-2,i} + \upsilon_x \phi_{3l,i})$. That means the farther between PTMD and mass center of the installed floor is, the more vibration reduction is obtained.

Critical seismic incident angle. The dynamic responses of a torsionally-coupled building also depend on the incident angle of earthquake excitation. To design optimal PTMDs, it is essential and necessary to find the critical seismic incident angle which induces the largest structural responses. In this paper, the critical seismic incident angle, β_{cr}, is determined such that the mean-square response of the desired controlled degree-of-freedom is maximum. For an N-story existing building, when only the first n_{ID} modal parameters are known by modal parameter identification techniques, its mean-square displacement response in y-direction of top floor is expressed as

$$E\left[y_N^2\right] = \int_0^\infty \left| \sum_{k=1}^{n_{ID}} \frac{-1}{m_k^*} \left[\left(-\omega^2 + 2i\xi_k\omega_k\omega + \omega_k^2\right)^{-1} \cdot \left(\phi_k^T \mathbf{Mr}\right) \right] \phi_{3N-1,k} \right|^2 \sin\beta S_{\ddot{u}_g}(\omega) d\omega \tag{10}$$

where $S_{\ddot{u}_g}(\omega)$ is the power spectral density of earthquakes.

OPTIMAL DESIGN OF SECOND PTMD

According to above design procedure for one PTMD, the response of controlled DOF is reduced. Since this PTMD is designed based on the dominant modal properties of this DOF and its corresponding seismic incident angle, its capability in reducing the responses of other DOFs under an earthquake from different angles should be further investigated. It is found in this paper that for a structure with nearly equal stiffnesses in x and y directions such as a square building, one PTMD designed for reducing y-responses is not able to decrease x-responses if the earthquake loading is applied from the critical angle of x-responses. Under this circumstance, a second PTMD becomes necessary.

Assumed that the first PTMD is optimally designed by previous procedure to control the *ith* mode. The second PTMD with mass, m_{sx}, damping, c_{sx}, and stiffness, k_{sx}, is also mounted at the *lth* floor of the N-story torsionally-coupled building to tune the *jth* mode and moving in x direction. The equation of motion for the *jth* mode and two PTMDs are expressed in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 \\ (\phi_{3l-2,j} + \upsilon_x \phi_{3l,j}) & 1 & 0 \\ (\phi_{3l-1,j} + \upsilon_y \phi_{3l,j}) & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_j \\ \ddot{v}_{sx} \\ \ddot{v}_{sy} \end{Bmatrix} + \begin{bmatrix} 2\xi_j\omega_j & -\mu_{jx}(2\xi_{sx}\omega_{sx}) & -\mu_{jy}(2\xi_{sy}\omega_{sy}) \\ 0 & 2\xi_{sx}\omega_{sx} & 0 \\ 0 & 0 & 2\xi_{sy}\omega_{sy} \end{bmatrix} \begin{Bmatrix} \dot{\eta}_j \\ \dot{v}_{sx} \\ \dot{v}_{sy} \end{Bmatrix}$$
$$+ \begin{bmatrix} \omega_j^2 & -\mu_{jx}\omega_{sx}^2 & -\mu_{jy}\omega_{sy}^2 \\ 0 & \omega_{sx}^2 & 0 \\ 0 & 0 & \omega_{sy}^2 \end{bmatrix} \begin{Bmatrix} \eta_j \\ v_{sx} \\ v_{sy} \end{Bmatrix} = - \begin{Bmatrix} \Gamma_j \\ \cos\beta \\ \sin\beta \end{Bmatrix} \ddot{u}_g \tag{11}$$

By matrix partition, Eq. (11) is separated into following two expressions

$$\begin{bmatrix} 1 & 0 \\ (\phi_{3l-2,j} + \upsilon_x \phi_{3l,j}) & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_j \\ \ddot{v}_{sx} \end{Bmatrix} + \begin{bmatrix} 2\xi_j\omega_j & -\mu_{jx}(2\xi_{sx}\omega_{sx}) \\ 0 & 2\xi_{sx}\omega_{sx} \end{bmatrix} \begin{Bmatrix} \dot{\eta}_j \\ \dot{v}_{sx} \end{Bmatrix} + \begin{Bmatrix} -\mu_{jy}(2\xi_{sy}\omega_{sy})\dot{v}_{sy} \\ 0 \end{Bmatrix}$$
$$+ \begin{bmatrix} \omega_j^2 & -\mu_{jx}\omega_{sx}^2 \\ 0 & \omega_{sx}^2 \end{bmatrix} \begin{Bmatrix} \eta_j \\ v_{sx} \end{Bmatrix} + \begin{Bmatrix} -\mu_{jy}\omega_{sy}^2 v_{sy} \\ 0 \end{Bmatrix} = - \begin{Bmatrix} \Gamma_j \\ \cos\beta \end{Bmatrix} \ddot{u}_g \tag{12}$$

$$(\phi_{3l-1,j} + \upsilon_y \phi_{3l,j})\ddot{\eta}_j + \ddot{v}_{sy} + 2\xi_{sy}\omega_{sy}\dot{v}_{sy} + \omega_{sy}^2 v_{sy} = -\sin\beta \ddot{u}_g \tag{13}$$

Taking Fourier transform of Eqs. (12) and (13) and substituting Eq. (13) into (12), we obtain the equation of motion of the *jth* mode and the second PTMD as

$$\begin{bmatrix} -\omega^2 + 2i\omega\xi_j\omega_j + \omega_j^2 + \dfrac{\omega^2(-2i\omega\mu_{jy}\xi_{sy}\omega_{sy} - \mu_{jy}\omega_{sy}^2)}{-\omega^2 + 2i\omega\xi_{sy}\omega_{sy} + \omega_{sy}^2}(\phi_{3l-1,j} + \upsilon_y\phi_{3l,j}) & -2i\omega\mu_{jx}\xi_{sx}\omega_{sx} - \mu_{jx}\omega_{sx}^2 \\ -\omega^2(\phi_{3l-2,j} + \upsilon_x\phi_{3l,j}) & -\omega^2 + 2i\omega\xi_{sx}\omega_{sx} + \omega_{sx}^2 \end{bmatrix} \begin{Bmatrix} \eta_j(i\omega) \\ v_{sx}(i\omega) \end{Bmatrix} = -\begin{Bmatrix} \Gamma_j + \sin\beta \dfrac{-2i\omega\mu_{jy}\xi_{sy}\omega_{sy} - \mu_{jy}\omega_{sy}^2}{-\omega^2 + 2i\omega\xi_{sy}\omega_{sy} + \omega_{sy}^2} \\ \cos\beta \end{Bmatrix} \ddot{u}_g(i\omega) \quad (14)$$

In Eq. (14), $\eta_j(i\omega)$ denotes the Fourier displacement response of the *jth* mode of a building with two PTMDs. As Eqs. (8)–(10), the optimal system parameters of the second PTMD are determined by minimizing the mean-square displacement response ratio of the *jth* mode, $R_{dE,j}$, between the building with two PTMDs and with one PTMD under an incident horizontal earthquake from the critical angle of x-responses, $\beta_{cr,x}$.

NUMERICAL VERIFICATIONS

Two five-story torsionally-coupled buildings are presented to demonstrate the proposed new design procedure and vibration control effectiveness of PTMDs. The first building (B1) has obvious weak stiffnesses in y-direction compared with those in x-direction such as long buildings. The second building (B2) has nearly equal stiffnesses in x and y directions. Table 1 lists their first three modal frequencies, damping ratios and mode shapes. It is seen that the modal orders are $y-\theta-x$ and $y-x-\theta$ for B1 and B2, respectively. In addition, B2 possesses very close translational modes which will cause high interaction between two modes. The total mass ratio of either one PTMD or two PTMDs to building total mass, is set to be 5% in the following numerical examples.

First PTMD. Because y-direction is weak for both buildings, y-direction of top floor is the controlled DOF and the first mode is thus the controlled mode. According to Eq. (10), the critical angles for both buildings are found to be 96° and 91° as shown in Figs. 2 and 3. Following the proposed design concept, the first PTMD is installed at top floor, moving in y direction to reduce the first modal displacement under earthquake from β = 96° or 91°. Since $\phi_{14,1}$ and $\phi_{15,1}$ have different sign, for instance −32.096 and 14.454 for B1 and −5.234 and 2.578 for B2, υ_y is chosen to be −1.25 for both buildings. The optimal PTMD's system parameters are then calculated by Eq. (9) and listed in Table 2.

The variation of mean square displacement response of top floor with β is shown in Fig. 2 for B1 with and without PTMD. It is seen that all responses (particularly y response) are reduced for earthquakes from any incident angle. Thus, it is concluded that only one optimal PTMD is adequate for the first type buildings. Fig. 4 depicts the transfer functions for β=6° and 96°. It is apparent that the first modal amplitude in all three directional responses is suppressed significantly as β=96°, which agrees with the theoretical results. The time history displacement responses at top floor of B1 under 1995 Kobe earthquake from β=96° are shown in Fig. 5. As we expect, both peak and root-mean-square responses are reduced up to 40%.

Second PTMD. However, it is found in Fig. 3 that the top floor mean square x-displacement response of B2 increases as β=(0–45)° and (145–180)°. This is attributed to the amplification of its dominant modal response (mode 2) after the installation of first PTMD. In this case, a second PTMD is required. To compare the vibration control effectiveness using one PTMD and two PTMDs, same total mass of PTMD is used for both cases. Through detailed numerical studies, we found that two PTMDs with equal mass will give the best control results. Therefore, the optimal locations and system parameters of two PTMDs for B2 are determined according to the procedure described in previous session and shown in Table 3. Fig. 6 illustrates the transfer functions of top floor displacement for B2 with one and two PTMDs under earthquakes from 9° ($\beta_{cr,x5}$) and 91° ($\beta_{cr,y5}$). The corresponding response time histories under 1995 Kobe earthquake from β=9° are given in Fig. 7. Their peak responses are summarized in Table 4. The number in parenthesis (•) denotes the percentage of response reduction. The fact of tremendous reduction of peak and root-mean-square responses again proves the necessity and importance of the second PTMD. All numerical results agree well with those of theoretical development.

CONCLUSIONS

This paper is a pioneer work dealing with the optimum installation location in plan and in elevation and moving direction of PTMDs for multi-story torsionally-coupled buildings under incidental horizontal earthquake excitations. The optimal PTMD's system parameters are calculated by minimizing the mean-square modal displacement response ratio of controlled mode between the building with and without PTMD under the earthquake excitation from critical direction. From theoretical developments and numerical results, the following conclusions are made: (1). the critical seismic incident angle is determined such that the mean-square response of the desired controlled DOF is maximum. (2). the dominant mode of desired controlled DOF is selected as the controlled mode of PTMD. (3). the floor corresponding to the tip of controlled mode shape is the optimum installed floor of PTMD. (4). the moving direction of PTMD is the same as the controlled DOF. (5). the farther between PTMD and mass center of the installed floor, the more vibration reduction. (6). one PTMD is adequate in reducing both translations and rotation of long buildings under earthquake excitations from any incident angle. However, a second PTMD is required for buildings with nearly equal stiffness in x and y directions. Numerical results of a long and a square five-story torsionally-coupled buildings under 1995 Kobe earthquake agree well with those of theoretical development. The proposed optimal PTMDs can reduce building responses up to 50%.

REFERENCES

1. Asami, T., et al. 1991. Optimal design of dynamic absorbers for a system subjected to random excitation.
JSME International Journal, Series III 34(2): 218-226.
2. Satareh, M. & R.D. Hanson 1992. Tuned mass dampers for balcony vibration control.
Journal of Structural Engineering, ASCE 118(3): 723-740.
3. Satareh, M. & R.D. Hanson, 1992. Tuned mass dampers to control floor vibration from humans.
Journal of Structural Engineering, ASCE 118(3): 741-762.
4. Lin, C.C., C.M. Hu, J.F. Wang & R.Y. Hu 1994. Vibration control effectiveness of passive tuned mass dampers.
Journal of the Chinese Institute of Engineers 17(3): 367-376.
5. Fujino, Y. & M. Abe 1993. Design formulas for tuned mass dampers based on a perterbation technique.
Earthquake Engineering and. Structural Dynamics 22: 833-854.
6. Xu, Y.L., & K.C.S. Kwok 1994. Semianalytical method for parameteric study of tuned mass dampers.
Journal of Structural Engineering, ASCE 120(3): 747-764.

7. Xu, Y.L., B. Samali & K.C.S. Kwok 1992. Control of along-wind response of structures by mass and liquid dampers.
Journal of Engineering Mechanics, ASCE 118(1): 20-39.
8. Kawaguchi, A., A. Teramura & Y. Omote 1992. Time history response of a tall building with a tuned mass damper under wind force.
Journal of Wind Engineering and Industrial Aerodynamics 41-44: 1949-1960.
9. Villaverde, R. & L.A. Koyama 1993. Damped resonant appendages to increase inherent damping in buildings.
Earthquake Engineering and Structural. Dynamics 22: 491-507.
10. Lin, C.C., J.F. Wang & Y.C. Gau 1993. System identification and vibration control of structures with tuned mass dampers.
3rd ROC-Japan Joint Seminar on Natural Hazards Mitigation: 436-450.

Table 1. The first three modal properties of B1 and B2

	Mode Shapes						ω (Hz)		ξ (%)	
	x-direction		*y*-direction		$r\theta$-direction					
Building	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2
Mode 1	$\begin{bmatrix}1.000\\1.987\\2.863\\3.533\\3.907\end{bmatrix}$	$\begin{bmatrix}1.000\\1.988\\2.890\\3.608\\4.026\end{bmatrix}$	$\begin{bmatrix}-7.894\\-15.848\\-23.086\\-28.786\\-32.096\end{bmatrix}$	$\begin{bmatrix}-1.298\\-2.583\\-3.761\\-4.700\\-5.234\end{bmatrix}$	$\begin{bmatrix}3.892\\7.636\\10.851\\13.206\\14.454\end{bmatrix}$	$\begin{bmatrix}0.659\\1.305\\1.877\\2.327\\2.578\end{bmatrix}$	0.769	0.730	2.00	2.00
Mode 2	$\begin{bmatrix}1.000\\2.005\\2.918\\3.635\\4.050\end{bmatrix}$	$\begin{bmatrix}1.000\\1.988\\2.894\\3.615\\4.037\end{bmatrix}$	$\begin{bmatrix}0.599\\1.228\\1.830\\1.330\\2.636\end{bmatrix}$	$\begin{bmatrix}0.803\\1.597\\2.328\\2.912\\3.244\end{bmatrix}$	$\begin{bmatrix}1.186\\2.322\\3.292\\3.998\\4.370\end{bmatrix}$	$\begin{bmatrix}0.073\\0.145\\0.209\\0.259\\0.287\end{bmatrix}$	0.980	0.758	2.00	2.00
Mode 3	$\begin{bmatrix}1.000\\2.022\\2.969\\3.731\\4.182\end{bmatrix}$	$\begin{bmatrix}1.000\\1.994\\2.923\\3.668\\4.114\end{bmatrix}$	$\begin{bmatrix}-0.211\\-0.442\\-0.674\\-0.876\\-1.004\end{bmatrix}$	$\begin{bmatrix}-0.934\\-1.865\\-2.738\\-3.442\\-3.849\end{bmatrix}$	$\begin{bmatrix}-0.844\\-1.650\\-2.335\\-2.830\\-3.090\end{bmatrix}$	$\begin{bmatrix}-3.566\\-7.055\\-10.142\\-12.566\\-13.919\end{bmatrix}$	1.105	0.910	2.04	2.04

Table 2. Optimal system parameters of first PTMD for B1 and B2

Building	Controlled Mode	Mass Ratio (%)	ξ_s (%)	γ_f	Installed Floor	Moving Direction	υ_y
B1	Mode 1	5	22.0	0.80	5F	*y*	-1.25
B2	Mode 1	5	18.0	0.84	5F	*y*	-1.25

Table 3. Optimal system parameters of two PTMDs for B2

PTMD	Controlled Mode	Mass Ratio (%)	ξ_s (%)	γ_f	Installed Floor	Moving Direction	υ_x or υ_y
First	Mode 1	2.5	13.0	0.91	5F	y	-1.25 (υ_y)
Second	Mode 2	2.5	10.0	0.95	5F	x	1.25 (υ_x)

Table 4. Peak responses of B2 under Kobe earthquake

	β=9°($\beta_{cr,x5}$)			β=91°($\beta_{cr,y5}$)		
	x_5(cm)	y_5(cm)	$(r\theta)_5$(cm)	x_5(cm)	y_5(cm)	$(r\theta)_5$(cm)
Uncontrolled	63.8	26.4	17.1	25.4	66.3	20.1
One PTMD	65.8 (+3%)	14.1 (-47%)	13.8 (-19%)	11.7 (-54%)	41.9 (-37%)	17.1 (-15%)
Two PTMDs	43.1 (-32%)	13.4 (-49%)	12.3 (-28%)	10.9 (-57%)	44.7 (-33%)	12.2 (-39%)

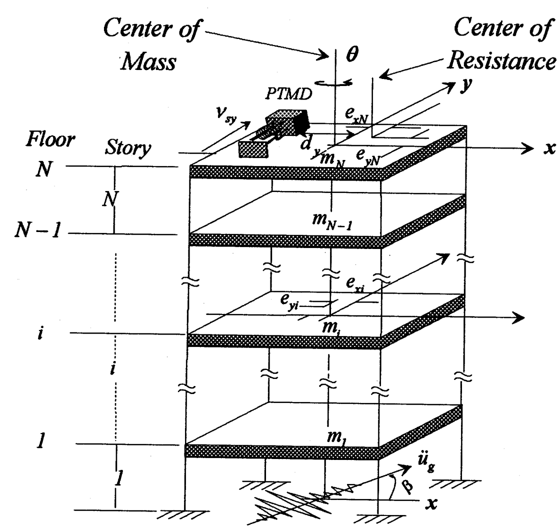
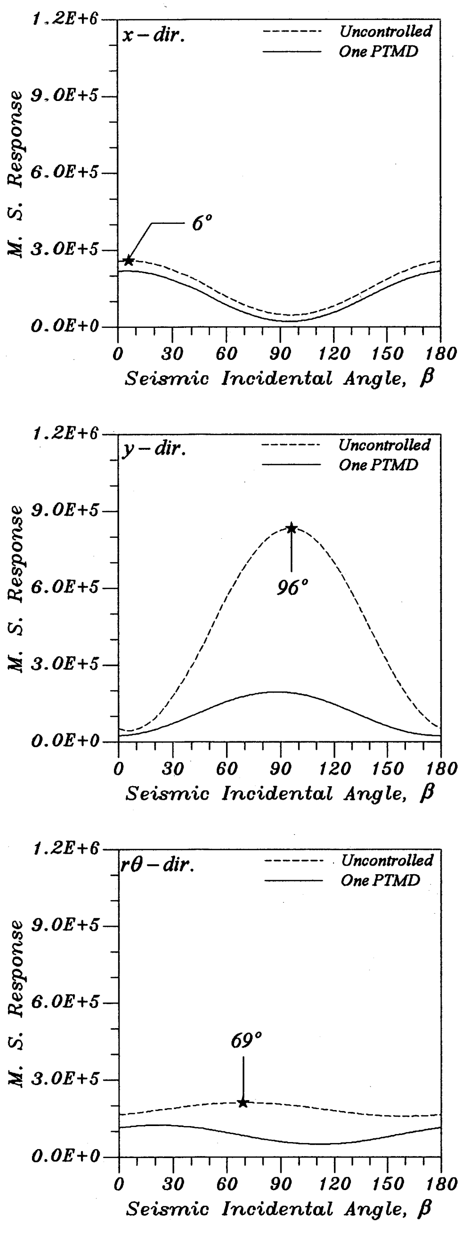


Fig. 1. N-Story General Torsionally-Coupled Building-TMD System



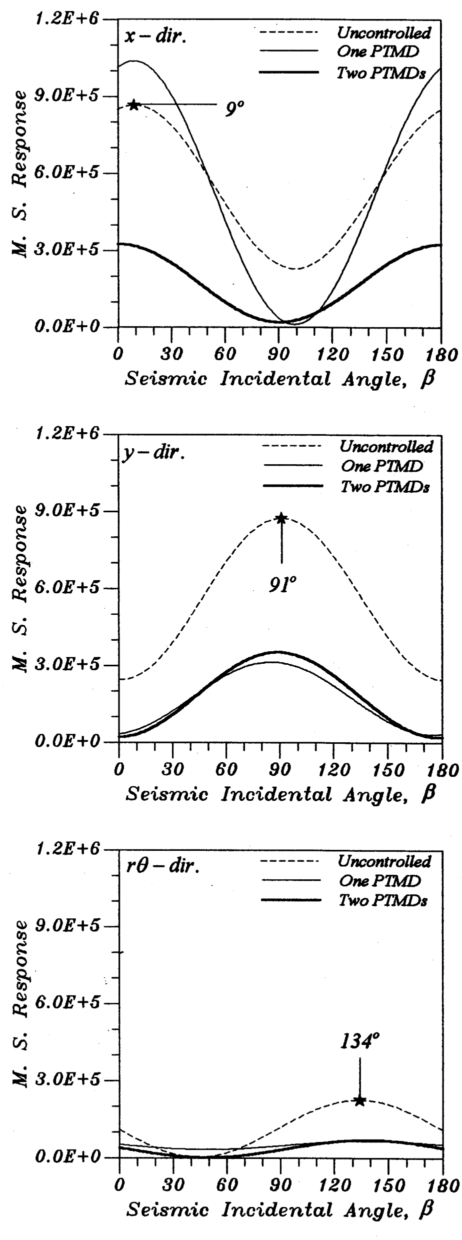


Fig. 2. Top floor Mean - Square displacement Response of B1 with and without PTMD

Fig. 3. Top floor Mean - Square displacement Response of B2 with and without PTMD

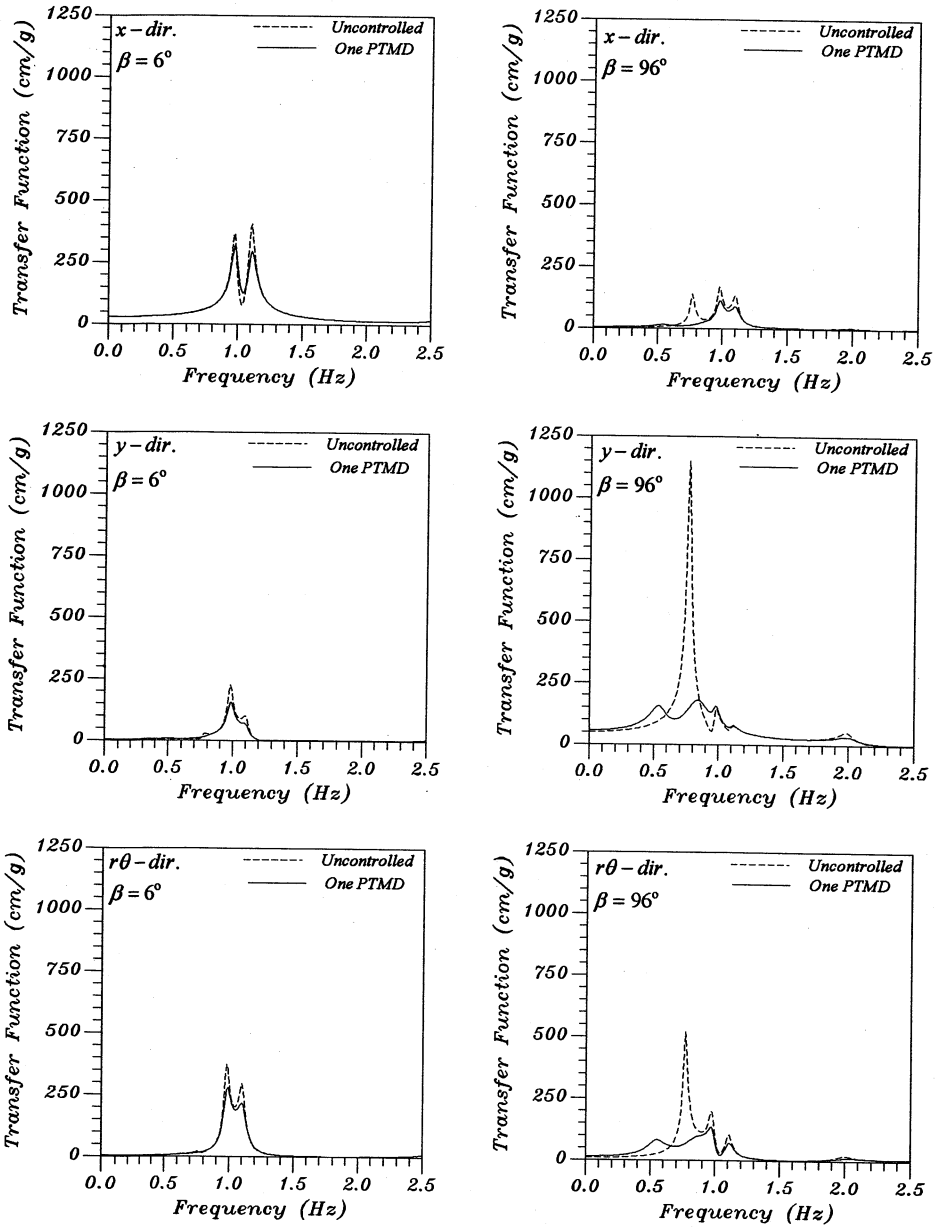


Fig. 4. Top floor displacement Transfer functions for B1 as $\beta = 6°$ and $\beta = 96°$

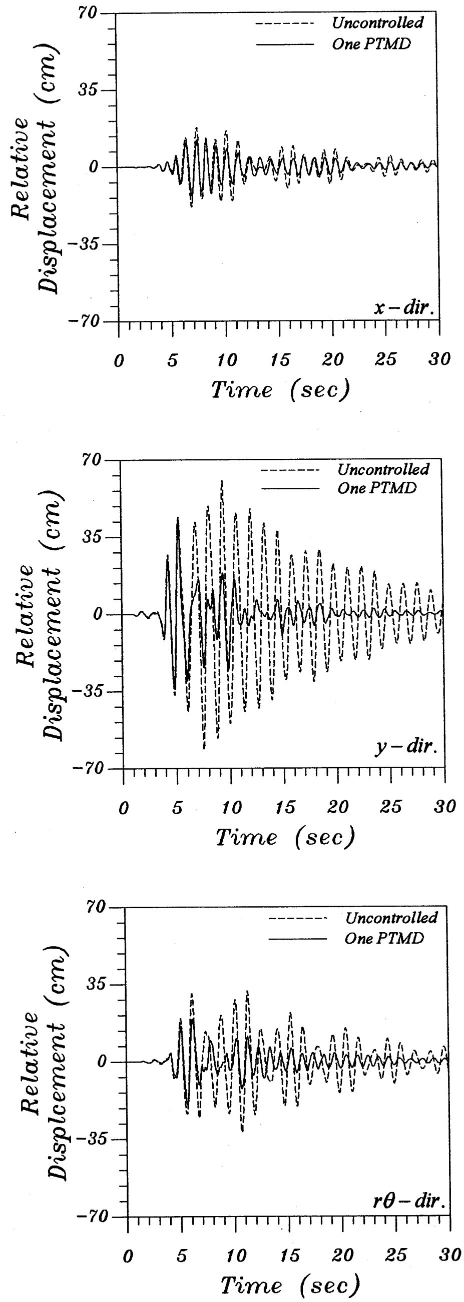


Fig. 5. Top floor displacement response of B1 under Kobe earthquake from $\beta = 96°$

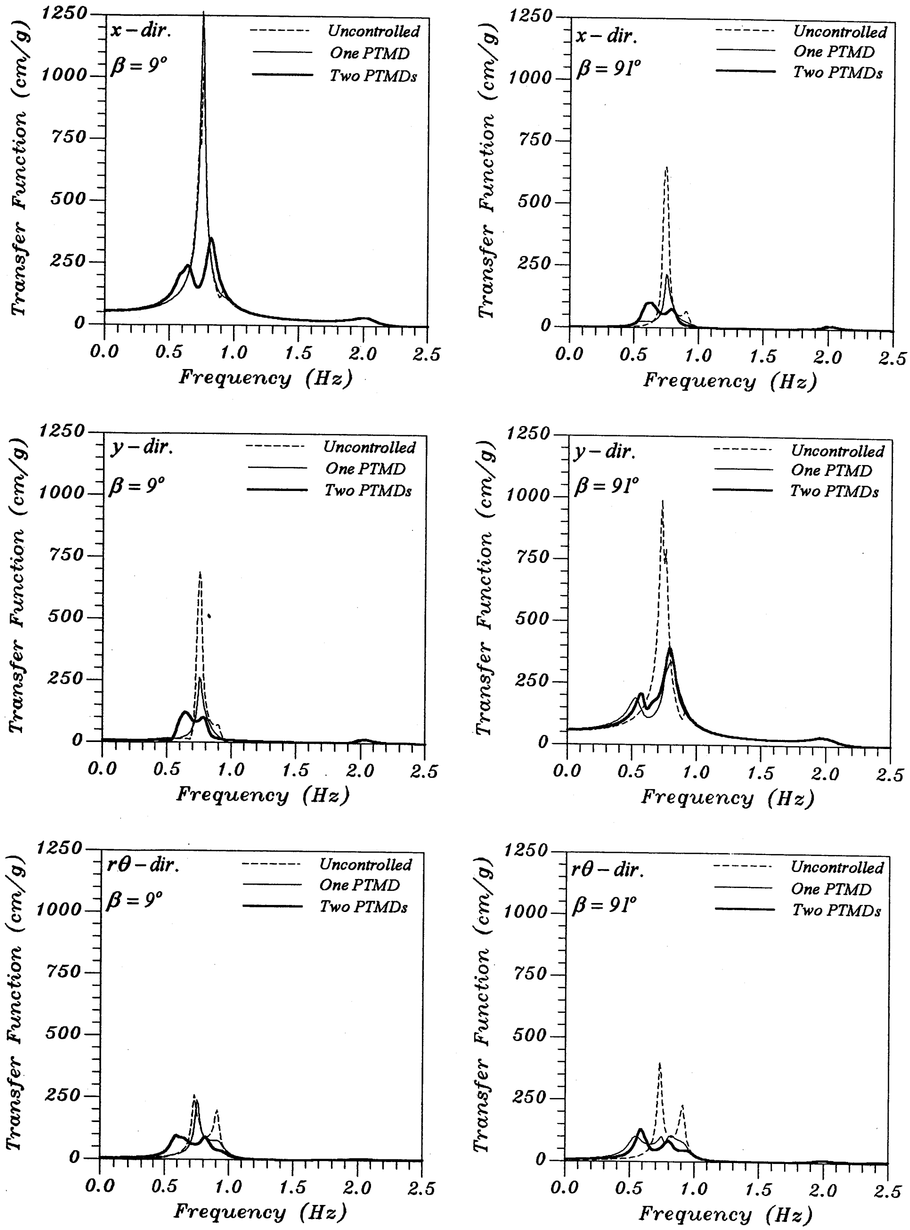


Fig. 6. Top floor displacement Transfer functions for B2 as $\beta = 9^\circ$ and $\beta = 91^\circ$

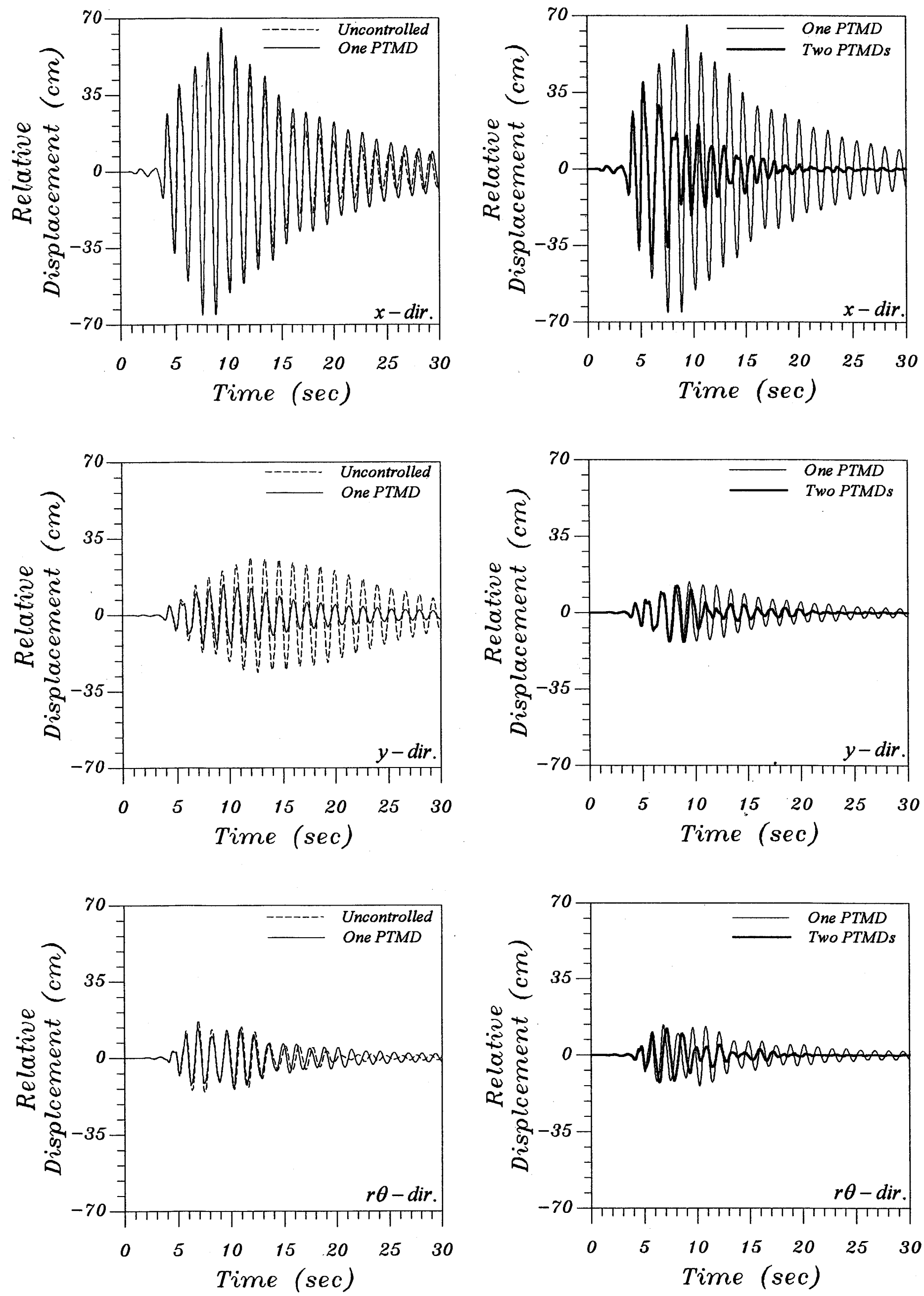


Fig. 7. Top floor displacement response of B2 under Kobe earthquake from $\beta = 9°$