On the influence of geometry change in the high temperature design of a cylinder-sphere intersection

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1 INTRODUCTION

The influence of geometry change on the value of the limit load of pressure vessels operating at temperatures sufficiently low for time dependent effects to be negligible has long been recognized (Allman and Gill 1968). It is generally observed that geometry change leads to an increase in the load carrying capacity of the structure. In these circunstances it is then safe to use the limit load of the undeformed structure as a basis for design unless the beneficial effects are too large in which cases the design may be excessively conservative.

For components operating at elevated temperatures consideration must be given to the time dependent effects of deformation and rupture. Since deformation occurs continuously with time it is reasonable to expect that the effects of geometry change assume even greater importance in high temperature design than they do in conventional design.

It is the objective of this work to assess the importance of such effect for the particular and important case of a cylinder-sphere intersection subjected to a constant internal pressure.

In the formulation adopted material behaviour at high temperatures is described by an elastocreep model in which the total strain rates and the damage rates are expressed by the generalized form of the Kachanov-Rabotnov equations proposed by Leckie and Hayhurst (1974). In this theory material damage is assumed to be an isotropic phenomenon and a full coupling between the deformation and damage processes is considered to take place.

The equations of motion in incremental form are derived from the principle of virtual work according to an updated Lagrangian formulation. Numerical solution is obtained through the finite element method.

The numerical lifetime predictions obtained in this work are compared with the approximated analytical solutions developed by Leckie (1982) based on energy bounding theorems.

2 PROBLEM DESCRIPTION

The shell considered in this study (Figure 1) is made of British Standard 1501-271 steel and is subjected to an internal pressure of 3.0 MPa (30.59 kgf/cm^2) and to a constant temperature field of 550 °C.

The B.S. 1501-271 steel obeys the effective stress deformation and rupture criterion and material deterioration occurs under both tension and

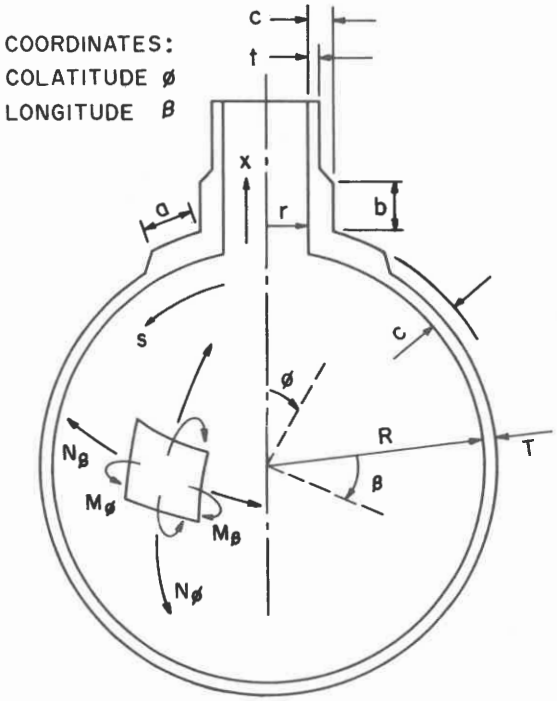

Figure 1. Shell geometry

compression regimes. Thus, the creep strain rate and the damage rate equations for this material can be written, considering the prevailing isothermal conditions, as

$$(1) \qquad \frac{\partial}{\partial t}(\varepsilon^{c}_{ij}) = \gamma \left\{ \bar{\sigma} / (1-\omega) \right\}^{N} \frac{d}{d\sigma_{ij}} (\bar{\sigma})$$

$$(2) \quad \text{and} \quad \frac{\partial}{\partial t}(\omega) = A \, \bar{\sigma}^{\chi} / (1-\omega)^{\phi}$$

where ε^{c}_{ij} is the creep strain tensor of infinitesimal strains, σ_{ij} the Cauchy stress tensor, $\bar{\sigma}$ the effective (Von Mises) stress, ω the damage scalar variable, and γ,N,A, χ and ϕ temperature dependent material properties. In the initial undamaged state $\omega = 0$ and rupture is deemed to occured at any point of the structure when ω reaches the critical value 1.

The temperature dependencies of the γ,N,A, χ and ϕ were determined by Wojedowzi (1978) from the results of uniaxial tests carried out by Glen and Hazra (1972) on specimens subjected to loads varying from 310 to 4340 kgf/cm^2 and to temperatures from 450 to 575 °C. The specific values of these parameters together with the material Young's modulus (E) and Poisson's ratio (ν) are compiled in Table 1 for the temperature of interest (550 °C).

Table 1. Material Properties for the British Standard 1501-271 steel at 550 °C

Property	Numerical value
E	1.5295×10 kgf/cm^2
ν	0.3
γ	3.0125×10^{-13} (kgf/cm^2)$^{-2.35}$ hr^{-1}
N	2.35
A	5.3800×10^{-11} (kgf/cm^2)$^{-1.96}$ hr^{-1}
$\chi = \phi$	1.96

The geometry of the shell shown in Figure 1 is determined by: R = 66.0 cm., T = t = c = 1.73 cm., r = 19.8 cm., H = 25.0 cm., h = 5.0 cm. and a = b = 0.0 (reinforcement-free structure).

3 FINITE ELEMENT ANALYSIS

The finite element (F.E.) analysis was carried out using the computer program EVP2D developed by this author (Gonçalves Filho 1984) for the elastoviscoplastic (creep) damage analysis of two-dimensional and axi-symmetric problems.

In the EVP2D code the F.E. incremental equations of motion are derived by an updated Lagrangian formulation based on the displacement method. Eight-noded isoparametric elements with quadratic interpolation functions are employed for the domain discretization and simple numerical procedures are developed for dealing with the presence of partially and/or fully ruptured elements in the mesh. For time integration of the creep strain rate equations an implicit time marching scheme is employed, which can be regarded as a Runge-Kutta method of second-order. The time integration of the coupled damage rate equations is performed using a first order predictor-corrector scheme with an automatic time step length control.

For the particular structure examined only one-fourth of the vessel was modelled due to the symmetry characteristics of the problem. The finite element mesh employed consisted of 74 elements and 301 nodes and is shown in Figure 2 together with the boundary conditions imposed. The problem was solved using the Euler backward scheme for integration of the strain rate equations combined with the 2x2 Gaussian quadrature rule for evaluation of the finite element matrices and vectors. Stringent accuracy tolerances were imposed with regard to the calculation of the stress and damage fields.

4 NUMERICAL RESULTS

The time to the appearance of the first macrocrack (t_1) and the total rupture time (t_R) determined in the geometric linear (GL) and nonlinear (GNL) analyses are compiled in Table 2.

In both analyses total failure occured closely after the appearance of the first macrocrack (approximately 10% later). The lifetime determined in the nonlinear analysis was 14% higher than that obtained in the linear case.

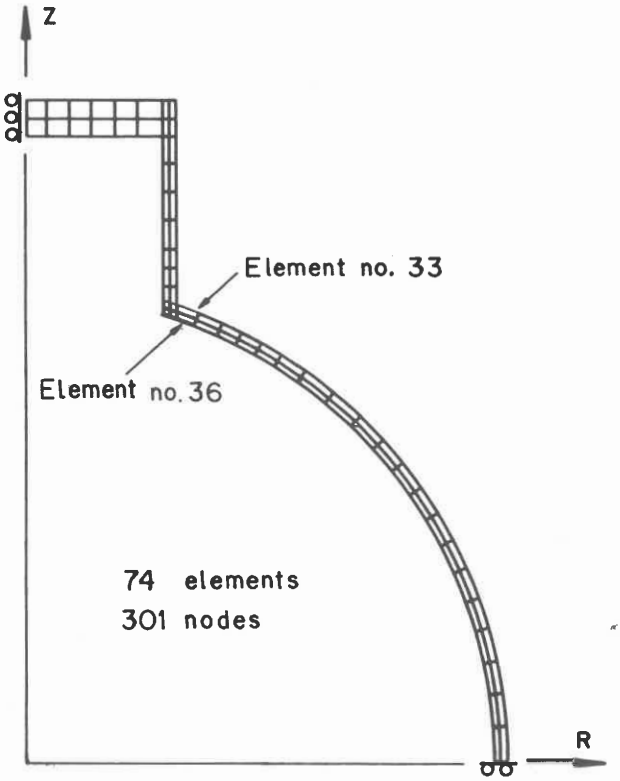

Figure 2. Finite element mesh

Table 2. Computed times to the appearance of the first macrocrack (t_1) and total collapse (t_R)

Analysis	t_1 (hours)	t_R (hours)	t_1/t_R
GL	6266	6770	0.93
GNL	6946	7741	0.90

The damage distribution computed was very much concentrated around the intersection zone as illustrated in Figure 3 for the geometric nonlinear analysis. The line(ring) of fracture was determined close to the nozzle juntion with the crack front moving from the outer surface inward. The vessel thus failed with the cylindrical portion of the nozzle shearing away from the main sphere.

Figure 4 shows the variation of the hoop stresses with time at the external (element 33/gauss point 4) and internal (element 36/gauss point 2) faces of the sphere close to the intersection zone for both the linear and nonlinear analyses. At these regions the primary and secondary phases of the creep deformation process were relatively short. At the onset of ter-

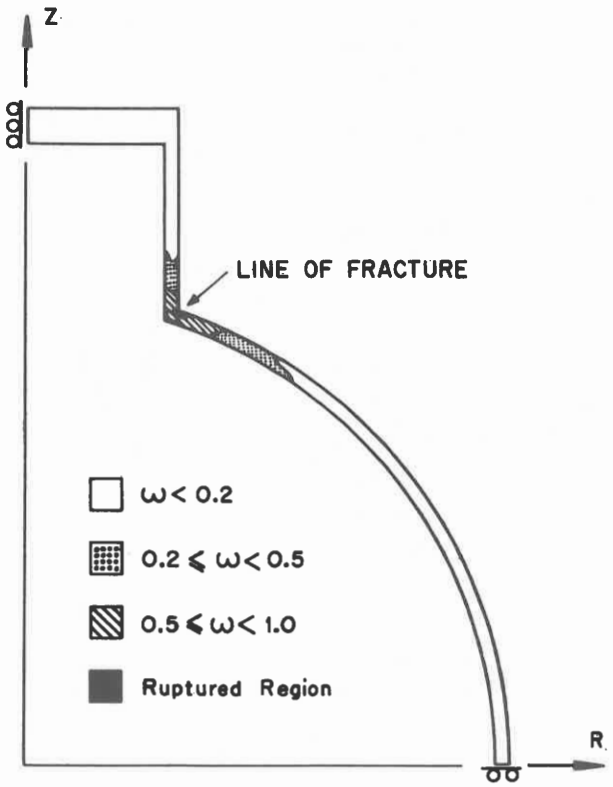

Figure 3. Damage contours at $t \triangleq t_R$

tiary creep a strong stress redistribution took place particularly at the outer surface. The fluctuations of stress observed after the first macrocrack are due to the propagation of the crack front.

Before discussing the implications in high temperature design of the previous results it is important to have them convalidated by an alternative solution. This was achieved by considering the approximated analytical study developed by Leckie (1982) based on energy bounding theorems. According to his study the ratio between the lifetimes predicted by the geometric nonlinear and linear analyses can be written as

$$(3) \qquad t_R^{NL}/t_R^{L} = \left\{(1+\alpha\ \varepsilon_h^c)^{N+1} - 1\right\} / \left\{(N+1)\alpha\ \varepsilon_h^c\right\}$$

where N is the creep index as in equation 1, ε_h^c the average value of the creep hoop strain and α is a parameter given by

$$(4) \qquad \alpha = e^{(k-1)}\ d(P^{NL}/P^{L})/d(\varepsilon_h^c)_{max.}$$

where k is the elastic concentration factor, P^{NL} the current limit pressure, P^{L} the limit load of the undeformed shell and $(\varepsilon_h^c)_{max.}$ the creep hoop strain at the point of intersection. The gradient term in equation 4 is given in parametric form by Allman and Gill (1968) for several vessels of similar geometry.

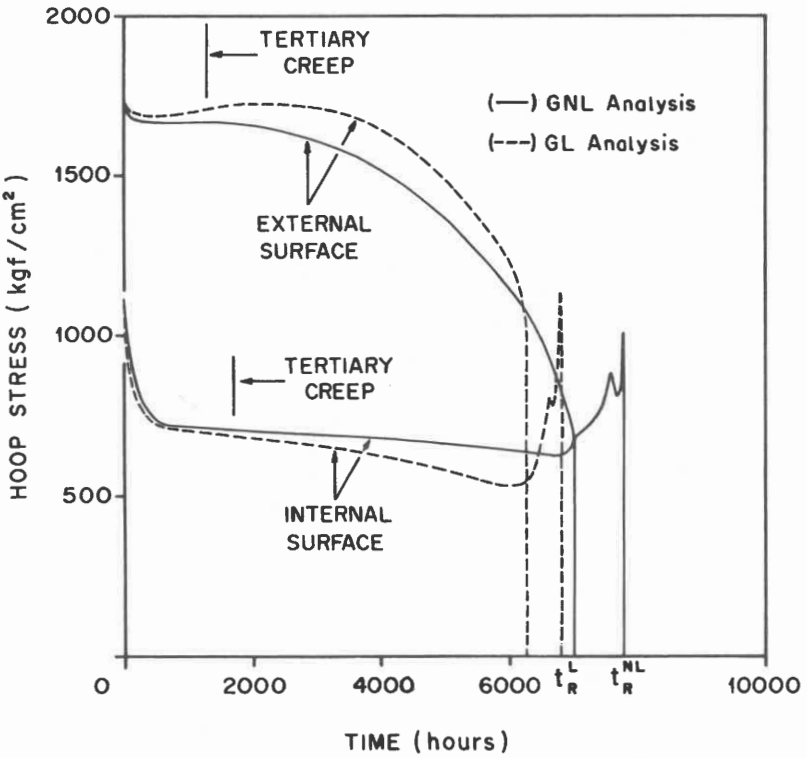

Figure 4. Variation of the hoop stress with time.

For the particular problem investigated with the stress concentration factor equal to 3.0, the average value of the creep hoop strain approximated by the effective creep strain value as 1.32%, and the derivative of the normalized limit pressure with regard to the maximum creep hoop strain determined as 3.0, it follows from equations 3 and 4 that

(5) $$t_R^{NL}/t_R^L = 1.39$$

This result, because of the approximations involved, can be said to reasonably convalidate the finite element based solution obtained in this work (Section 4), namely,

(6) $$t_R^{NL}/t_R^L = 1.14$$

5 DISCUSSION AND CONCLUDING REMARKS

In high temperature design of pressure vessels it is usual practice to assume that the geometry remains unaltered during the life of the component. The numerical results on the cylinder-sphere shell intersection obtained in this work provided further evidence that this procedure is conservative since the effect of geometry change is to cause an increase

in the lifetime of the vessel.

The relatively low beneficial effect observed in this study, however, is attributed to the low stress sensitivity of the creep and damage processes of the material selected ($N = 2.35; \chi = 1.96$). This assumption was investigated by performing a second analysis on a vessel of similar geometry but made of a hypothetical creep material with high stress dependency on both the deformation and damage processes ($N = 7.0; \chi = 20.0$). The material selected in this case obeys the Von Mises deformation criterion and fails according to the maximum principal tensile stress law.

In the geometric linear analysis a lifetime of 17034 hours was computed with an average effective creep strain of less than 1%. Despite this small level of average strain when geometry changes were considered the stresses were affected sufficiently enough to cause a substancial increase in the estimation of the vessel lifetime. In fact a life span 5.2 times greater than in the geometric linear analysis was then determined ($t_R^{NL} = 89025$ hours). As a consequence of the increase in the lifetime prediction the average value of the effective creep strain also increased up to 2.5% approximately.

In short it can be concluded that in the case of a pressurized cylinder-sphere intersection operating at elevated temperatures geometry changes do induce a beneficial effect on the vessel lifetime which is likely to be considerable for materials in which the deformation and damage processes are highly sensitive to stress.

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Constitutive equations II