Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

Analysis of thermal stresses and deformations of plates of variable thickness

Biswas, P.
P.D. Women's College, Reader in Mathematics, West Bengal, India

ABSTRACT: The thermo-elastic behaviour of a solid circular plate of exponentially-varying thickness under stationary temperature distribution has been investigated. Thermal deformation and thermal moments per unit length and also the bending stresses have been determined for both clamped and simply-supported plates.

1. INTRODUCTION:

Thermal stresses and deformation in plates of variable thickness have wide applications in structural mechanics, particularly in turbine discs, shafts and in the design of machine parts. Although a number of linear and nonlinear problems of plates of variable thichness without thermal loading have been investigated [1 - 4], very few papers have been devoted for thermally-stressed plates of variable thickness. The present analysis includes the effect of steady temperature distribution in an axisymmetric circular plate of exponentially-varying thickness. Such variation of thickness has been fully discussed in [5].

2. DERIVATION OF BASIC EQUATION AND ANALYSIS:

For a circular plate of radius $r = a$, the thickness h is assumed as

$$h = h_0 e^{kr} \quad (h_0, k \text{ are constants }).$$

In the steady state, the temperature distribution $T(r,z)$ satisfies the Laplace's heat conduction equation

$$\nabla^2 T = 0 \quad \text{where} \quad \nabla^2 = \text{Laplacian Operator.} \cdots \quad (1)$$

For thin plates the solution of the above equation is approximated as

$$T(r, z) = T_0 (r) + z T_1(r) \quad \cdots \quad (2)$$

of which the first part resulting in no stress, will not be taken into consideration, and the second part, giving rise to the bending problem is obtained in the form :

$$z T_1(r) = z (A + B \log r) \quad \ldots \quad (3)$$

where A and B are constants to be determined.

Considering the equation of equilibrium and the expressions for bending moments involving the thermal stress. couple one may arrive at the following differential equation :

$$r^2 \phi_{,rr} + (3kr^2 + r) \phi_{,r} +$$

$$+ (3k \nu r - 1) \phi = 3k A \alpha_t (1 + \nu) r^2 \quad \dots \quad (4)$$

where $\phi(r) = w_{,r}$, $w(r)$ being the normal displacent.

The two successive substitutions

$$\phi(r) = r\, u(r) \text{ and } u(r) = v(t), \quad t = -3kr \quad \dots \quad (5)$$

will reduce the equation (4) to a Confluent Hyper-geometric differential equation of the form

$$t\, v_{,tt} + (3 - t)\, v_{,t} - (1 + \nu)\, v = \lambda t^2 \quad \dots \quad (6)$$

where $\lambda = (1 + \nu) \alpha_t A / 3k \quad \dots \quad (7)$

The complementary function of equation (6) is given by

$$v_c(t) = C_1 \, {}^1F_1 (1 + \nu ; 3 ; t) \quad \dots \quad (8)$$

where C_1 is a constant.

The second solution involves logarithmic term and is dis - carded being indeterminate at the origin.

The particular integral $v_p(t)$ is also obtained as an infinite series.

Therefore, $\phi(r) = C_1 \, r \, {}^1F_1(1 + \nu ; 3 ; -3kr) + r v_{p(-3kr)}$

Since the above series for $\phi(r)$ is uniformly convergent, term-by-term integration is permissible. Accordingly the deflection $w(r)$ is obtained in the form of an infinite series with the addition of a constant of integration C_2.

The constants of integration C_1 and C_2 can be obtained by using the boundary conditions for both clamped and simply- supported plates.

The bending moments M_r and M_θ are obtained by using their usual expressions.

As $k \to 0$, the thickness becomes constant and in this limiting case the deflection vanishes and the bending moments become constants and given by

$$M_r = M_\theta = - D_0 (1 + \nu) \alpha_t A \quad \dots (9)$$

which are the same results as obtained by Nowacki in his famous treatise 'Thermo-elasticity' (P.467) [6] while determining the thermal deflection of a clamped circular plate of constant thickness.

For simply-supported plates and in the limiting case $k \to 0$ the deflectin and the bending moments are given by

$$w(r) = \frac{1}{2} A \alpha_t (a^2 - r^2), \quad M_r = M_\theta = 0 \quad \dots (10)$$

The results cited in (10) are in exact agreement with those given by Nowacki [6] for simply-supported plates of constant thickness.

3. CALCULATION OF BENDING STRESSES:

M_r and M_θ being known, the bending streses σ_r and σ_θ can be calculated from the following expressions

$$\sigma_r = 6\, M_r/h^2 , \quad \sigma_\theta = 6\, M_\theta/h^2 \qquad \ldots \quad (11)$$

Assum ng the set of values

$$3ka = 1, \quad 3kr = 0.5 , \quad \nu = 0.3 , \quad r = 1$$

for the case of clamped plates, one gets the results

$$\frac{(\sigma_r)_{r=1}}{(\sigma_r)_{origin}} = 0.76 , \quad \frac{(\sigma_\theta)_{r=1}}{(\sigma_\theta)_{origin}} = 0.80 \quad \ldots (12)$$

From equations (12) it can be concluded that although the bending stresses are constants at the origin, they assume different values at any other point depending on the values of k.

REFERENCES

Gran Olasson, R.& 1934 Ingeniur Archiv. 5 : 363

Reissner, E. 1937 Maths. Phys. J.(Cambridge,Mass).16:43

Mansfield, EH. 1962 Proc. Royal Soc.,Edin. Sect. A. 268 : 316

Banerjee, M M. 1976 Nonlinear vibrations of a circular plate of variable thickness , J. Sound and Vibration. 57:341

Timoshenko and Krieger. 1957 Theory of Plates and Shells, Mc Graw Hill Pub. Co., New York.

Nowacki, W.1962 Thermo-elasticity, Addision -Wesley,467 & 469.

ACKNOWLEDGEMENT :

The Author wishes to express his grateful thanks to the University Grants Commission of India, Department of Science and Technology Department, Govt. of India, and the Council of Scientific and Industrial Research(CSIR), New Delhi, for possible sanction of partial travel grant to enable the author to attend and present the paper at the 13- SMiRT Conference, Porto Alegre, Brazil, August, 1995.