SURFACE FLAW IN A THERMALLY SHOCKED HOLLOW CYLINDER

A.S. KOBAYASHI, A.F. EMERY, N. POLVANICH, W.J. LOVE

Department of Mechanical Engineering, University of Washington, Seattle Washington 98195, U.S.A.

SUMMARY

The objective of this paper is to illustrate a procedure for estimating the stress intensity factors of a semi-elliptical crack located in the inner or outer surface of a thermally shocked hollow cylinder.

The first step in this procedure is to estimate the transient thermal elastic stresses induced by sudden cooling of an uncracked cylinder by numerically evaluating standard heat transfer and thermal stress formulae. The stresses at the location of the crack surface in the uncracked cylinder are eliminated by the method of superposition in order to obtain a stress free crack surface.

The stress intensity factors are then determined by a judicious use of two sets of solutions, one set involving stress intensity factors for a semi-elliptical crack in a flat plate and subjected to a polynomial distribution of pressure loading, and another set involving single-edge notched plates with prescribed edge-displacements and single-edge internally or externally notched cylinders with thermal shock loading. The former solutions are determined by the alternating technique in three-dimensional fracture mechanics with a fourth order polynomial pressure distribution on the crack surface where both the front and back surface effects are accounted for. The latter solutions involve two-dimensional finite element solutions of single-edge notched plates with prescribed edge-displacements and single-edge notched cylinders with thermal shock loading. By comparing these two two-dimensional solutions, an estimate of the effect of the cylindrical curvature on an edge-cracked plate is obtained. The combination of these two sets of solutions thus yields an estimate of the stress intensity factor in an internal and external semi-elliptical crack in a thermally shocked cylinder.

Due to length limitation, the compact will provide only stress intensity factor solutions necessary to construct final stress intensity factor solutions for hollow cylinder with inner-to-outer diameter ratios of 1:2 and 4:5 and with internal and external semi-elliptical cracks of crack aspect ratios of 0.1 and 0.5 at crack depths of 0.4 and 0.6.

INTRODUCTION

A major problem which is receiving considerable attention in thermal cracking is the fracture resistance of a nuclear pressure vessel subjected to transient thermal stresses in the event of an emergency core cooling. In this process, the entire heated pressure vessel is flooded by cold water, thus subjecting the vessel wall to severe thermal shocking. The induced thermal stresses which are momentarily superimposed upon the mechanical stresses could act to trigger brittle fracture if a crack of sizeable dimension pre-existed in the pressure vessel.

A procedure for estimating the fracture resistance of a nuclear pressure vessel in the event of emergency core cooling is to postulate a hypothetical surface crack, normally of a crack aspect ratio of $1/10 \sim 1/20$ at the inner surface of the vessel. Such surface crack can be generated by low cycle fatigue (due to startups and shutdowns) along the irradiation embrittled inner wall. Having postulated a surface crack of sufficient magnitude, linear elastic fracture mechanics can then be used to estimate the onset of fracture under the combined thermal and mechanical loadings provided the respective stress intensity factors are known.

Unfortunately, the relatively simpler inner and outer cylindrical boundaries impose severe complexity on the already complex analytical techniques in three-dimensional fracture mechanics. Two-dimensional, plane strain solutions for extremely oblong surface flaws in pressurized cylinders [1] and thermally shocked cylinders [2] have been obtained by various numerical techniques. Only recently did stress intensity factors for surface crack with finite crack aspect ratio, in pressurized cylinders, become available [3]. Two-dimensional stress intensity factors of an internal flaw of a partially filled, thermally shocked cylinder were also determined by photothermoelasticity [4]. More recently Blauel et al. used glass cylinders to investigate the behavior of surface cracks under transient thermal load and noted that crack arrest should occur as the surface crack penetrated into the original compression region of the uncracked ligament of the cylinder [5]. This qualitative conclusion is of practical significance and warrants further consideration since recent investigations in crack arrest indicate that the static stress intensity factor must be $1/2 \sim 1/3$ of the fracture toughness in order to arrest a rapidly propagating crack [6].

The purpose of this paper is to estimate the stress intensity factors of semi-elliptical surface cracks at the inner surface of a cylinder suddenly chilled at its inner surface. The analytical procedure is to first determine thermal stresses in a thermally shocked uncracked cylinder. These thermal stresses on the crack surface are then freed by imposing opposite stresses on the crack surface. The latter step is accomplished by using the alternating technique in three-dimensional fracture mechanics. In the following, brief outlines of each step in this procedure are described.

THERMAL STRESSES IN AN UNCRACKED CYLINDER SUDDENLY CHILLED AT ITS INSIDE SURFACE

For a hollow cylinder which is restrained against axial displacement, the hoop stress, $\sigma_{\theta\theta}$, is given by [7]

$$\sigma_{\theta\theta} = \frac{\alpha E}{r^2} \left\{ \frac{r^2 + R_i^2}{R_o^2 - R_i^2} \int_{R_i}^{R_o} Tr\,dr + \int_{R_i}^{r} Tr\,dr - Tr \right\} \qquad (1)$$

where α is the linear coefficient of thermal expansion

E is the modulus of elasticity

R_i and R_o are the internal and external radii of the cylinder, respectively

T is the temperature

r is the radial distance from the cylinder axis.

Emery [8] has shown that this expression is accurate to within 1/2% for cylinders with thickness to external radius ratios smaller than one. The temperature distribution in a cylinder initially at T_o whose inner surface ($r = R_i$) is instantaneously reduced to T_∞ is thus given by [9]:

$$\frac{T-T_\infty}{T_o-T_\infty} = -\pi \sum_{n=1}^{\infty} e^{-\kappa\alpha_n^2 t} \frac{J_o(R_i\alpha_n)J_o(R_o\alpha_n)\, U_o\,(r\alpha_n)}{J_o^{\,2}(R_i\alpha_n) - J_o^{\,2}(R_o\alpha_n)} + \frac{\ln r/R_i}{\ln R_o/R_i} \qquad (2)$$

where $U_o(r\alpha_n) = J_o(r\alpha_n)\, Y_o(R_o\alpha_n) - J_o(R_o\alpha_n)\, Y_o(r\alpha_n)$

J_o and Y_o are Bessel functions of the first and third kind, respectively

α_n are the roots of $U_o(R_i\alpha_n) = 0$ (3)

κ is the thermal diffusivity.

Note that infinite surface heat transfer coefficients are not attainable in real systems and therefore the temperature response is slower than the values predicted by Equation (2). However, the effect of a delayed response only reduces the stress intensity factors [8] and does not change the temporal characteristics of the fracture response.

Once the thermal stresses due to sudden chilling are known, the stress intensity factor in the cracked cylinder can be determined by simple superposition.

SURFACE FLAW WITH PRESCRIBED PRESSURE DISTRIBUTION

The method of approach used in this section is the iterative procedure based on the alternating technique for solving three-dimensional problems in fracture mechanics. This procedure is well documented in the papers by F.W. Smith [10] and Shah [11] and will not be repeated here. The numerical difficulties in computing the front-surface stress intensity magnification factor for a semi-elliptical crack as described in Reference [11] were removed by prescribing appropriate fictitious pressure on the fictitious one-half part of the elliptical crack which protrudes into the free half space [12,13]. Such properly prescribed fictitious pressure enhanced the numerical convergence of the iterative procedures and thus made it possible to solve a series of problems heretofore unattempted except for the recent effort by F.W. Smith [14].

In addition to improving the numerical convergence of the three-dimensional alternating procedure, the efficiency of the numerical procedure was improved by partial replacement of the third order derivatives of the stress functions with numerical differentiation and judiciously reducing the number of repetitive computations in freeing the front and back surfaces. Thus, a workable numerical procedure in which the stress intensity magnification factors of shallow to deep surface flaws in a structural member with free front and back surface was finally developed.

The alternating technique, in its current state of development, with fictitious pressure cannot, however, be used to solve surface crack problems in cylinders since one of the two solutions used in this alternating process is the half-space solution with a flat free surface boundary. A comparable Love's solution for a cylindrical surface in an infinite solid or an infinite cylinder does not exist at this time. As a result, surface crack solutions in flat plates with thermal stress loading on the crack surfaces were first derived by using the improved alternating technique. These flat plate solutions are then corrected by curvature correction factors derived from two-dimensional analogs of edge-cracked, thermally shocked cylinders and single edge-cracked plates with identical thermal stresses prescribed on the crack surface. Figure 1 shows schematically this solution procedure which combines the results of three-dimensional surface-cracked flat plates and the curvature corrections derived from two-dimensional analogs. The solution procedure used in this analysis thus consists of two major parts: solutions for a semi-elliptical crack in a plate and two-dimensional solutions for internal edge cracks in cylinders and single edge-cracked plates. Further details of this solution procedure are given in Reference [13].

Two-dimensional Analogs

Two-dimensional solutions of an internally edge-cracked, thermally shocked cylinder can be obtained by prescribing the appropriate thermal stresses on the crack surface of an isothermal cylinder. The flat plate approximation of this problem is a single edge-notched plate, which is restrained from end rotations by prescribed edge displacements, and with a prescribed pressure equal to the thermal stress in the uncracked cylinder. These two sets of solutions were obtained by using the finite element program with embedded singularities of unknown stress intensity factors recently developed by Emery et al. [15]. The ratios of the stress intensity factors obtained by comparing the corresponding two sets of solutions yield the curvature correction factor sought for converting the plate solution into that of a cylinder as illustrated schematically in Figure 1.

Semi-elliptical Crack in an Infinite Plate

The pressure profile prescribed on the surface of a semi-elliptical crack in a pressurized cylinder is represented by the following third order polynomial:

$$\sigma_{zz}(x,y) = B_{00} + B_{01}(1-y/b) + B_{02}(1-y/b)^2 + B_{03}(1-y/b)^3 \qquad (4)$$

where the Cartesian coordinate system and the geometry of the semi-elliptical crack are given in Figure 1. B_{ij}'s are constant coefficients and b is the semi-minor diameter of the ellipse. This pressure profile is least square fitted to each normalized thermal stress at a given time. The pressure distribution represented by Equation (4) suggests a decomposition of the pressure distribution into four components of $\sigma_{zz}(x,y) = 1$, $(1-y/b)$, $(1-y/b)^2$ and $(1-y/b)^3$ which can be linearly combined to represent a general thermal stress distribution on the crack surface.

The three-dimensional alternating technique was then used to determine the stress intensity factor for a deep semi-elliptical crack in a finite thickness plate and subjected to the above four pressure loadings on the crack surface. Some discussions on these computation details for related surface crack problems are found in References [11,12,13].

RESULTS

Figure 2 shows typical thermal stress distributions at several time intervals in a thermally shocked cylinder of $R_o/R_i = 10/9$. At a normalized time of about $\kappa t/R_i^2 = 0.05$ the thermal stress distribution reaches a steady state condition.

Table I shows the curvature correction factors derived for the two-dimensional analog. Instead of deriving these correction factors for all possible distributions of thermal stresses, curvature corrections were derived for the four polynomial components of σ_o,

$\sigma_o(1-y/b)$, $\sigma_o(1-y/b)^2$, and $\sigma_o(1-y/b)^3$. These curvature correction factors can then be superposed to obtain the necessary correction for each thermal stress or any stress distribution represented by Equation (4).

Figures 3 and 4 show the stress intensity magnification factors for two elliptical cracks, b/a = 0.2 and 0.98, respectively, in a flat plate and at crack depth of b/h = 0.6, and for four applied crack pressures of σ_o, $\sigma_o(1-y/b)$, $\sigma_o(1-y/b)^2$ and $\sigma_o(1-y/b)^3$. Similar curves for elliptical crack with b/a = 0.2 at crack depth of b/h = 0.4 are shown in Figure 5. Five iterations in the alternating technique were executed for each loading condition, requiring approximately 1500 seconds of CPU time on CDC 6400 computer. The residual surface tractions on the crack surface ranged from a high of 0.10 at an isolated region where the crack periphery intersects the free front surface to $\sigma_{zz} = 10^{-3}$ for b/a = 0.98 and $\sigma_{zz} = 1$. Generally, the maximum residual tractions for other crack problems were of the order of 0.02 again at the isolated region where the crack periphery intersects the free front surface. The maximum residual surface tractions on the free front bounding surface were of the order of 10^{-2} and 10^{-3} on the free back bounding surface.

One-time eliminations of the isolated high residual surface tractions, following the procedure outlined in Reference [13], were then conducted. The relatively small values of residual surface tractions on the crack surfaces, however, did not produce a pronounced downward trend in the stress intensity magnification factors near the free front bounding surface as reported in Reference [13] and originally in Reference [16.]

The polynomial representation of Equation (4) was then least square fitted with appropriate weighting functions to the thermal stress distributions shown in Figure 2 and the coefficients of B_{ij} were determined. The stress intensity magnification factor for each pressure distribution of $B_{ij}(1-y/b)^j$ was then corrected with the respective curvature correction factors shown in Table I. The curvature-corrected stress intensity magnification factors were then superposed to yield the stress intensity magnification factors for the semi-elliptical crack at various time intervals as shown in Figures 6, 7 and 8. These curves show the characteristic decrease in stress intensity magnification factor due to decreasing thermal stresses and front surface effect of the internal cylindrical surface.

CONCLUSIONS AND DISCUSSION

1. Stress intensity magnification factors for semi-elliptical cracks of b/a = 0.2 and 0.98 at crack depth of b/h = 0.6 and 0.4 in a plate subjected to crack pressure loadings of σ_o, $\sigma_o(1-y/b)$, $\sigma_o(1-y/b)^2$, and $\sigma_o(1-y/b)^3$ were determined by using an improved three-dimensional alternating technique.
2. Using a curvature correction factor, the above stress intensity magnification factors were converted into stress intensity magnification factors for the corresponding semi-elliptical cracks in pressurized cylinders of R_o/R_i = 10/9, shown in this paper, and R_o/R_i = 7/6, 5/4 and 3/2 which are not shown due to the conference-imposed page limitations.

ACKNOWLEDGEMENT

The work reported in this paper is sponsored by the Electric Power Research Institute under Contract No. RP231-0-0. The authors wish to thank Drs. Conway Chan and A. Gopalkrishnan of EPRI for their encouragement throughout the course of this research program.

REFERENCES

1. Kobayashi, A.S., "A Simple Procedure for Estimating Stress Intensity Factors in a Region of High Stress Gradient," Significance of Defects in Welded Structures (ed. by T. Kanazawa and A.S. Kobayashi), Univ. of Tokyo Press, 1974, pp 127-143.
2. Emery, A.F., "Stress Intensity Factors for Thermal Stresses in Thick Hollow Cylinders," J. of Basic Engineering, Trans. of ASME, Series D, vol 87, 1966, pp 45-52.
3. Kobayashi, A.S., Polvanich, N., Emery, A.F., Love, W.J., "Stress Intensity Factor of a Surface Crack in a Pressurized Cylinder," to be published in Symposium Proceedings of the 2nd National Conf. on Pressure Vessels and Piping, San Francisco, June 24-26, 1975.
4. Emery, A.F., Williams, J.A., Avery, J., "Thermal-stress Concentration Caused by Structural Discontinuities," Experimental Mechanics, vol 9, no 12, Dec. 1969, pp 558-564.
5. Blauel, J.G., Kalthoff, J.F., Stahn, D., "Model Experiments for Thermal Shock Fracture Behavior," to be published in the J. of Engineering Materials and Technology, Trans. of ASME.
6. Kobayashi, A.S., "Criterion for Crack Branching and Crack Arrest," Progress in Experimental Mechanics - Durelli Anniversary Volume, 1975.
7. Boley, B.A., Wiener, J.H., Theory of Thermal Stresses, John Wiley & Sons, New York, 1960.
8. Emery, A.F., Walker, G.F., Jr., Williams, J.A., "A Green's Function for the Stress Intensity Factor and Its Applications to Thermal Stresses," J. of Basic Engineering, Trans. of ASME, Series D, vol 91, Dec. 1969, pp 618-624.
9. Carslaw, H.S., Jaeger, J.C., Conduction of Heat in Solids, Oxford Press, Oxford, 1959.
10. Smith, F.W., "The Elastic Analysis of the Part-circular Surface Flaw Problem by the Alternating Method," The Surface Crack: Physical Problems and Computational Solutions (ed. by J.L. Swedlow), ASME, 1972, pp 125-152.
11. Shah, R.C., Kobayashi, A.S., "On the Surface Flaw Problem, ibid, loc cit, pp 79-124.
12. Kobayashi, A.S., Enetanya, A.N., Shah, R.C., "Stress Intensity Factors of Elliptical Cracks," to be published in the Proc. of the Conf. on the Prospects of Fracture Mechanics, Delft, The Netherlands, June 24-28, 1974.
13. Kobayashi, A.S., Enetanya, A.N., "Stress Intensity Factor of a Corner Crack," to be published in the Proc. of the 8th National Symposium on Fracture Mechanics, ASTM STP.
14. Smith, F.W., Sorensen, D.R., "Mixed Mode Stress Intensity Factors for Semi-Elliptical Surface Cracks," NASA CR-134684 prepared under NASA LeRC Grant NGL-06-002-063, June 1974.
15. Emery, A.F., Cupps, F.J., Neighbors, P.K., "The Use of Singularity Programming in Finite Element Calculations of Elastic Stress Intensity Factors, Plane and Axisymmetric - Applied to Thermal Stress Fracture," to be published in the Symposium Proc. of the 2nd National Conf. on Pressure Vessels and Piping, San Francisco, June 24-26, 1975.
16. Hartranft, R.J., Sih, G.C., "Alternating Method Applied to Edge and Surface Crack Problems," Mechanics of Fracture I - Methods of Analysis and Solutions of Crack Problems (ed. by G.C. Sih), Noordhoff Intl, 1973, pp 179-238.

TABLE I

CURVATURE CORRECTION FACTOR

Crack Depth b/h	Stress Intensity Factor psi$\sqrt{in}$	CRACK PRESSURE			
		σ_o	$\sigma_o(1-y/b)$	$\sigma_o(1-y/b)^2$	$\sigma_o(1-y/b)^3$
0.4	$K_{cylinder}$	1.778	.847	.650	.497
	K_{plate}	1.926	.927	.713	.546
	Curvature Correction Factor M_c	.923	.913	.912	.910
0.6	$K_{cylinder}$	3.418	1.906	1.409	1.038
	K_{plate}	3.930	3.228	1.659	1.228
	Curvature Correction Factor M_c	.870	.856	.849	.846

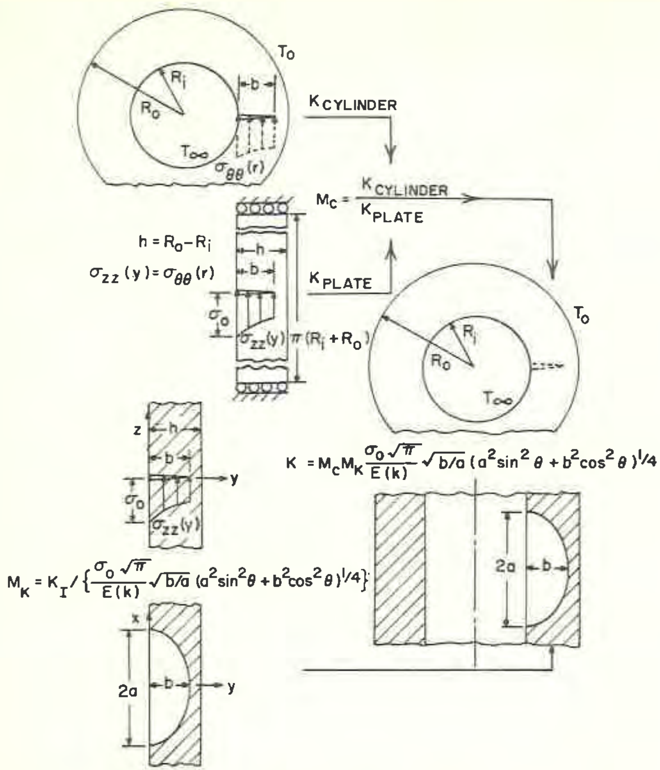

FIGURE 1. PROCEDURE FOR ESTIMATING STRESS INTENSITY FACTOR OF AN INTERNAL SEMI-ELLIPTICAL CRACK IN A THERMALLY SHOCKED CYLINDER.

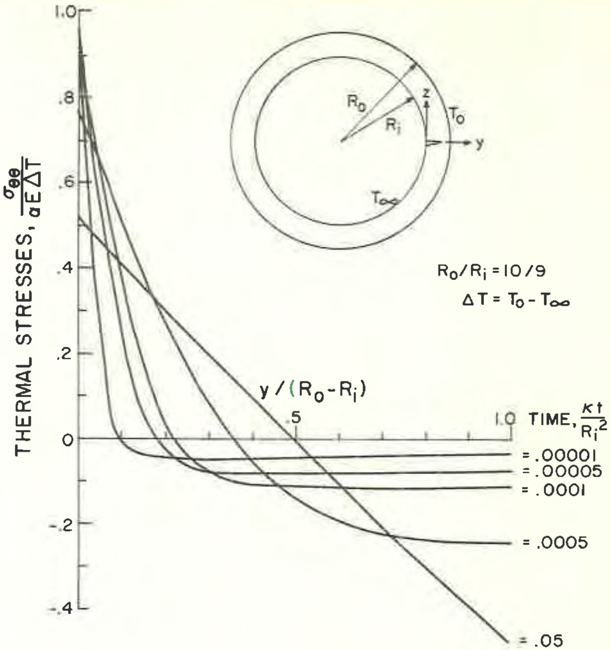

FIGURE 2. THERMAL STRESSES IN A CYLINDER SUDDENLY CHILLED AT THE INTERNAL SURFACE.

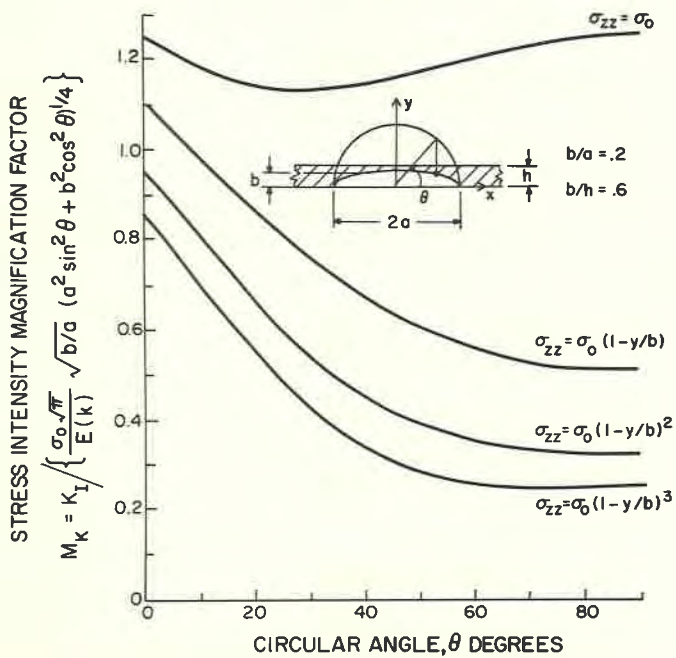

FIGURE 3. STRESS INTENSITY FACTOR OF A SEMI-ELLIPTICAL CRACK IN A PLATE AND SUBJECTED TO FOUR PRESSURE LOADINGS.

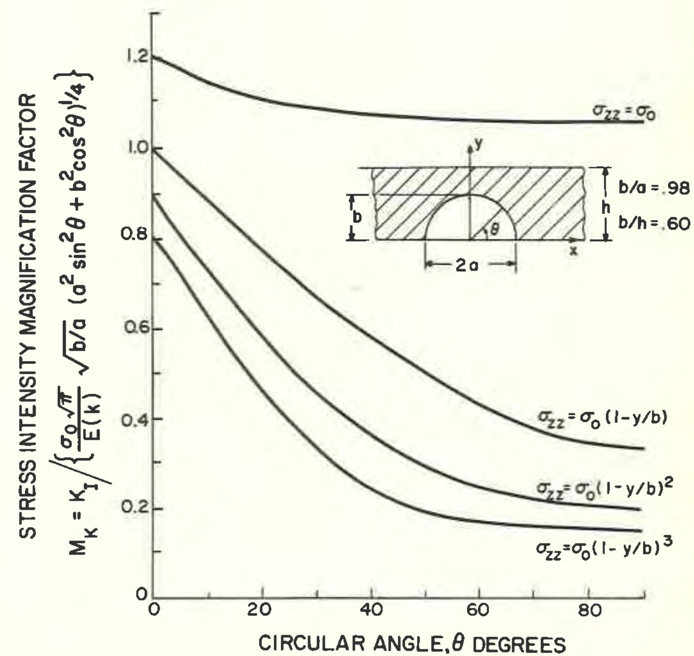

FIGURE 4. STRESS INTENSITY FACTOR OF A SEMI-ELLIPTICAL CRACK IN A PLATE AND SUBJECTED TO FOUR PRESSURE LOADINGS

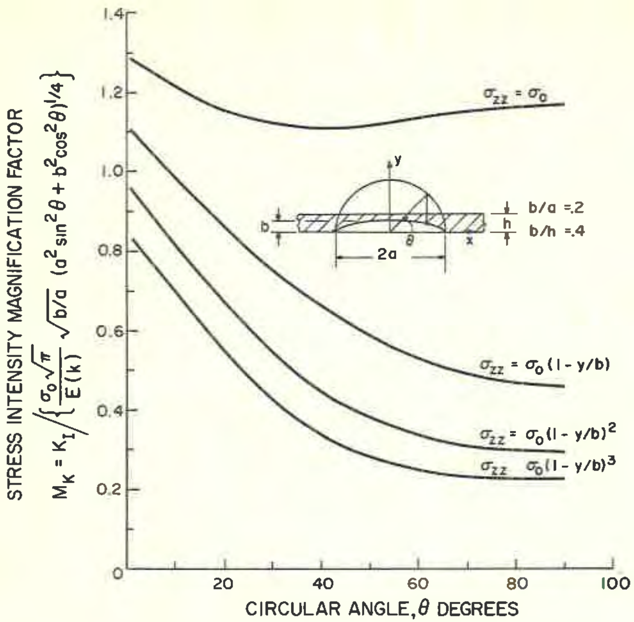

FIGURE 5. STRESS INTENSITY FACTOR OF A SEMI- ELLIPTICAL CRACK IN A PLATE AND SUBJECTED TO FOUR PRESSURE LOADINGS.

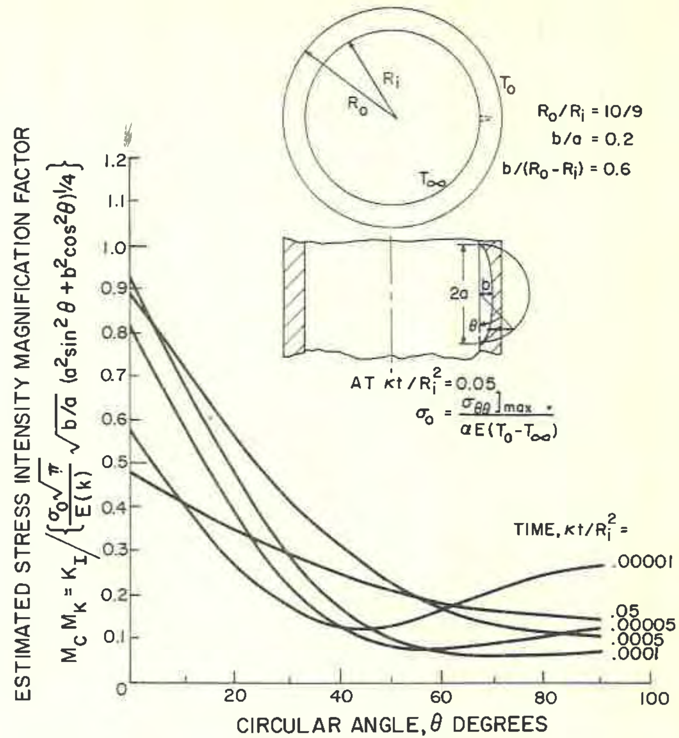

FIGURE 6. STRESS INTENSITY FACTOR OF AN INTERNAL SEMI-ELLIPTICAL CRACK IN A THERMALLY SHOCKED CYLINDER

G 4/3

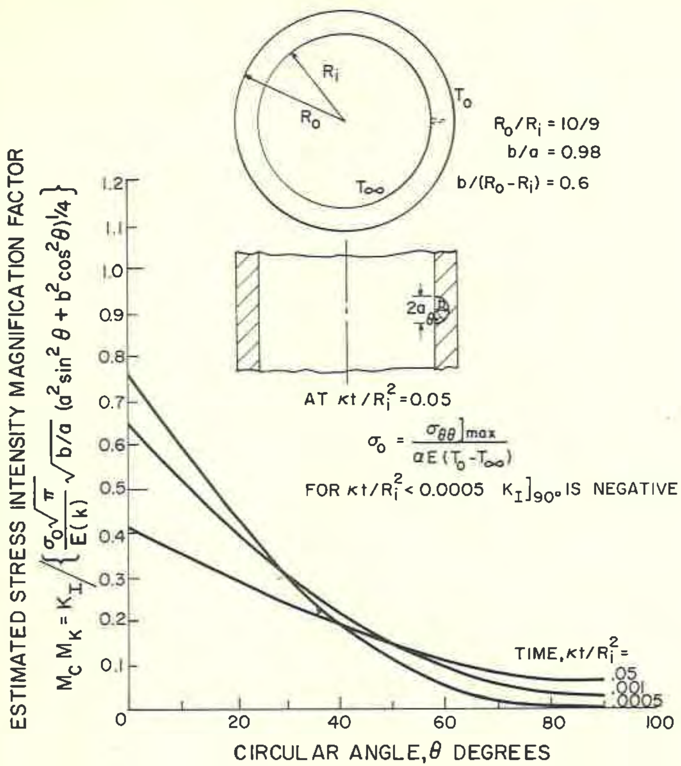

FIGURE 7. STRESS INTENSITY FACTOR OF AN INTERNAL SEMI-CIRCULAR CRACK IN A THERMALLY SHOCKED CYLINDER.

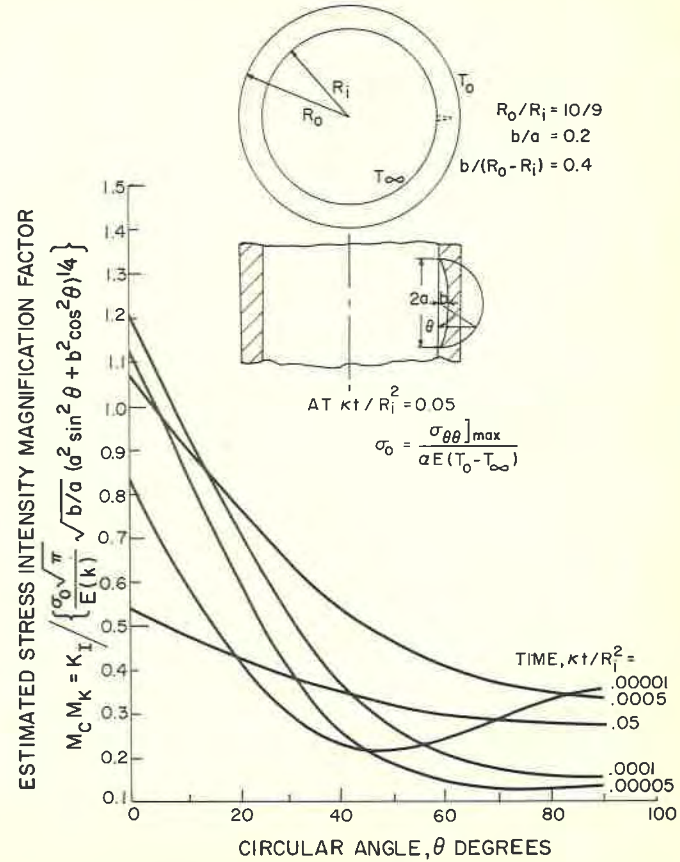

FIGURE 8. STRESS INTENSITY FACTOR OF AN INTERNAL SEMI-ELLIPTICAL CRACK IN A THERMALLY SHOCKED CYLINDER.

G 4/3