FORMATION OF LOCA JETS AND INDUCED FLOWS DURING WATER CLEARING

C. K. CHU, T. T. LEE
Burns & Roe, Incorporation, 185 Crossways Park Drive, Woodbury, New-York 11797, U.S.A.

H. C. LUI
Plasma Physics Laboratory, Columbia University, New York, New York 10027, U.S.A.

Abstract

The formation of the head of the jet during the water clearing phase of LOCA, as well as the induced flow, are obtained for the axisymmetric case including boundary effects. The numerical method used is as follows: discrete vorticity is shed from the jet-fluid interface, the velocity field is solved from Poisson's equation for the stream function with the vorticity as the right hand side, and the vortices are then convected with the local velocities. Computed results are compared with model tests.

1. Introduction

During the vent clearing phase of LOCA operation, water jets enter the suppression pool from the downcomers with a mushroom-shaped vortex ring at the head of each jet. Although the phenomenon has been known qualitatively for some time, a detailed calculation of the ring vortex has thus far not been achieved. For estimating loads on submerged structures and on boundaries, it is essential that the shape of the mushroom be accurately determined.

This paper presents a numerical method to calculate the mushroom and the entire flow field in the high Reynolds number limit. The method is self-consistent and has no adjustable parameters, and results agree very well with experiment. Finite boundaries are automatically incorporated, but axisymmetry must be assumed - the axis of the downcomer and jet must be the axis of symmetry of the entire geometry.

2. Mathematical Formulation

The Euler equations for an incompressible inviscid fluid are replaced by the equivalent set of equations for the Stokes stream function ψ and vorticity ζ in axisymmetric geometry:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial z} \quad , \quad v_z = -\frac{1}{r}\frac{\partial\psi}{\partial r} \tag{1}$$

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right) + \frac{\partial^2\psi}{\partial z^2} = -r\zeta \tag{2}$$

$$\frac{D}{Dt}\left(\frac{\zeta}{r}\right) = 0 \tag{3}$$

Figure 1 shows the domain in which the problem is to be solved. Boundary values are prescribed for ψ (or its normal derivative) at all boundary points, while boundary values for ζ are prescribed at the water entry only.

We assume the downcomer velocity, U(t), a given function of time, to be uniformly distributed across the downcomer exit section, and we assume the free surface to be sufficiently far up so that the velocity U' there is already uniform (the latter assumption is equivalent to assuming the free surface to be level and at constant pressure). By continuity,

$$U' = Ua^2/(R^2-a^2) \tag{4}$$

The boundary conditions for ψ are then

$$\begin{aligned} \psi &= 0 && \text{on the center line, side wall and floor} \\ \psi &= -\tfrac{1}{2}Ua^2 && \text{on downcomer wall} \\ \psi &= -\tfrac{1}{2}Ur^2 && \text{at the downcomer exit} \\ \psi &= -\tfrac{1}{2}U'(R^2-r^2) && \text{at the free surface} \end{aligned} \tag{5}$$

In solving equation (2) for ψ, time t enters the problem through the boundary conditions only, since (2) does not contain t as an independent variable.

The boundary conditions for ζ can be given only at the downcomer exit. We stipulate a uniform jet, and the vorticity is then concentrated at the edge of the jet, point A of Figure 1. Then

$$\zeta = 0 , \quad Z = 0 , \quad 0 \leq r < a \qquad (6)$$

$$\zeta = \text{a delta function}, \quad z = 0 , \quad r = a$$

The latter is handled in the following manner. A flow of velocity U on one side of the vortex sheet and zero on the other side (in this case, the boundary layer in the downcomer) has a circulation of U per unit length (the vorticity is infinite since the sheet is of zero thickness). Moreover, the vortex sheet travels at exactly U/2. Hence, we prescribe both the strength and the rate of entry of the concentrated vortex sheet at the point A.

3. Solution Procedure

The set of differential equations (1) through (3), subject to the boundary conditions stipulated, are solved in the domain by finite difference; one may also prefer a finite element procedure, but the difference is inessential. The domain is subdivided into a grid of widths Δr and Δz. To solve (2), ζ on the right-hand side must be known on the grid points, but strictly speaking, ζ is zero everywhere and infinite on the vortex sheet. A spreading procedure is described below; but accepting that for the time being, (2) is differenced and solved in the standard manner at each time step using, for example, the alternating direction implicit method (or one may use a direct Poisson solver for computational economy, but boundary conditions are harder to treat). Once ψ is found, the velocities v_r and v_z are found by simple differencing.

It only remains to describe the treatment of the vortex sheet, which we discretize into many elemental vortex rings. Thus, if there are n rings injected into the domain per Δz, the circulation of each ring is $U\Delta z/n$, since U is the circulation per unit length. The vortex is injected initially at the point A with the velocity U/2, but afterwards moves with the self-consistent fluid velocity at each location. The self-consistent velocity of each vortex location is found by double linear interpolation from the velocities of the four corner grid points of the cell in which the particular ring is resident. This procedure is the consistent approximation for equation (3). The positions of the vortices are obtained from the velocities by simple forward integration.

The final point is the spreading of the circulation of the concentrated vortices onto the gird points as vorticity. Since a vorticity ζ at a grid point carries a circulation $\zeta \Delta r \Delta z$, we can use inverse linear interpolation to spread the circulation Γ of a concentrated vortex to the four corners in the following way. Referring to Figure 2, where A_1 through A_4 represent the shaded areas, we simply prescribe

$$\zeta_1 = \frac{\Gamma}{(\Delta r \Delta z)^2} \cdot A_1 \quad , \quad \zeta_2 = \frac{\Gamma}{(\Delta r \Delta z)^2} \cdot A_2 \quad , \quad \cdot \cdot \cdot \cdot \tag{7}$$

The total circulation of the four corners is then exactly Γ, since

$$\begin{aligned} &\zeta_1 \Delta r \Delta z + \zeta_2 \Delta r \Delta z + \dots \\ &= \frac{\Gamma}{\Delta r \Delta z} (A_1 + A_2 + A_3 + A_4) \\ &= \Gamma \end{aligned} \tag{8}$$

This spreading is not diffusive, i.e., the vorticity does not continue to spread outward, since we always maintain the identity of each elemental ring, doing the spreading only in each time step to solve equation (2). This procedure is somewhat similar to giving each elemental vortex with a finite but fixed cross section.

In addition to the vortices, fluid "particles" are also ejected from the downcomer in order to simulate the dye markers in a real experiment. They are moved in exactly the same way as the elemental vortices, but they do not participate in any dynamic calculation.

4. Results

Figure 3 shows typical mushroom vortices so calculated. Figure 4 and 5 show the time growth of the penetration and width when the calculation procedure is applied to a test performed by SRI/EPRI, [1]. The agreement lends confidence to the basic correctness of this model.

Reference

[1] KIANG, R. L., JEUCK, P. R., "A Study of Pool Swell Dynamics in a Mark II Single-Cell Model," Stanford Research Institute, Menlo Park, California.

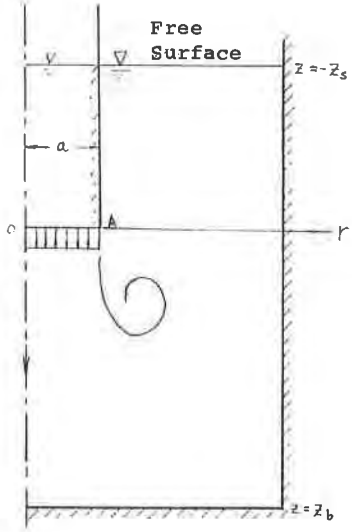

Figure 1: Domain of Calculation

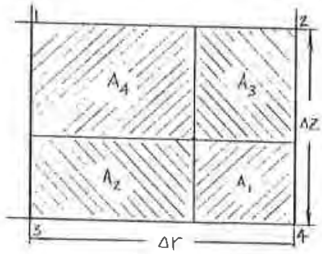

Figure 2: Spreading Scheme

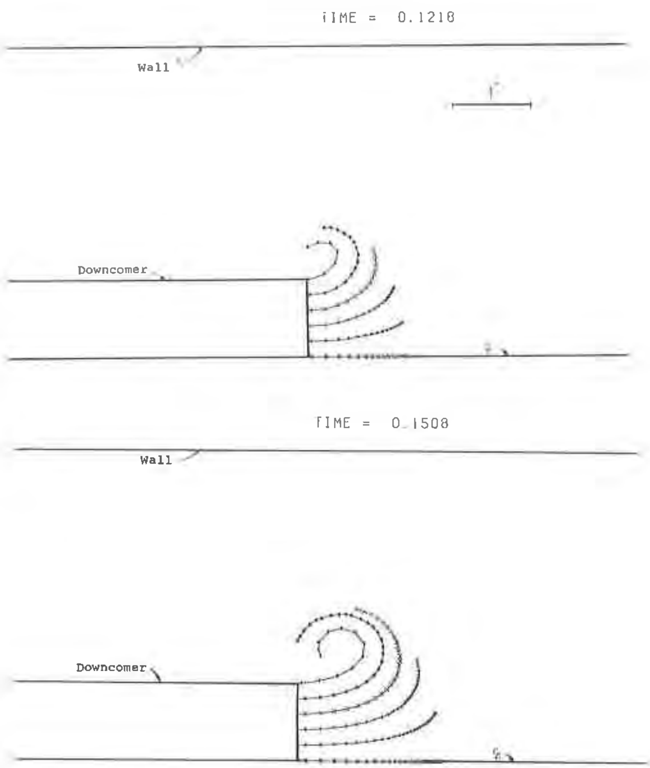

Figure 3 Formation of Mushroom

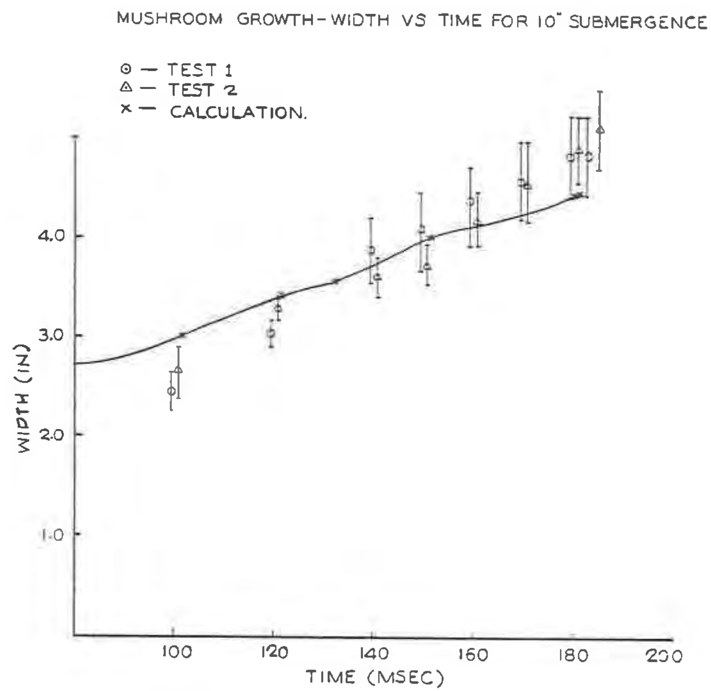

Figure 4 Mushroom Growth - Penetration vs. Time

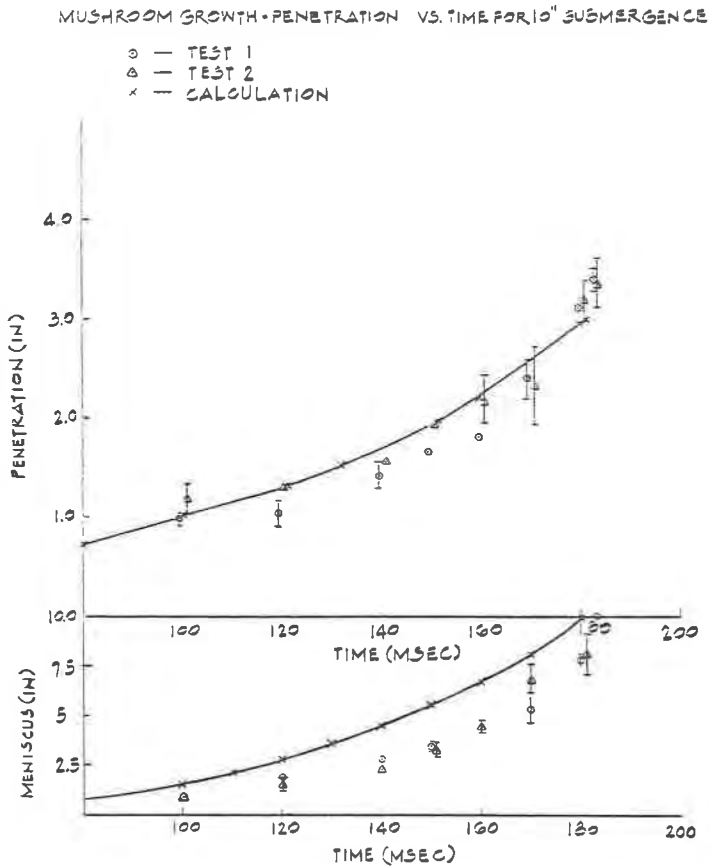

Figure 5 Mushroom Growth - Width vs. Time