A CRITICAL PLANE BASED MODEL FOR FATIGUE ASSESSMENT UNDER FIXED AND ROTATING PRINCIPAL DIRECTION LOADING

Suneel K. Gupta[1], T. M. Fesich[2], X. Schuler[3], V. Bhasin[1], K. K. Vaze[1] and E. Roos[3],
[1]Reactor Safety Division, Bhabha Atomic Research Centre, Mumbai, India
[2]IMWF, University of Stuttgart, Stuttgart, Germany; [3]MPA, University of Stuttgart, Stuttgart, Germany
E-mail of corresponding author: suneelkg@barc.gov.in

ABSTRACT

Nuclear Power Plant Mechanical Components such as piping, vessels etc. are subjected to complex multi-axial cyclic loading during normal operation and other transients and events considered in design. Fatigue damage is generally evaluated according to the design codes, where Rankine, Tresca or von-Mises theory along with fatigue life curves (S-N curve based on axial tests) and linear damage accumulation rules are used. However, due to complex geometries and loading, at structural discontinuity locations, the induced stresses/ strains are multi-axial and the principal directions rotate during a loading cycle. The rotation of principal direction is known to cause additional fatigue damage. In view of this, a systematic investigation on multi-axial fatigue is underway where a series of fatigue tests is being conducted on SA333Gr6 carbon steel material under tension-torsion loading, proportional and non-proportional loading conditions. A Multi-axial fatigue analysis software has been developed for fatigue evaluation using various critical plane based models and ASME Code procedure (Tresca Criteria, von-Mises models). In addition to in-house multi-axial fatigue test data on SA333Gr6, test data from literature also was analysed using these fatigue models. It is observed that the fatigue life observed in tests under varying principal direction loading are shorter when compared to predictions of most of the fatigue models using the best fit fatigue life curve generated from uniaxial tests. Based on the test data and their analyses, a critical plane based fatigue model is proposed which provides better and conservative correlation of fatigue life under varying principal direction loading with respect to the best fit axial fatigue life curve. The new model was validated for 5 different materials and under multi-axial – non proportional loading conditions.

INTRODUCTION

Under cyclic loading, fatigue is the most common cause for failure initiation in these components. Nuclear Power Plant Mechanical Components such as piping, vessels etc. are subjected to cyclic stresses/strains during the day to day operation loading and several other transients and events considered in designing of these components. Due to complex geometries like structural discontinuity locations, and asynchronous loadings, the induced stresses/ strains are multiaxial and non-proportional. The non-proportional loading is one when, the principal directions rotate during a loading cycle. The fatigue damage is more under non-proportional loading. In most of the design codes, fatigue damage is evaluated using a multi-axial hypothesis such as Tresca or von-Mises along with fatigue life curves (S-N curves) evaluated from the material's uniaxial fatigue tests and safety factors are used to account for ignorance / uncertainties. The fatigue damage assessment under multi-axial non-proportional loading is not well understood and continues to be investigated by many researchers. A literature study showed availability of more than 40 models for multi-axial fatigue analyses. These models have used different definitions for a fatigue equivalent parameter or Fatigue Parameter (FP). In the present work, systematic investigations have been carried out at LCF conditions. Different fatigue assessment models such as ASME Sec.III NB procedure, von-Mises /ASME Sec.III NH Appendix-T procedure, selected critical plane based models like Fatemi-Socie, Smith Watson-Topper (SWT) etc. have been used to predict the fatigue crack initiation life. A Multi-axial Fatigue Analysis (MFA) software has been written for multi-axial fatigue assessment using ASME Sec III NB /NH fatigue models and several critical plane based fatigue models. Results from 230 fatigue tests which include pure axial, pure torsion and axial-torsion with different phase shifts loading conditions, have been used to evaluate these fatigue models. In this, about 50 numbers of fatigue tests were from in-house tests on SA333Gr6 Carbon steel (Indian PHWR piping material) by Gupta [1]. The remaining data of about 180 multi-axial fatigue tests on SS347 austenite stainless steel (see Hoffmeyer [2, 3]), SA406N high strength structural steel (see Hoffmeyer [2, 3], Jiang [4]), Al 5083 aluminium alloy see (Hoffmeyer [2,3]) and SA302B carbon steel (see Gao [5]), have been taken from literature and used for evaluation of above fatigue models. The ASME Sec III procedure, along with the best fit fatigue life curve (without use of safety factors, obtained from axial fatigue tests), yields reasonable prediction results for the proportional loading axial-torsion tests. However, for the varying principal stress directions cases, longer lives than recorded in the experiments are predicted. Therefore, there arises a need for defining the fatigue damage parameter to provide good correlation with the proportional as well as non-proportional multi-axial loading conditions. The critical plane based models have been evaluated with respect to this data base. The critical plane based approaches provide information of the cracking plane and thus the direction in which the fatigue crack initiates. The Fatemi-Socie model gives reasonable prediction, except for austenite stainless steel. In the present work, a critical plane based fatigue model, which is a modification of the well-known SWT model, has been proposed. The new fatigue parameter yields good and mostly conservative predictions of fatigue lives for several tension-torsion fatigue tests on five different materials.

OVERVIEW OF FATIGUE MODELS

A literature study showed that the fatigue under complex loadings continues to be investigated by many researchers and presently more than 40 models for multi-axial fatigue analyses are available in literature [4-44]. These proposals have used different definition for fatigue equivalent parameter or Fatigue Damage Parameter (FP). The FP in these approaches is based on the macro parameters such as stresses, strains, their invariants or strain energy or multiple of stress and strain cycle parameters such as amplitudes or maximum values etc. The Table 1 provides a summary of the fatigue models based on their definition of equivalent Fatigue Parameter (FP). FP may be based on the principal components of stress and strains; however in critical plane models FP is defined using stress/strain on an oblique plane considered to have maximum fatigue damage. Some models are based on the stress invariants or second order stress terms. In some models the FP depends on the average / integral value of the certain stress or strain components at material point.

Table 1: Summary of various fatigue life evaluation approaches available in literature and design codes

Fatigue Model / Approach → Parameter Included in defining Fatigue Equivalent (FP) ↓	Principle Stress (Rankine)	Principle Strain (B-C-M)	Shear Strain γ	ASME / KTA / Tresca	Equivalent ε_{eq} (Mises)	Equivalent σ_{eq} (Mises)	Marin 1956	Sines 1959	Crossland 1970	Kakuno & Kawada 1979	Deperrois 1991	Findley 1959	Matake 1980	Robert 1997	McDiarmid 1991	Gough & Pollard	Dang Van 1989	Lee 1989	Lee & Chnag 1991	L. Susmel 2002	Brown & Miller 1973	Glinka 1995	Fatmi & Socie 1988	Smith Watson & Topper SWT	Dietmann 1991	Chu C C 1995	Popadopoulos 1995/97	Y. Jiang 1999	Qylafku 1999	Chamat et. al. 2007	Grubisic & Simbuger 1976	Zenner & Liu 2000	Popadopoulos 2001	Park & Nelson 2000	Varvani-Ferhani 2000	Morel 2000	Carpinteri (WMPS) 2001	Liu Y 2005	Ninic D. 2007
Strains ε_{ij}		■	■		■																■	■	■	■		■		■						■	■	■			
Stresses σ_{ij}	■			■		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■			■	■	■	■	■		■	■	■
σ Invariants / Ellipse						■	■	■	■	■	■					■		■	■										■	■							■	■	■
Strain Energy / $\sigma\varepsilon$																												■						■	■				
Critical Plane Based												■	■	■	■					■	■	■	■	■	■	■		■					■		■	■	■	■	■
Fracture Plane																																					■	■	
Integral / Averages																											■		■	■	■	■	■				■		

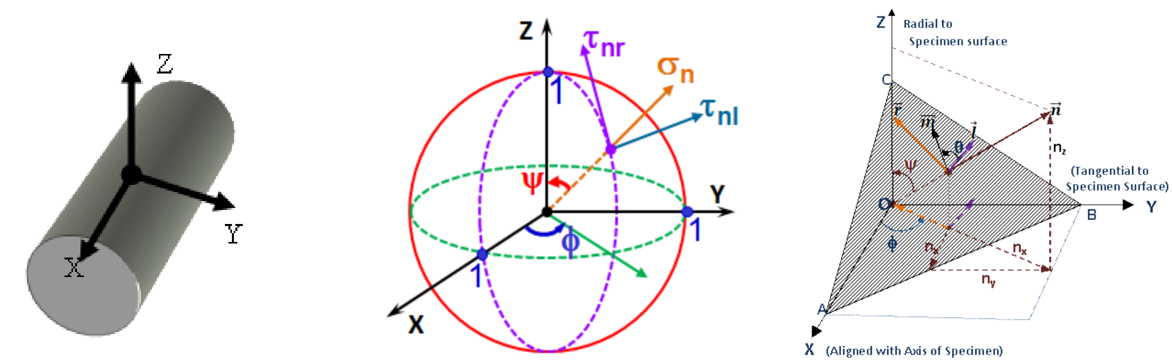
The critical plane based approaches use resolved stress / strains (normal and shear components) on the an oblique plane and obtained by maximizing a parameter such as shear stress or shear strain or Fatigue Parameter FP (function of stress, strain) etc. by rotating the oblique plane in both azimuth and meridian direction. Hence, the critical plane models also provide the information of direction and plane in which the fatigue crack initiates. The invariants and energy-based approaches represent the damage, over all the planes, in an average sense. These approaches correlate the stain energy dissipation with the fatigue damage. However they do not provide information about the plane and direction of crack initiation. Recently [6- 8] scientists are trying to combine the strain energy and critical plane concepts to develop fatigue parameter based on terms representative of strain energy and evaluated on critical plane. The commonly used fatigue life estimation techniques are based on strain or combination of stress and strain, for example, equivalent strain, maximum principal strain, the critical plane based criteria proposed by Smith Watson & Topper [9], Brown & Miller [10], Fatemi & Socie [11, 12], Glinka [13] , Chu [14], Jiang [6], Varvani-Ferhani [7, 8] as well as the energy criteria given by Ellyin [15, 16], Park [17] etc. These criteria are sometimes extended to high-cycle fatigue, where the plastic strain contribution becomes negligible. On the contrary, most of the multi-axial fatigue criteria devoted solely to high-cycle fatigue are based exclusively on stress, such as, maximum shear stress, normal stress, von-Mises stress, the criteria proposed by Sines, Gough & Pollard, Dang Van, Lee, Grubisic & Simburger, Zenner [18, 19], Papadopoulos [20], and the critical-plane-based criteria proposed by Findley, Matake, McDiarmid [21], Papadopoulos [22-24], Dietmann, Susmel [25], Carpinteri & Spagnoli [26, 27], Liu [28, 29], Ninic [30, 31], Morel [32] etc. In the present article, fatigue evaluation using only few models, like ASME NB and NH procedure, SWT model, Fatemi-Socie etc. have been presented.

MULTI-AXIAL FATIGUE EVALUATIONS

Because of the high number of fatigue theories reviewed that use of different input parameters for defining a FP, it is found useful to give first a complete set of definitions of the stress and strain quantities arising in the various criteria. Consider a material point on the specimen surface and assume an Orthogonal Cartesian Coordinate System $O._{xyz}$, such that the X-axis is aligned with the specimen axis, Z- axis points radial outward and Y-axis is perpendicular to X and Z (the tangential direction to the specimen surface), as shown in figure 1a. The material point 'O' is subjected to periodic stresses and at time't' the stress tensor is $\sigma_{ij}(t)$.

$$\sigma_{ij}(t) = [\sigma(t)] = \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_y(t) & \tau_{yz}(t) \\ \tau_{zx}(t) & \tau_{zy}(t) & \sigma_z(t) \end{bmatrix} \tag{1}$$

Where, $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$ *and* $\tau_{yz} = \tau_{zy}$, however, they act in different planes and different directions. Consider an oblique plane at above material point 'O', defined by unit vector $\vec{n}$ which makes an angle ψ with direction Z, as shown in figure 1c. The projection of vector $\vec{n}$ on XY plane, vector $\vec{n}_{xy}$ makes an angle ϕ with direction X. The orientation of the plane can be visualized in spherical system as shown in figure 1b &1c .The new Coordinate system $O._{nlr}$ is right-handed Orthogonal Cartesian.



a) Material point on specimen surface

b) The ϕ and ψ angles defining local System O_{nlr} with respect to System O_{xyz}

c) Oblique Plane O_{nlr} system, and project of its axis vectors on O_{xyz} System

Fig. 1: Oblique plane and local coordinate system used in Fatigue analysis

The rotation tensor / matrix for rotation of stress/strain tensors from O.xyz system to O.nlr system is given by eq. 2.

$$Q_{ij} = [Q] = \begin{Bmatrix} n \\ l \\ r \end{Bmatrix} = \begin{bmatrix} n_x & n_y & n_z \\ l_x & l_y & l_z \\ r_x & r_y & r_z \end{bmatrix} = \begin{bmatrix} \sin\psi\cos\phi & \sin\psi\sin\phi & \cos\psi \\ -\sin\phi & \cos\phi & 0 \\ -\cos\psi\cos\phi & -\cos\psi\sin\phi & \sin\psi \end{bmatrix} \tag{2}$$

The subscript i and j are free indices in the index notation scheme of rotation tensor Q. The Traction Vector (or Stress Vector) $\vec{T_n}$ on the oblique material plane, will be

$$T_{ni} = \vec{T_n} = \begin{Bmatrix} T_{nx} \\ T_{ny} \\ T_{nz} \end{Bmatrix} = \sigma_{ji}n_j = \lfloor n \rfloor [\sigma] = \begin{Bmatrix} \sigma_x n_x + \tau_{yx} n_y + \tau_{zx} n_z \\ \tau_{xy} n_x + \sigma_y n_y + \tau_{zy} n_z \\ \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z \end{Bmatrix} \tag{3}$$

The projection of traction vector $\vec{T_n}$ on to vector $\vec{n}$ will provide the normal stress acting on the oblique plane, given by eq. 4. Similarly the Traction Vector $\vec{T_n}$ can be projected on direction $\vec{l}$ and $\vec{r}$ to obtain the two orthogonal shear stress components on oblique-plane, given by eq. 5&6. The resultant shear stress $\tau_{n\theta}$, and its angle, θ_n , with direction $\vec{l}$ is given by eq. 7 & 8

$$\sigma_n = \vec{T_n}.\vec{n} = T_{nx}n_x + T_{ny}n_y + T_{nz}n_z = \lfloor n \rfloor [\sigma] \{n\} = \sigma_{ji}n_j n_i \tag{4}$$

$$\tau_{nl} = \overrightarrow{T_n}.\vec{l} = T_{nx}l_x + T_{ny}l_y + T_{nz}l_z = \lfloor n \rfloor[\sigma]\{l\} = \sigma_{ji}n_jl_i \tag{5}$$

$$\tau_{nr} = \overrightarrow{T_n}.\vec{r} = T_{nx}r_x + T_{ny}r_y + T_{nz}r_z = \lfloor n \rfloor[\sigma]\{r\} = \sigma_{ji}n_jr_i \tag{6}$$

$$\tau_{n\theta} = \sqrt{\tau_{nl}^{\,2} + \tau_{nr}^{\,2}} \tag{7}$$

$$\theta_n = \tan^{-1}\left(\frac{\tau_{nr}}{\tau_{nl}}\right) \tag{8}$$

The quantities, $\overrightarrow{T_n}$, σ_n, τ_{nl}, τ_{nr}, $\tau_{n\theta}$ and θ_n are varying with time and are also dependent on the orientation of the material plane that is defined by angle ψ and ϕ. In multi-axial fatigue models, the mean and amplitude components of the normal and shear stress on oblique plane are needed. The figure 2a plots of the variation of tip of the total traction vector $\overrightarrow{T_n}$ in one time period. It is a closed 3D curve. The projection of $\overrightarrow{T_n}$ curve on plane $\vec{l} \times \vec{r}$ gives variation of total shear stress $\tau_{n\theta}$. This clearly shows that the direction of the shear stress continuously changing with time and hence the principal direction will also rotate. The projection of tip of $\overrightarrow{T_n}$ on to the normal vector $\vec{n}$ provided the periodic variation of normal stress σ_n. The tip moves between two extreme, minimum and maximum, points and does not changes it direction. Hence the mean and amplitude components of the normal stress σ_n can be evaluated using following equations.

$$\sigma_n^a(\emptyset, \psi) = \frac{1}{2}[\,\max_t \sigma_n(\emptyset, \psi, t) - \min_t \sigma_n(\emptyset, \psi, t)] \tag{9}$$

$$\sigma_n^m(\emptyset, \psi) = \frac{1}{2}[\max_t \sigma_n(\emptyset, \psi, t) + \min_t \sigma_n(\emptyset, \psi, t)] \tag{10}$$

$\tau_{n\theta}$ is a periodic vector which is changing its direction θ_n with time t. The tip of the vector $\tau_{n\theta}$ if plotted will describe a closed curve in the $\vec{l} \times \vec{r}$ plane as shown in figure 2b. Evaluation of Mean and Amplitude of $\tau_{n\theta}$ is complex since both magnitude and direction changes under non-proportional loading conditions. The longest chord (LC) approach is one of the well-known approaches as summarized by Papadopoulos [24], which defines the shear stress amplitude as half of the longest chord of the loading path. The Minimum Circumscribing Circle (MCC) approach of Papadopoulos [24] defines the shear stress amplitude as the radius of the minimum radius circle circumscribing to the loading path and has given algorithm to obtaining centre and radius of the MCC. The figure 2b plots the LC and MCC methods of determining mean and amplitude components of shear stress on the oblique plane. The definition of the fatigue damage parameter, FP, of few selected multi-axial fatigue models has been given below.

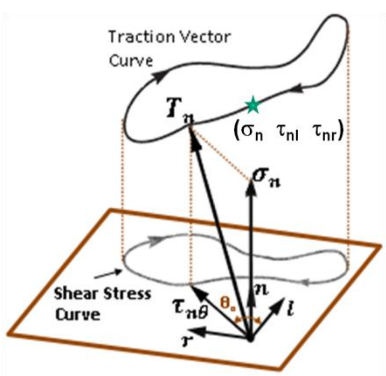


Fig. 2a: Traction vector T_n and shear stress $\tau_{n\theta}$ in $O._{nlm}$ coordinate system

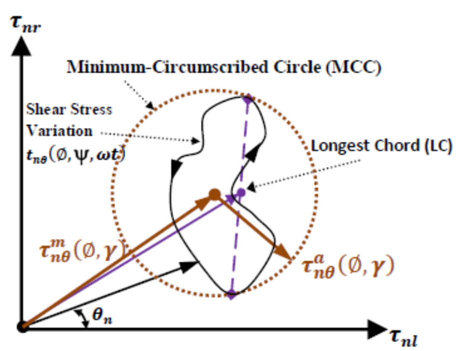


Fig. 2b: $\tau_{n\theta}$ Mean & Amplitude evaluation using minimum-circumscribed circle & longest chord method

SWT Model: Smith Watson Topper [9], has defined the fatigue damage parameter, FP, as given below

Critical Plane $(\emptyset_c, \psi_c)$: $\max\limits_{(\emptyset,\psi)}[\varepsilon_n^a]$; the oblique plane where normal strain amplitude is maximum

Fatigue Damage Parameter, FP : $FP_{SWT} = \varepsilon_c^a \sigma_c^{max}$ (evaluated on critical plane) (11)

Fatigue Life Curve : Best fit curve between FP_{SWT} vs. N_f using pure axial and pure torsion tests

Fatemi-Socie Model: Fatemi and Socie [12] performed multiaxial fatigue tests with Inconel 718 and proposed the following fatigue model

Critical Plane $(\emptyset_c, \psi_c)$: $\max\limits_{(\emptyset,\psi)}[\gamma_n^{max}]$; the oblique plane where shear strain amplitude is maximum

Fatigue Damage Parameter, FP : $FP_{FS} = \gamma_c^{max}\left(1 + k\frac{\sigma_c^{max}}{\sigma_{ys}}\right)$; (evaluated on critical plane) (12)

Fatigue Life Curve : Best fit curve between FP_{FS} vs. N_f using pure axial and pure torsion tests

The equation by Fatemi and Socie accommodates the effect of normal mean stress. The material constant 'k' is a tuning / adjustment parameter and is determined from a set of uniaxial and pure torsion tests such that the fatigue parameter FP_{FS} versus fatigue life N_f data from these tests falls in narrow band.

ASME Sec-III NB procedure:

The ASME Sec. III NB procedure is based on maximum shear stress (Tresca) theory. The fatigue damage parameter, FP, is taken as half of the largest range of stress intensity. The ASME code has given procedure for fixed-principal direction (that is proportional loading) and varying principal directions (out of phase or non-proportional loading) conditions. Both these procedures are valid with pseudo elastic stress reflecting the actual strains at the material point. The pseudo elastic (pe) stresses can be evaluated from the applied strain tensor history $\varepsilon_{ij}(t)$ using the Lame's equations of elasticity as given by eq.

$$\sigma_{ij}^{pe}(t) = \lambda\varepsilon_{kk}(t)\delta_{ij} + 2G\varepsilon_{ij}(t) \qquad (13)$$

In above equation λ is Lame's constant and G is Shear modulus of elasticity. It may be noted that for proportional loading, both the procedures give same result, hence only varying principal direction procedure was programmed. Based on the pseudo elastic stress tensor history, $\sigma_{ij}^{pe}(t)$, the alternating stress intensity is evaluated using the procedure for varying principal direction loading as given below

- Choose a reference point, t_i in time, generally correspond to extreme of the cycle. The stress components (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz} & τ_{zx}) at time 't_i' are (σ_x^{ti}, σ_y^{ti}, σ_z^{ti}, τ_{xy}^{ti}, τ_{yz}^{ti} & τ_{zx}^{ti})
- Evaluate offset stresses: $\sigma'_x = \sigma_x^{ti} - \sigma_x$, $\sigma'_y = \sigma_y^{ti} - \sigma_y$ etc. vs time
- Find principal stresses (σ'_1, σ'_2, σ'_3) vs. time from σ'_x, σ'_y, σ'_z, τ'_{xy}, τ'_{yz} & τ'_{zx}
- Determine Stress Differences vs time: $S'_{12} = \sigma'_1 - \sigma'_2$; $S'_{23} = \sigma'_2 - \sigma'_3$; $S'_{31} = \sigma'_3 - \sigma'_1$, for complete cycle
- Alternating Stress Intensity (S_{alt}) is one half of the largest absolute value of any stress difference at any time.

In the Varying Principal Stress Direction procedure, guideline for choosing the reference time which gives largest Alternating Stress Intensity (S_{alt}) is not available for complex loading condition, hence every point is taken as reference, and the largest value of alternating stress intensity evaluated. The fatigue life curve is evaluated by best fitting the FP (that is S_{alt}) vs. fatigue life (N_f) data derived from the uniaxial fatigue tests data for each material.

von-Mises/ASME Sec-III NH procedure:

Evaluation of the equivalent strain amplitudes in case of varying principal direction condition, has been given in Appendix-T of ASME Section-III, subsection NH

$$\varepsilon_{eq}^{a} = \max_{\substack{t_i \in t \\ t_j \in t}}\left(\frac{1}{2\sqrt{2}(1+\nu')}\left\{\left[\left(\varepsilon_x^{t_j} - \varepsilon_x^{t_i}\right) - \left(\varepsilon_y^{t_j} - \varepsilon_y^{t_i}\right)\right]^2 + \left[\left(\varepsilon_y^{t_j} - \varepsilon_y^{t_i}\right) - \left(\varepsilon_z^{t_j} - \varepsilon_z^{t_i}\right)\right]^2 + \left[\left(\varepsilon_z^{t_j} - \varepsilon_z^{t_i}\right) - \left(\varepsilon_x^{t_j} - \varepsilon_x^{t_i}\right)\right]^2 + 6\left[\left(\varepsilon_{xy}^{t_j} - \varepsilon_{xy}^{t_i}\right)^2 + \left(\varepsilon_{yz}^{t_j} - \varepsilon_{yz}^{t_i}\right)^2 + \left(\varepsilon_{zx}^{t_j} - \varepsilon_{zx}^{t_i}\right)^2\right]\right\}^{1/2}\right) \qquad (14)$$

Here 't_i' and 't_j' are different time instances in one time period. The fatigue life curve, here is obtained by best fitting the FP (that is ε_{eq}^{a}) Vs. fatigue life (N_f) data derived from uniaxial fatigue tests data for each material.

Multi-Axial Fatigue Analyser (MFA) Software:

The calculation procedure of different fatigue models has been split into smaller modules such that, the common steps, resolved σ-ε parameters, can be evaluated by common script/function and be shared by different models. These modules are, Input module for inputting the material constants and the complex cyclic loading in various ways like time history, cycle parameters etc., Oblique Plane Analyses Module (OPAM) to evaluate resolved strain-stress cycles, their amplitude and means on the rotated planes, Fatigue Analyses Module (FAM) to perform assessment of fatigue life, crack initiation plane orientation etc. using different fatigue models, and Visualization Module (VM) to visualize, 2D, 3D plots of parameters in different modules. The variation of the Poisson ratio, with changing proportion of elastic and plastic strains has been incorporated using an iterative procedure to evaluate effective Poisson ratio at each calculation point. The figure-3 show the organization and flow of the developed Multiaxial fatigue Analysis (MFA) Software.

Fatigue Tests Analysed:

The calculations using MFA software for ASME NB/NH, SWT and Fatemi-Socie models were made for about 230 fatigue test data sets, which include results from pure axial, pure torsion and axial-torsion tests with different phase shifts. Out of these, about 50 fatigue tests were from in-house tests on SA333Gr6 Carbon steel (Indian PHWR piping material) by Gupta [1]. The remaining data of about 180 multi-axial fatigue tests on SS347 austenite stainless steel (see Hoffmeyer [2, 3]), SA406N high strength structural steel (see Hoffmeyer [2, 3], Jiang [4]), Al 5083 aluminium alloy (see Hoffmeyer [2,3]) and SA302B carbon steel (see Gao [5]), have been taken from literature and used for evaluation of above fatigue models. The basic material properties of these materials are given in table 2. Gao [5] has conducted a series of fatigue experiments on 16MnR, typical low carbon manganese steel (similar to ASME SA302B) in ambient air. Hoffmeyer et al. [2, 3] presented tension and torsion tests on tubular specimen made of three materials, the micro alloyed fine grained structural steel S460N (FeE460), aluminium Al5083 (AlMg4.5Mn) and SS347 stainless steel (X6CrNiNb18-10). Jiang et. al. [4] presented fatigue calculations on the S460N tests data.

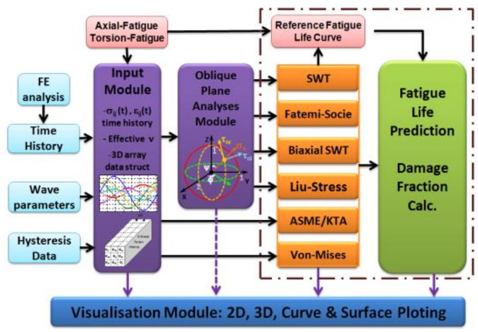


Fig. 3: Multiaxial fatigue Analysis (MFA) Software organization

Table 2: Summary of Material properties and tests summary used in multi-axial fatigue analyses

Sr. No.	Material	Young's Modulus E(GPA)	Poisson Ratio ν	Yield Strength σ_{ys} (MPa)	Ultimate Strength σ_{uts} (MPa)	No of Tests / Loading condition				Reference
						Axial	Torsion	Axial + Torsion	Phase Angles Degree	
1	SA333Gr6	203	0.3	304.5	502.6	22	6	22	0, 45, 90	Gupta [1]
2	SS347	200	0.3	251	588	30	3	19	0, 45, 90	Hoffmeyer et al. [2], [3] Jiang [4]
3	S460N	208.5	0.3	500	643	5	4	35	0, 45, 90	
4	Al5083	68	0.33	169	340	5	4	36	0, 45, 90	
5	SA302B	212.5	0.31	324.4	544.5	12	10	15	0, 90	Gao [5]

ANALYSES RESULTS AND DISCUSSIONS

Using the MFA software, analysis of all tests, table-2, was performed for ASME NB and NH procedure, SWT model, Fatemi-Socie etc. The fatigue damage parameters, FP, were evaluated for all tests including pure axial, pure torsion. Using the pure axial fatigue tests, that is fatigue damage parameter, FP vs. fatigue life N_f , a best fit curve (Axial Fatigue Fit Curve) was obtained for each material and fatigue model. This axial fatigue fit curve, was subsequently used to predict fatigue life for axial-torsion and pure torsion load cases. This is in line with the current design philosophy. It may be noted that no safety factor on the fatigue life or on FP was used. The fatigue tests generally have a data scatter band of 2 (with respect to mean) even under controlled test conditions like material heat, polished specimen, same size, air environment, room temperature etc. In view of it, the upper and lower bound data scatter bounds (factor of 2 on fatigue life cycles) have also been plotted about the fitted mean axial fatigue life curve. For stainless steel SS347, and carbon steel SA333Gr6 material, the ASME mean curve (without safety factors) and mean curve recently proposed by, Argon National Laboratory (ANL), Chopra [42] has also been plotted. The SS347 (X6CrNiNb18-10), ASME SA302B and SA333Gr6 steels are used in various nuclear components. Figure 4 & 5 plots the FP Vs. N_f, using ASME NB and von-Mises (ASME Sec III NH Appendix-T) fatigue procedures. These clearly show that the 90° and 45° phase-shift cases are on the non-conservative side and mostly outside the scatter bounds. The use of the ASME Sec III NB elastic alternating stress intensity and von-Mises strain procedures for the materials investigated may lead to over prediction of fatigue life under out of phase non-proportional loading conditions. Hence, for realistic fatigue assessment under multi-axial non-proportional loading, critical plane based models need to be investigated. The critical plane models are realistic since the fatigue crack initiates in a certain plane and certain direction, generally associated with Persistent Slip Bands (PSB) formation. The PSB forms along the planes with highest shear strain cycling and leads to initiation of microstructure cracks which further grow to mechanical size where the normal stress also assists. The fatigue evaluations were made using the two most popular critical plane based models, SWT and Fatemi-Socie and the resulting FP Vs. N_f were plotted in figure 6 and 7, respectively. Figure 6 show that the SWT model poorly correlates the combined axial torsion and pure torsion tests. This is due to that fact that it does not use the maxima shear strain / stress which are important parameter of fatigue crack initiation.

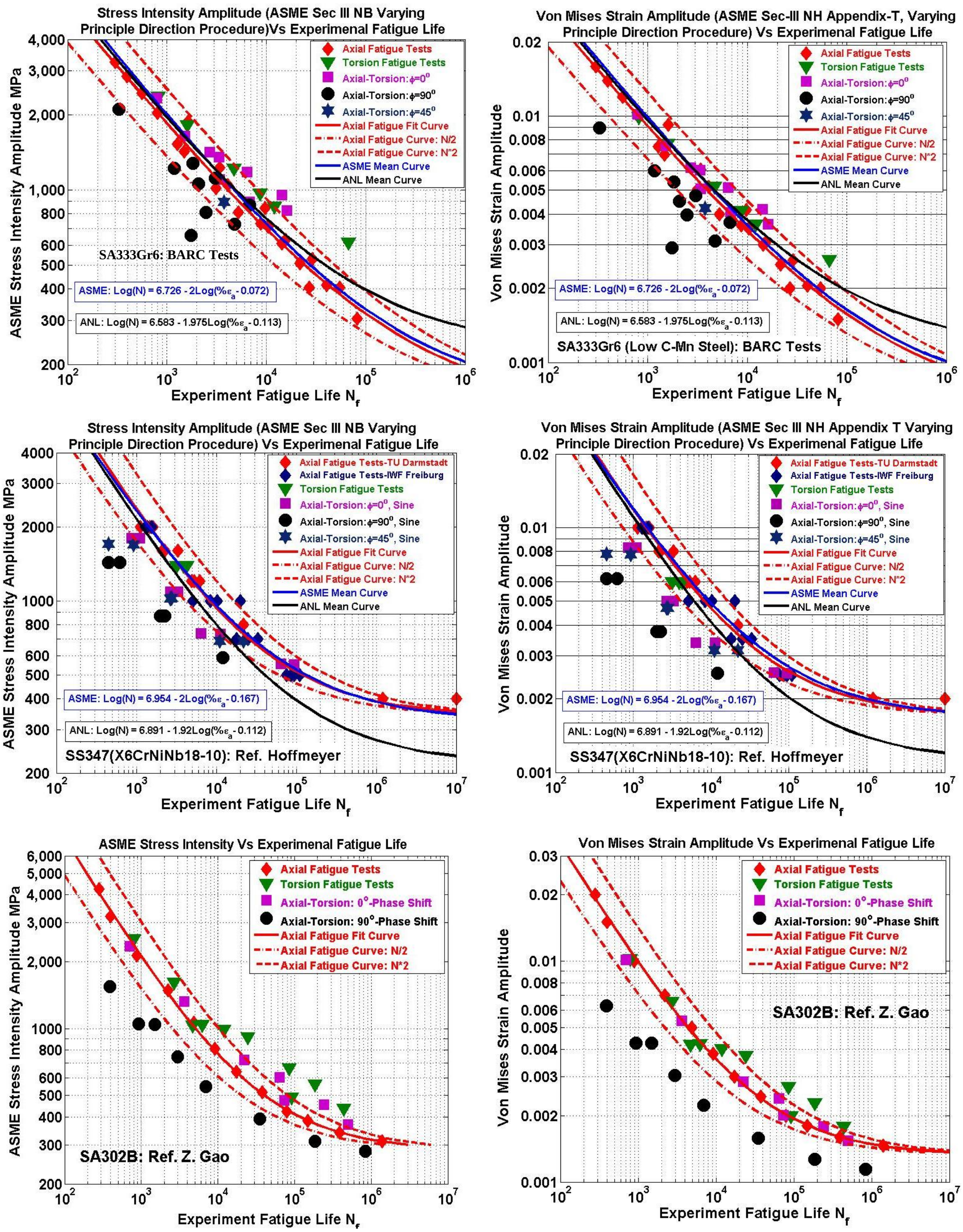


Fig. 4: FP Vs. Nf, for ASME NB and von-Mises model for SS347, SA333Gr6 and SA302B material

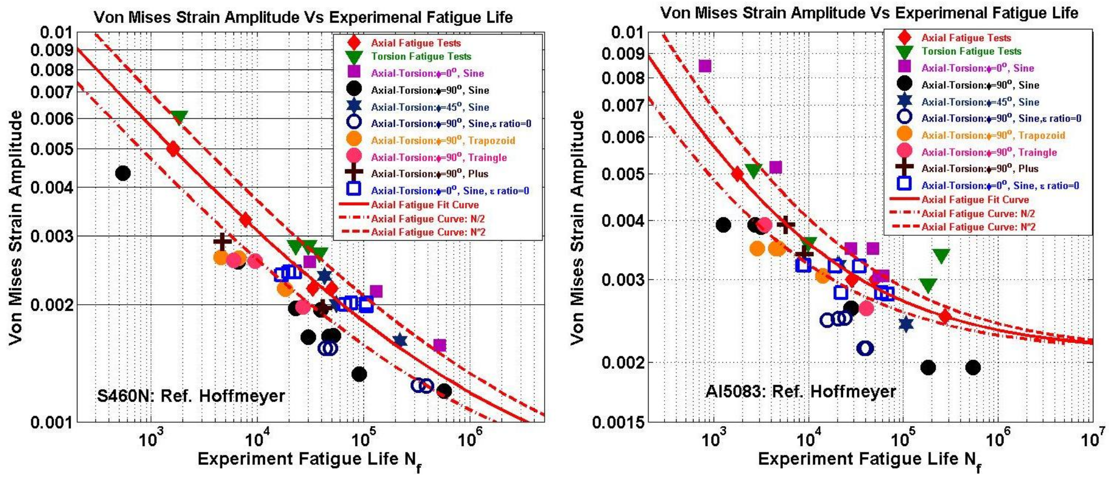


Fig. 5: FP vs. Nf, for von-Mises model for S460N and Al5083 material

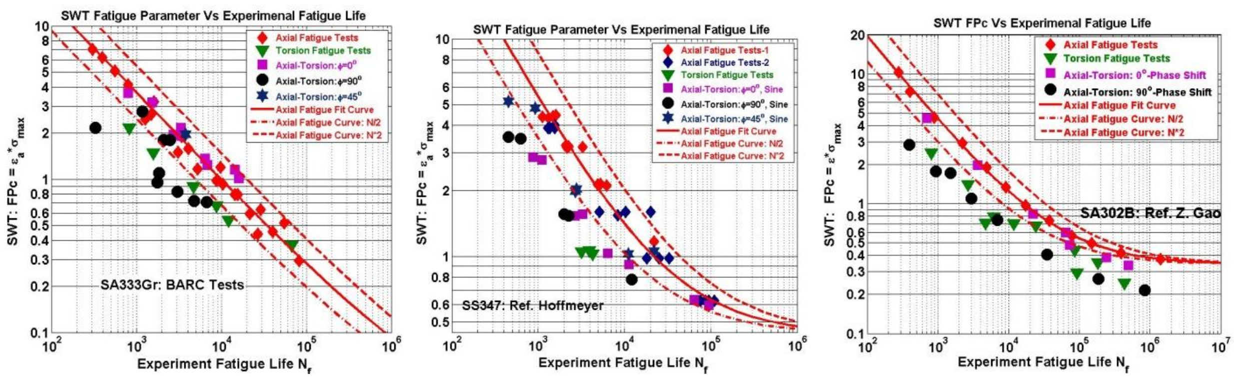


Fig. 6: FP vs. Nf, using SWT model

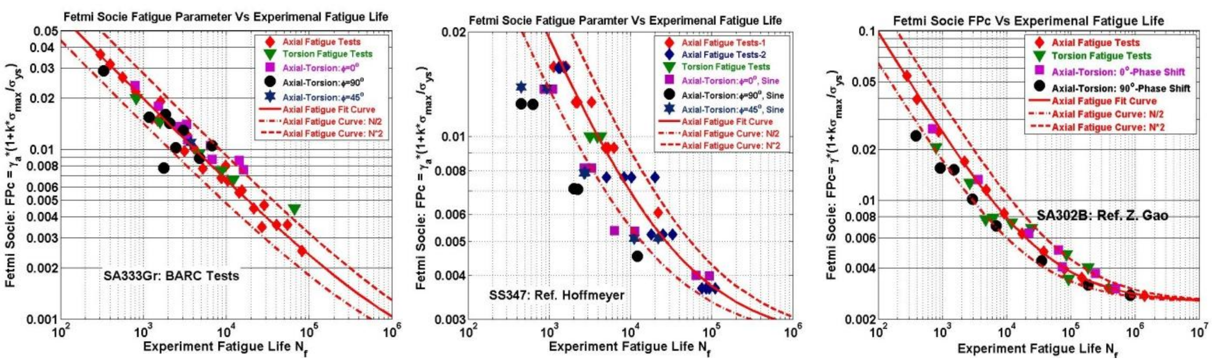


Fig. 7: FP vs. Nf, using Fatemi-Socie model

Figure 7 show that the Fatemi-Socie model fairly correlates all data sets except the SS347 stainless steel. Although on a closer look, it was noticed that mostly, the non-proportional cases lie below the best fit curve. To understand it, the σ-ε hysteresis loops under different non-proportional loading conditions were investigated. Figure 8 plots the σ-ε loop corresponding to half life of axial-torsion fatigue tests on SA333Gr6 material with 0°, 90° and 45° phase-shifts. This figure clearly shows the extra hardening due to non-proportionality/ phase-shift in the loading. The von-Mises stresses versus strain amplitudes evaluated for axial, torsion and axial-torsion fatigue tests have been plotted in figure 9a &9b for SA333gr6 and SS347 material. The stress-strain data of pure axial and pure torsion tests are in good agreement. However, the axial-torsion phase shift tests clearly show extra hardening due to non-proportion (out of phase loading). Since the ASME Sec III NB that is Tresca theory (with pseudo elastic stress evaluated from strain tensor) and von-Mises models (ASME Sec III NH Appendix-T), use only strain components, the additional cyclic hardening due to the rotation of the principal axes during non-proportional loading cannot be accounted for. The Fatemi-Socie model, since real normal stress and shear strain components used, better accounts for the non-proportional cyclic hardening. However, it does not include shear stress and normal strain, which would be needed to completely represent non-proportional hardening. Also, the $\gamma_c^{max}\sigma_c^{max}$ quantity does not convey any physical meaning as in case of SWT model, quantity $\varepsilon_c^a\sigma_c^{max}$ is representative strain energy due to normal deformation. This argument is further extended to shear process and a new fatigue damage parameter FP, is defined as $\varepsilon_c^a\sigma_c^{max} + \gamma_c^a\tau_c^a$. The new model, Biaxial SWT (BSWT), fatigue parameter, FP_{BSWT}, is representative of the total strain energy that is sum of strain energy dissipation in normal and shear deformation modes, on the critical plane. Since the proposed FP, includes all the resolved normal and shear stresses/strains, it can fully reflect the non-proportional strain hardening phenomenon. The critical plane is defined as a plane where the FP_{BSWT} takes its maximum value.

Critical Plane $\left(\emptyset_c, \psi_c\right)$: $\max\limits_{(\emptyset,\psi)}\left[\varepsilon^a\,\sigma^{max} + \gamma^a\,\tau^a\right]$

Fatigue Parameter, FP : $FP_{BSWT} = \varepsilon_c^a\sigma_c^{max} + \gamma_c^a\tau_c^a$; (evaluated on critical plane) (15)

Fatigue Life Curve : Best fit curve between FP_{BSWT} vs. N_f using pure axial and pure torsion tests

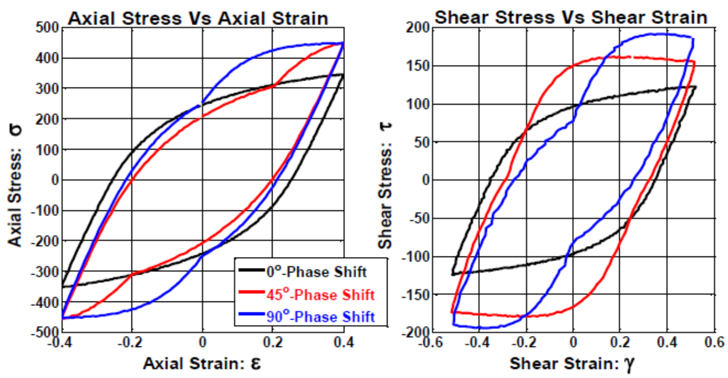
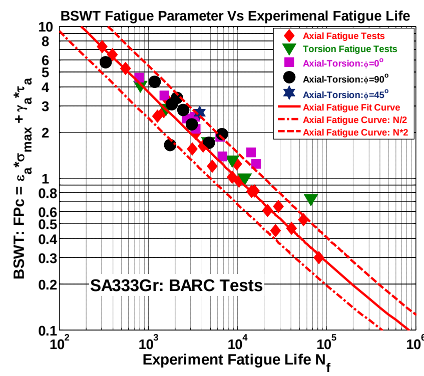


Fig. 8. Axial and Shear Stress Vs Strain hysteresis with different phase-shifts



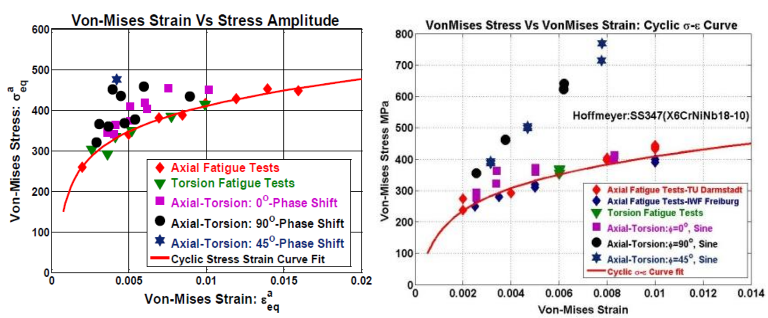


Fig. 9: Equivalent Stress Vs Strain amplitude for axial, torsion and axial-torsion tests under different phase-shifts

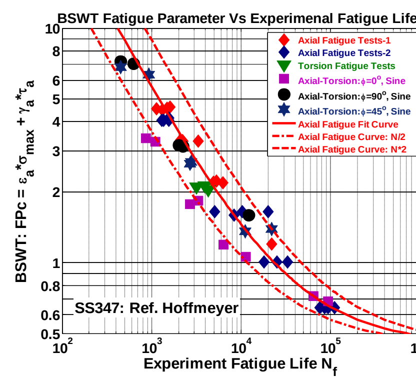


Fig. 10a: FP Vs. N_f, using new Biaxial SWT model

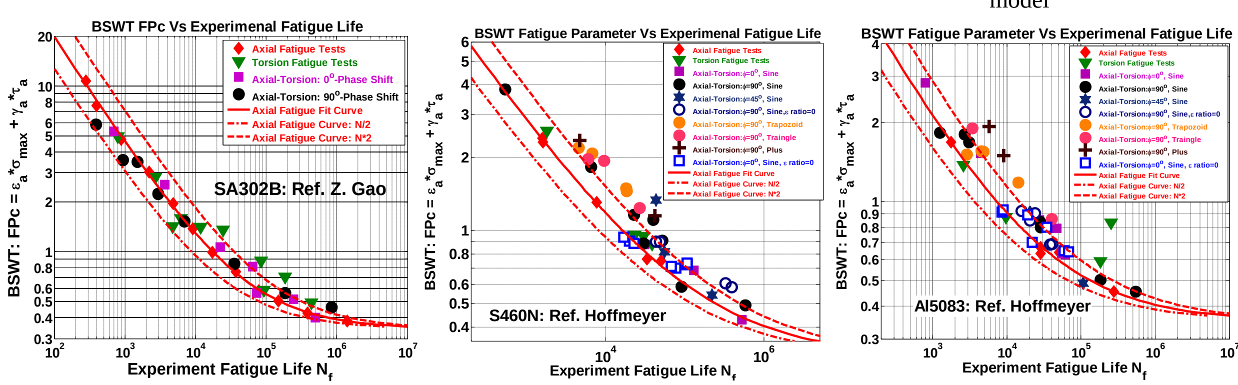


Fig. 10b: FP Vs. N_f, using new Biaxial SWT model

Figure 10a and 10b plot the FP Vs. N_f, results from the new proposal, biaxial SWT model, for all the five materials and about 230 tests data. It clearly shows that all tests data including the 45° and 90° phase shift tests, are better correlated and mostly are on the conservative side with respect to the axial fatigue data for all the 5 materials. The reason is that, in addition to strain, the inclusion of both normal and shear stress components account for the non-proportional cyclic hardening if necessary, and the FP would increase accordingly. Higher FP will subsequently predict shorter fatigue life when used with axial fatigue fit curve, which was also observed in tests.

CONCLUSION

It was established that the fatigue life under non-proportional loading conditions depends on the non-proportional hardening characteristic of the material and hence the solely pseudo-elastic stress or strain based models like Tresca / von-Mises are in-adequate for fatigue life assessment under such conditions. This study was further extended to develop a critical plane based fatigue damage parameter which accounts for the additional hardening/ damage caused in case of varying principal direction loading. A simple parameter representation the sum of shear and normal energy dissipation was identified as fatigue damage parameter and a critical plane based procedure was used for evaluation of its maximum value for given complex loading conditions. This parameter is an extension of the idea used in SWT models. The predicted fatigue life using the developed parameter, for varying principal direction tension-torsion tests, is found in good agreement and mostly on the conservative side. The validation was performed on about 230 fatigue tests data of five different materials, which includes pure axial, pure torsion and axial-torsion test with different phase shifts and strain paths. The new fatigue parameter yields better and conservative correlation of calculated and experimental fatigue lives for proportional as well as non-proportional tension-torsion loading tests on all five materials investigated. Hence the proposed biaxial SWT model can be adopted for improving the fatigue design of power plant mechanical components.

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