Probabilistic Evaluation of Lateral Extent of Liquefaction under Seismic Loading

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ABSTRACT

A method to compute the probability of the onset of liquefaction at a given depth in a layered soil deposit is extended to compute the probability that liquefaction in the layer will extend over a specified area. The probability that liquefaction at the critical depth will extend over a specified area, plotted as a function of the intensity of the ground motion is denoted the liquefaction fragility for the soil deposit.

1.0 INTRODUCTION

Liquefaction of cohesionless soils during earthquakes can be major cause of failure or malfunction of lifeline systems (e.g., power supply systems) during and following earthquakes. For buried structures (e.g., conduits and tunnels) liquefaction-induced forces and damage will depend on the volume of soil surrounding the structure that will liquefy[1]. A method to calculate the probability of the onset of liquefaction at a given depth in a soil deposit[2,3] is extended to assess the probability that a specified volume of soil will liquefy when liquefaction occurs at the given depth. To account for the variability of soil properties with depth the soil deposit is divided into horizontal layers and the volume of liquefied soil in each layer is calculated as the product of the layer thickness and the lateral extent of liquefaction. The probability that the volume of liquefied soil will exceed some specified volume is the probability that liquefaction occurs everywhere in the specified soil volume.

Assumptions underlying the methodology for calculating the probability of the lateral extent of liquefaction are: (i) the spatial variability of the soil resistance against liquefaction in the layer is described by an homogeneous and axisymmetric random field; and (ii) the ground motions in the horizontal direction are assumed to be perfectly correlated.

2.0 LATERAL EXTENT OF LIQUEFACTION

Consider the soil deposit shown in Fig. 1a. The probability that liquefaction will occur at any point in the layer (say point O), independently of what happens anywhere in the layer, is given by [2,3]

$$P[E_0] = \int_0^\infty F_S(q) f_Q(q) dq \tag{1}$$

where S is a random variable that describes the soil resistance against liquefaction for any point in the layer, and Q is a random variable that describes the seismic load effect in the layer for a given intensity of peak ground acceleration (Q is obtained from random vibration analysis); $F_S(q)$ is the cumulative probability distribution function of S and $f_Q(q)$ is the probability density function of Q.

To compute the probability that liquefaction will extend over a circle with radius R, the circle is divided into segments of length D, as shown in Fig. 1b, for 5 segments numbered $0, 1, ..., 4$. The soil resistances against liquefaction at the center of each segment are random variables denoted by $S_i, i = 0, 1, ..., 4$. The random variables S_i are identically distributed but are not statistically independent. The correlation coefficients between any pair of random variables (S_i, S_j) is calculated from the autocorrelation function $R_{SS}(x)$ of the random field $S(x)$ as described below.

Suppose that S_i and S_j are separated by a distance kD, where k is a positive integer, then the desired correlation coefficient is

$$\rho_{SS}(kD) = \frac{R_{SS}(kD)}{\sigma_{S_i}\sigma_{S_j}} \tag{2}$$

where σ_{S_i} is the standard deviation of S_i, which is equal to the standard deviation of S_j.

The probability that liquefaction will extend over the entire circle with radius $R = 4.5D$ is the probability that all segments in the circle will liquefy. Therefore,

$$P_L(R = 4.5D) = Probability[(S_0 < Q) \cap (S_1 < Q) \cap (S_2 < Q) \cap (S_3 < Q) \cap (S_4 < Q)] \tag{3}$$

which can be writen as

$$P_L(R = 4.5D) = \int_0^{\infty} F_{S_0S_1S_2S_3S_4}(q, q, q, q, q) f_Q(q) dq \tag{4}$$

where

$$F_{S_0S_1S_2S_3S_4}(q, q, q, q, q) = \int_0^q \cdots \int_0^q f_{S_0S_1S_2S_3S_4}(s_0, s_1, s_2, s_3, s_4) dS_0 dS_1 dS_2 dS_3 dS_4 \tag{5}$$

is the probability that liquefaction will extend over the entire area when $Q = q$ and is denoted $P_L(R, q)$.

The random variables S_i may be assumed to follow a lognormal distribution with median S_m and standard deviation σ_{S_i}[2,3]. The matrix of the correlation coefficients for the random variables S_i is given by

$$[\rho] = \begin{bmatrix} 1.0 & \rho_{SS}(D) & \rho_{SS}(2D) & \rho_{SS}(3D) & \rho_{SS}(4D) \\ & 1.0 & \rho_{SS}(D) & \rho_{SS}(2D) & \rho_{SS}(3D) \\ & & 1.0 & \rho_{SS}(D) & \rho_{SS}(2D) \\ & Symmetric & & 1.0 & \rho_{SS}(D) \\ & & & & 1.0 \end{bmatrix} \tag{6}$$

Evaluation of the multiple integral in Eq. 5 requires numerical integration. All numerical integrations were performed using an adaptive algorithm for numerical integration over an N-dimensional rectangular region[4]. With this algorithm, the numerical integrations can be performed without major difficulties except when the correlation matrix becomes nearly singular.

The correlation function of the soil resistance against liquefaction is assumed to be of the form

$$\rho_{SS}(x) = exp[-(x/b)^2] \tag{7}$$

where b is a positive parameter. This form of the correlation function has been suggested for some soil properties (e.g., relative density, grain-size distribution and shear strength)[5,6,7]. Other possible forms for the correlation function have also been used [6,7]; e.,g.

$$\rho_{SS}(x) = exp(-|x|/a) \tag{8}$$

with $a > 0.0$. This second form has the disadvantage that its second derivative at $x = 0.0$ does not exist which implies that the random field describing the soil property is not differentiable. This problem is not present with the form of Eq. 7 which was adopted in this study. The values of the parameter b will depend on the particular soil deposit being analyzed. In this study, b was treated as a parameter.

3.0 EXAMPLE APPLICATION

The method described in the previous sections is used to compute the newly defined liquefaction fragilities for the soil deposit described in Fig. 2. Each liquefaction fragility gives the probability of liquefaction over a circle with a specified radius as a function of the peak ground acceleration. First, the radius of the circle is fixed and the sensitivity of the results to the number of segments in which the circle is subdivided is investigated. Next, the liquefaction fragilities are computed for different radii of the circle. Finally, results showing the probability of liquefaction over a circle with a given radius conditional on the occurrence of liquefaction at the center of the circle are also presented and discussed.

To account for the variability of the soil properties with depth (layering) the soil profile was modeled with 10 elements as shown in Fig. 2. The input ground motion (loading) is characterized by its power spectral density function, intensity and duration, and is specified at the bottom of the soil deposit. The power spectral density is modeled with the Clough-Penzien power spectrum[8], the intensity is measured by the expected maximum ground acceleration and a strong-motion duration of 8.0 seconds is used. The liquefaction fragility curve describing the probability of liquefaction for a point at the critical depth in the soil deposit computed with the methodology described in Refs. 1 and 2 is shown in Fig. 3 for reference purposes.

The circle with radius $R = 1.69b$ was divided into segments with lengths D as indicated in Fig. 1b. To investigate the senstivity of the results to the number of segments into which the circle is subdivided several values of the length D were considered. Liquefaction fragilities computed for D equal to $0.676b$, $0.483b$ and $0.375b$ are shown in Fig. 3. It can be seen that the probabilities of liquefaction decrease as the number of segments increases and seem to converge for $D = 0.375b$. For $D = 0.307b$ the correlation matrix becomes almost singular and its lowest eigenvalue is very small. For this case, the integral in Eq. 5 has to be evaluated approximately. Nevertheless, the approximate values were found to be almost identical to the values for $D = 0.375b$. Therefore, $D = 0.375b$ was selected. As expected, the probability of liquefaction over a circle with radius $R = 1.69b$ is, of course, significantly lower than the probability of liquefaction for a single point in the layer.

To compute the probability of liquefaction over a circle with radius $R_n = (n + 0.5)D$, where n is a positive integer, an (n+1)-fold integral needs to be evaluated numerically (see

Eq. 5). As the radius of the circle increases to $R_{m+n} = (m+n+0.5)D$, a numerical integration of an (m+n+1)-fold integral is involved. If many different radii need to be considered, the numerical integrations can become very time consuming. Therefore, it is desirable to find a simplified method to compute the probability of liquefaction for a circle of a given radius, say $R_k = (k+0.5)D$, given that the probability of liquefaction for a circle with radius R_n is known. The following simplified method is proposed:

(i) assume that the probability of liquefaction for a circle with radius $R_n = (n+0.5)D$ and for a load effect with intensity q can be written as

$$P_L(R_n, q) = F_{S_0}(q)exp[-n\alpha(q)] \tag{9}$$

which can be solved for $\alpha(q)$, obtaining

$$\alpha(q) = \ln[P_L(R_n, q)/F_{S_0}(q)]/(-n) \tag{10}$$

and

(ii) for other radii, $R_k = (k+0.5)D$, where k is a positive integer, the value of $P_L(R_k, q)$ is

$$P_L(R_k, q) = F_{S_0}(q)exp[-\alpha(q)k] \tag{11}$$

Figure 4 shows liquefaction fragilities for circles with radii equal to 0.94b and 2.44b computed by Eqs 4 and 5 and by Eqs. 9-11 with $R = 1.69b$, i.e., n=4, as the reference values to compute the quantity α in Eqs. 9-11. The approximate method appears to be sufficiently accurate.

Probabilities of liquefaction at the critical depth in the soil deposit for expected maximum ground accelerations of 0.24g and 0.159g are shown in Fig. 5 for various radii of the liquefied areas. Figure 5 also shows the conditional probabilities of liquefaction for circles of different radii assuming that liquefaction occurs at the center of the circles. These conditional probabilities are computed by

$$P_{L|O}(R) = \int_0^\infty \frac{P_L(R, q)}{F_{S_0}(q)} f_Q(q)dq \tag{12}$$

The conditional probabilities are equal to 1.0 at the center of the circle and decrease as the radius of the liquefied zone increases. It can be seen that, as the radius increases, the probabilities shown in Fig. 5 decrease more rapidly for the smaller peak ground acceleration, i.e., when the probabilities of liquefaction anywhere in the layer are smaller. This implies that the median size of the liquefied zone will increase if the probability of liquefaction anywhere in the layer also increases.

4.0 CONCLUSIONS

A method to calculate the probability that a specified volume of soil will liquefy when liquefaction occurs at a given depth in the soil deposit is presented. The method is an extension of a previously developed methodology to calculate the probability of the onset of liquefaction at a given depth in a horizontally layered soil deposit under random seismic loads. With the proposed extension the liquefaction fragility curves represent the probability of liquefaction over a circle with a specified radius as a function of the intensity of the earthquake ground shaking. Since the adverse consequences of liquefaction on buried lifelines depend on the lateral extent of liquefaction, a realistic definition of liquefaction ought to include this lateral spread. It was observed that the median size of the lateral extent of liquefaction increases as the probability of liquefaction at the site increases.

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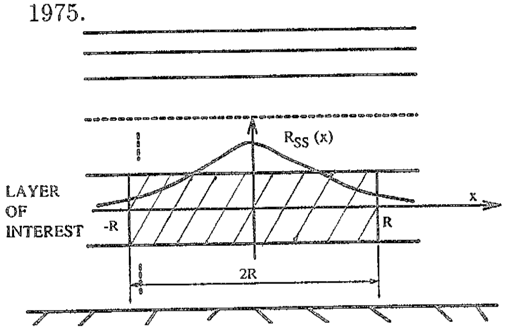
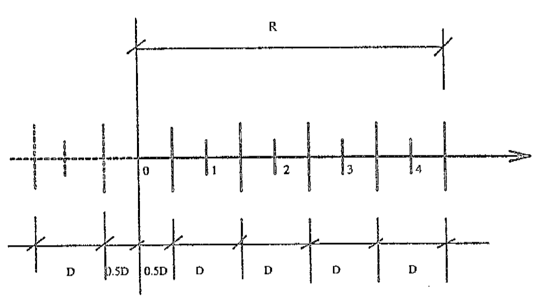


a) Lateral Spread of Liquefaction



b) Discretization of Layer

Figure 1.

ELEMENT THICKNESS (m)	SOIL TYPE	N-VALUE SPT (Uncorrected)	SHEAR WAVE VELOCITY (m/sec)	UNIT WEIGHT (tonf/m**3)	PERCENT OF FINES
1.5	LOAM	4	160	1.42	
1.5	LOAM	4	160	1.42	
2.0	LOAM	4	160	1.42	
2.0	LOAM	4	160	1.42	
2.5	LOAM WITH CLAY	5	173	1.56	
2.5	LOAM WITH CLAY	5	173	1.56	
2.5	CLAYED SAND	22	225	1.82	32
2.5	CLAYED SAND	22	225	1.82	32
3.0	SILT	18	263	1.68	
3.0	SILT	18	263	1.68	

*TRANSMITTING BOUNDARY

Figure 2. Soil Deposit Analyzed

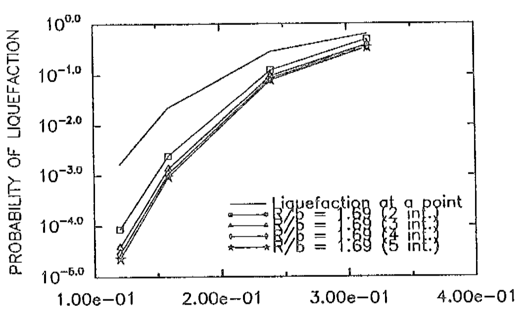


Figure 3. Effect of Discretization on Liquefaction Fragilities

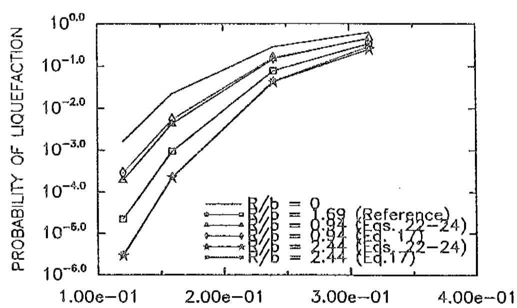


Figure 4. Probability of Liquefaction Spread Using Approximate Method

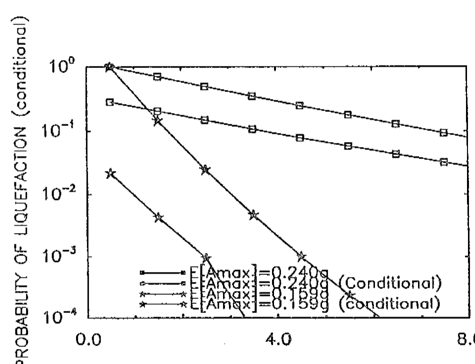


Figure 5. Probability of Liquefaction Spread for Various Radii