THE EFFECT OF TRANSVERSE SHEAR AND MATERIAL ORTHOTROPY IN CRACKED CYLINDRICAL AND SPHERICAL SHELLS

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In this paper the effect of transverse shear and material orthotropy on the stress intensity factors in cylindrical and spherical shells containing a crack is considered. Unlike the classical shell theory in which the twisting moment and the transverse shear resultants are combined on the shell boundary through a Kirchhoff-type assumption, the particular theory used in this study permits the consideration of all five physical boundary conditions regarding moment and stress resultants on the crack surface separately. As a consequence of this it is found that the membrane and bending components of the asymptotic stress field near the crack tip become identical. In shell problems since the two fields are coupled, the difficulty in justifying the superposition of membrane and bending stresses and particularly of the related stress intensity factors when applying the results to fracture problems has thus been removed. In the solution given in this paper, in addition to the standard shell parameter $a/(Rh)^{\frac{1}{2}}$, the stress intensity factors are found to depend on the thickness parameter h/a where 2a, h, and R respectively are the crack length, the shell thickness, and the mean radius of curvature. The stress intensity factors are given for an isotropic shell and for two specially orthotropic shells. In isotropic shells the effect of Poisson's ratio is separately studied and is shown to be relatively insignificant.

The next problem considered is that of a part-through circumferential crack in a cylinder. This problem may be important in piping subjected to fluctuating axial loads. A limiting case of the problem is a relatively thin-walled cylinder with a deep crack in which the net ligament and a certain portion of the shell wall around the crack tips are fully yielded. The problem is solved by using the shell theory which takes into account the effect of transverse shear. In this solution the plastic deformations are taken into consideration through an appropriate plastic strip model and the crack opening stretch along the crack periphery is calculated.

The bulk of the results given in this paper refers to elastic stress intensity factors for through cracks in cylindrical and spherical shells. Due to space limitation only some sample results are given for the elastic-plastic part-through crack problem.

1. Introduction

In designing pressure vessels and piping, it is often necessary to consider fatigue crack propagation and fracture among the possible modes of failure. From a practical viewpoint the reasons for this are that in such structural components the applied loads are generally cyclic in nature and the component itself is seldom completely defect-free. In order to demonstrate the fact that a given component would be safe under the worst possible loading and environmental conditions that could act on the component, each stage of the failure process namely, the formation and the subcritical propagation of a part-through crack, the rupture of the weakened wall, and the ensuing phenomenon of "leak" or "break", as well as the time profile of the applied loads needs to be modeled as realistically as possible. For the initial phase of the subcritical crack propagation the associated problem giving the stress intensity factor along the crack front is basically a three-dimensional elasticity problem and can only be solved by using a numerical method such as finite element, finite difference, or boundary integral equations (see, for example, [1] and [2]). In the advanced stages of propagating part-through crack and in the case of through crack in relatively high toughness materials, around the crack region the plastic deformations would spread through the entire wall. For these as well as the elastic through crack problems the pipe or the vessel may be modeled by a cracked shell with or without the plastic strip approximating the yield zones.

In addition to the effect of the geometrical variables such as thickness, radii of curvature and crack dimensions on the fracture related parameters such as stress intensity factors and crack opening displacements, in pipes and pressure vessels another factor which may be worthwhile to investigate is the effect of material anisotropy resulting, for example, from rolling or extrusion. In this paper we will therefore consider the problem for specially orthotropic spherical and cylindrical shells by using an improved shell theory which permits the use of all five physical boundary conditions on the crack surfaces separately.

2. On The General Formulation of the Problem

The basic formulation of the problem and the method of solution follows the procedure outlined in [3-5]. The isotropic shell theory given in [3] which takes into account the effect of transverse shear deformations can be extended to orthotropic materials. However, this leads to a system of differential equations which appears to be analytically intractable. On the other hand it can be shown that [6] if the elastic constants E_1, E_2, ν_1, ν_2, and G_{12} of the orthotropic shell satisfies the condition

$$2G_{12} = E/(1+\nu),\ E = \sqrt{E_1E_2}\ ,\ \nu = \sqrt{\nu_1\nu_2}\ , \tag{1}$$

i.e., if the material is specially orthotropic, then the related differential operators can be factorized and the analytical solution becomes feasible. Using a superposition technique and standard Fourier transforms the shell problem may be reduced to a mixed boundary value problem. If the plane of the crack is a plane of symmetry in loading as well as in geometry, then the following functions could be considered as the natural unknown functions of the mixed boundary value problem:

$$G_1(y) = \frac{\partial}{\partial y}\, u(+0,y),\ G_2(y) = \frac{\partial}{\partial y}\, \beta_x(+0,y) \tag{2}$$

where $u(x,y) = \sqrt{c}\ U_1/a$ is the normalized crack surface displacement in the neutral surface of the shell, $\beta(x,y) = \beta_1\sqrt{c}$ is the normalized angle of rotation of the crack surface in x z plane, and (see Figures 1 and 2)

$$x = X_1/a\sqrt{c}\ ,\ y = X_2\sqrt{c}/a,\ z = X_3/a,\ c = (E_1/E_2)^{\frac{1}{4}}. \tag{3}$$

By defining

$$t = y/\sqrt{c}\ ,\ s = x/\sqrt{c}\ ,\ H_i(t) = G_i(\sqrt{c}\ t),\ i = 1,2, \tag{4}$$

after a lengthy analysis the mixed boundary conditions

$$\lim_{x\to+0} N_{xx}(x,y) = F_1(y), \lim_{x\to+0} M_{xx}(x,y) = F_2(y),\ |y|<\sqrt{c}\ ,$$

$$u(0,y) = 0,\ \beta_x(0,y) = 0,\ \sqrt{c}<|y|<\infty, \tag{5}$$

may be reduced to the following system of singular integral equations and auxiliary conditions:

$$\int_{-1}^{1} \frac{H_1(\tau)}{\tau-t}\, d\tau+\int_{-1}^{1}\sum_{1}^{2} k_{1j}(t,\tau)H_j(\tau)d\tau=2\pi F_1(\sqrt{c}\ t),$$

$$\int_{-1}^{1}\frac{H_2(\tau)}{\tau-t}d\tau+\int_{-1}^{1} \sum_{1}^{2} k_{2j}(t,\tau)H_j(\tau)d\tau=2\pi\frac{h}{a}\, F_2(\sqrt{c}\ t),\ |t|<1, \tag{6}$$

$$\int_{-1}^{1} H_j(t)dt = 0,\ j = 1,2. \tag{7}$$

where h is the shell thickness and 2a is the crack length. After determining H_1 and H_2 from equations (6) and (7), displacements U_1, U_2, W, stress resultants N_{11}, N_{12}, N_{22}, moment resultants M_{11}, M_{12}, M_{22}, and transverse shear resultants V_1, V_2 may all be expressed in terms of these density functions and appropriate known kernels. Also, defining the polar coordinates r, θ in t, s plane by

$$r \cos \theta = t-1,\ r \sin \theta = s, \tag{8}$$

the asymptotic distribution of the stresses around the crack tip t=1, s=0 may be expressed as

$$\sigma_{xx} \cong - \frac{h_1(1)+zh_2(1)}{2\sqrt{2r}} \left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{5\theta}{2}\right),$$

$$\sigma_{yy} \cong - \frac{h_1(1)+zh_2(1)}{2\sqrt{2r}} \left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2}\right),$$

$$\sigma_{xy} \cong - \frac{h_1(1)+zh_2(1)}{2\sqrt{2r}} \left(- \frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{5\theta}{2}\right), \tag{9}$$

$$V_x \cong [- \frac{\kappa}{2}\, h_1(1)(\lambda\lambda_2)^2+h_2(1)]\sqrt{r/2}\, \sin \theta \cos \frac{\theta}{2}\ , \tag{10}$$

where

$$H_i(t) = h_i(t)(1-t^2)^{-\frac{1}{2}},\ i = 1,2, \tag{11}$$

$$\sigma_{ij} = N_{ij} + \frac{12az}{h} M_{ij},\ (i,j)=(x,y),\ V_x=V_1/(\sqrt{c}\ h5G/6), \tag{12}$$

$$N_{xx} = N_{11}/(chE),\ N_{yy} = cN_{22}/(hE),\ N_{xy} = N_{12}/(hE),$$

$$M_{xx} = M_{11}/(ch^2E),\ M_{yy} = cM_{22}/(h^2E),\ M_{xy} = M_{12}/(h^2E), \tag{13}$$

$$\lambda^4 = 12(1-\nu^2)a^2/h^2,\ \kappa = h^2/(5(1-\nu)a^2),$$

$$\lambda_1^4 = 12(1-\nu^2)c^2a^4/(h^2R_1^2),\ \lambda_2 = \lambda_1(R_1/R_2)^{\frac{1}{2}}/c, \tag{14}$$

R_1 and R_2 being the radii of curvatures in X_1X_3 and X_2X_3 planes, respectively (Figures 1 and 2). For the symmetric problem under consideration the Mode I stress intensity factor may then be defined and evaluated as follows:

$$k_1(X_3) = \lim_{X_2 \to a} \sqrt{2(X_2-a)}\, \sigma_{11}(0,X_2,X_3) = -\frac{cE\sqrt{a}}{2}\,[h_1(1) + \frac{X_3}{a} h_2(1)]. \qquad (15)$$

The results for the isotropic shell may be obtained by substituting c=1 and by observing that E and ν are the isotropic material constants. Equations (9), (12) and (13) show that the asymptotic stress distributions for membrane and bending stresses are identical. The effect of plastic deformations for part-through as well as through cracks in shells may be considered by introducing a plastic strip model. Aside from the particular shell theory used in the analysis, the method of solution would be identical to that followed in [7].

3. The Results and Discussion

In this paper a summary of the elastic solutions for isotropic and specially orthotropic cylindrical and spherical shells containing a through crack is given. Since in isotropic shells the results depend on the Poisson's ratio ν explicitly, the stress intensity factors given in the tables are obtained for a fixed value of ν=0.3. For the specially orthotropic shells, two different materials are considered. The first is a mildly orthotropic material for which titanium is taken as an example. The second is graphite and represents a strongly orthotropic material. Table 1 shows the properties of these two materials where G_{12} is the measured shear modulus and $G_{av.}$ is calculated from (1). The fact that these two values are sufficiently close may justify for considering these materials as specially orthotropic.

Table 1. Elastic constants of orthotropic materials

	Titanium	Graphite
E_1 N/m² (psi)	1.039×10^{11} (1.507×10^{7})	1.034×10^{10} (1.5×10^{6})
E_2 N/m² (psi)	1.434×10^{11} (2.08×10^{7})	2.758×10^{11} (4×10^{7})
ν_1	0.1966	0.0075
ν_2	0.2714	0.2000
G_{12} N/m² (psi)	4.675×10^{10} (6.78×10^{6})	2.758×10^{10} (4.0×10^{6})
G_{av}. N/m² (psi)	4.930×10^{10} (7.15×10^{6})	2.572×10^{10} (3.73×10^{6})

The results are given for shells under uniform membrane loading N_{11} or bending moment M_{11}. The normalized stress intensity factors shown in Tables 2-4 are defined by

$$k_{mj} = \frac{k_1(0)}{\sigma_j\sqrt{a}}\,,\quad k_{bj} = \frac{k_1(h/2)-k_1(0)}{\sigma_j\sqrt{a}},\ (j=m,b),\ \sigma_m = \frac{N_{11}}{h}\,,\quad \sigma_b = \frac{6M_{11}}{h^2}. \qquad (16)$$

It should be noted that λ_1 and λ_2 which are used in the tables as the variables representing the crack length are functions of $c=(E_1/E_2)^{\frac{1}{4}}$ (see eq. (14)). Hence, the effect of the material orientation on the stress intensity factors, particularly in strongly orthotropic materials is much more severe than indicated by the tables. In the tables the results given for E_1/E_2=1 correspond to isotropic shells with ν=0.3. The remaining two columns give the results for the same orthotropic shell with two different material orientations.

Some sample results from the plasticity solutions are given in Figures 3, 4, and 5. For a circumferential through crack in an isotropic cylindrical shell (ν=0.3), Figure 3 shows the crack opening stretch calculated at the crack tips $X_2=\mp a$, $X_1=X_3=0$, and Figure 4 shows the crack opening displacement at the midpoint $X_1=X_2=X_3=0$ (see Figure 1). For the same cylinder having an external part-through crack of depth d Figure 5 shows the crack opening stretch in the X_1X_3 plane at the leading edge of the crack. In these examples the membrane load $N_{11}=N_0$ applied to the cylinder away from the crack region is the only external load, σ_F is the flow stress representing the yield behavior of the material, and $d_1=a\sigma_F/E$ is the normalization

Table 2. Normalized Stress intensity factors for isotropic and specially orthotropic cylindrical shells with a circumferential through crack.

a/h	10			5			2			1		
E_1/E_2	0.725	1.380	1.000	0.725	1.380	1.000	0.725	1.380	1.000	0.725	1.380	1.000
λ_2						k_{mm}						
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.012	1.012	1.012	1.012	1.012	1.012	1.013	1.013	1.012	1.013	1.013	1.013
1.0	1.047	1.047	1.048	1.047	1.047	1.048	1.049	1.049	1.050	1.053	1.053	1.055
1.5	1.100	1.099	1.102	1.101	1.100	1.103	1.105	1.104	1.108			
2.0	1.164	1.164	1.168	1.166	1.165	1.169	1.174	1.173	1.179			
3.0	1.308	1.308	1.314	1.312	1.311	1.317						
4.0	1.457	1.456	1.462	1.462	1.461	1.467						
5.0	1.599	1.598	1.604									
						k_{bm}						
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.036	0.036	0.041	0.036	0.036	0.041	0.038	0.038	0.043	0.040	0.040	0.044
1.0	0.081	0.082	0.092	0.081	0.081	0.092	0.082	0.082	0.092	0.084	0.083	0.093
1.5	0.110	0.111	0.123	0.108	0.109	0.119	0.104	0.105	0.114			
2.0	0.115	0.117	0.125	0.110	0.111	0.119	0.100	0.101	0.107			
3.0	0.072	0.075	0.071	0.059	0.062	0.057						
4.0	-0.009	-0.006	-0.024	-0.026	-0.024	-0.042						
5.0	-0.100	-0.096	-0.126									
						k_{bb}						
0.0	0.632	0.634	0.652	0.651	0.650	0.667	0.691	0.687	0.704	0.742	0.735	0.752
0.5	0.614	0.617	0.631	0.632	0.631	0.644	0.668	0.664	0.676	0.712	0.706	0.718
1.0	0.572	0.574	0.581	0.585	0.585	0.590	0.611	0.609	0.613	0.647	0.642	0.646
1.5	0.519	0.522	0.520	0.528	0.528	0.526	0.547	0.545	0.542			
2.0	0.467	0.470	0.463	0.473	0.474	0.466	0.490	0.488	0.481			
3.0	0.383	0.385	0.373	0.387	0.388	0.376						
4.0	0.327	0.329	0.317	0.332	0.332	0.322						
5.0	0.290	0.292	0.281									
						k_{mb}						
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.008	0.008	0.010	0.009	0.009	0.010	0.010	0.010	0.012	0.012	0.012	0.014
1.0	0.019	0.019	0.022	0.021	0.020	0.024	0.024	0.023	0.027	0.027	0.026	0.030
1.5	0.028	0.028	0.032	0.030	0.029	0.034	0.033	0.033	0.037			
2.0	0.033	0.033	0.037	0.035	0.035	0.039	0.038	0.038	0.042			
3.0	0.036	0.036	0.040	0.038	0.037	0.041						
4.0	0.034	0.034	0.037	0.036	0.035	0.039						
5.0	0.032	0.031	0.034									

Table 3. Normalized stress intensity factors for isotropic and specially orthotropic cylindrical shells with an axial through crack.

a/h	10			5			2			1		
E_1/E_2	0.725	1.380	1.000	0.725	1.380	1.000	0.725	1.380	1.000	0.725	1.380	1.000
λ_1						k_{mm}						
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.048	1.064	1.057	1.048	1.065	1.057	1.049	1.066	1.058	1.051	1.069	1.061
1.0	1.171	1.226	1.199	1.172	1.227	1.200	1.178	1.236	1.208	1.197	1.261	1.233
1.5	1.341	1.442	1.394	1.344	1.446	1.398	1.362	1.468	1.420			
2.0	1.539	1.687	1.618	1.545	1.695	1.625	1.580	1.738	1.668			
3.0	1.976	2.213	2.105	1.991	2.230	2.122						
4.0	2.430	2.748	2.603	2.458	2.780	2.634						
5.0	2.887	3.283	3.096	2.934	3.337	3.146						
6.0	3.347	3.827	3.580									
8.0	4.327	-	4.515									
10.0	-	-	5.422									

Table 3 -Cont.

λ_1						k_{bm}						
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.041	0.051	0.054	0.042	0.052	0.055	0.044	0.053	0.057	0.046	0.056	0.060
1.0	0.095	0.113	0.120	0.095	0.112	0.119	0.098	0.113	0.121	0.102	0.117	0.124
1.5	0.138	0.158	0.167	0.137	0.154	0.164	0.137	0.150	0.160			
2.0	0.168	0.185	0.194	0.164	0.176	0.185	0.159	0.165	0.175			
3.0	0.186	0.184	0.189	0.173	0.159	0.166						
4.0	0.154	0.117	0.116	0.127	0.073	0.076						
5.0	0.076	-0.008	-0.014	0.034	-0.073	-0.070						
6.0	-0.041	-0.183	-0.187									
8.0	-0.388	-	-0.634									
10.0	-	-	-1.183									
E_1/E_2	0.037	26.67	1.000	0.037	26.67	1.000	0.037	26.67	1.000	0.037	26.67	1.000
λ_1						k_{mm}						
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.012	1.238	1.057	1.012	1.238	1.057	1.012	1.242	1.058	1.012	1.253	1.061
1.0	1.043	1.714	1.199	1.043	1.717	1.200	1.045	1.736	1.208	1.048	1.795	1.233
1.5	1.093	2.244	1.394	1.093	2.252	1.398	1.047	2.298	1.420			
2.0	1.155	2.779	1.618	1.156	2.794	1.625	1.164	2.877	1.668			
3.0	1.309	3.826	2.105	1.313	3.865	2.122						
4.0	1.490	4.852	2.603	1.498	4.955	2.634						
5.0	1.686	-	3.096	1.699	-	3.146						
6.0	1.891	-	3.580									
8.0	2.311	-	4.515									
10.0	2.737	-	5.422									
						k_{bm}						
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.008	0.075	0.054	0.008	0.075	0.055	0.009	0.075	0.057	0.010	0.077	0.060
1.0	0.023	0.150	0.120	0.023	0.145	0.119	0.024	0.139	0.121	0.027	0.137	0.124
1.5	0.039	0.186	0.167	0.040	1.172	0.164	0.042	0.151	0.160			
2.0	0.056	0.175	0.194	0.056	0.148	0.185	0.059	0.105	0.175			
3.0	0.089	0.017	0.189	0.088	-0.045	0.166						
4.0	0.117	-0.287	0.166	0.115	-0.395	0.076						
5.0	0.141	-	-0.014	0.136	-	-0.070						
6.0	0.158	-	-0.187									
8.0	0.169	-	-0.634									
10.0	0.148	-	-1.183									

Table 4. Normalized stress intensity factors for isotropic and specially orthotropic spherical shells with a meridional through crack.

a/h	10			5			2			1		
E_1/E_2	0.725	1.380	1.000	0.725	1.380	1.000	0.725	1.380	1.000	0.725	1.380	1.000
λ_2						k_{mm}						
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.016	1.021	1.018	1.016	1.021	1.018	1.016	1.021	1.019	1.017	1.022	1.019
0.50	1.060	1.077	1.069	1.060	1.077	1.069	1.062	1.079	1.071	1.065	1.084	1.076
0.75	1.129	1.164	1.149	1.130	1.165	1.150	1.135	1.171	1.156	1.148	1.187	1.173
1.0	1.220	1.276	1.252	1.222	1.279	1.255	1.232	1.291	1.268	1.262	1.327	1.305
1.5	1.450	1.555	1.512	1.456	1.561	1.519	1.486	1.596	1.556			
2.0	1.733	1.891	1.828	1.744	1.903	1.841	1.807	1.976	1.918			
2.5	2.056	2.270	2.186	2.076	2.291	2.208						
3.0	2.415	2.685	2.579	2.446	2.718	2.615						
3.5	2.806	3.132	3.004	2.852	3.183	3.058						
4.0	3.226	3.611	3.460	3.296	3.687	3.539						

Table 4 - Cont.

λ_2						k_{bm}						
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.25	0.025	0.029	0.033	0.026	0.030	0.034	0.027	0.031	0.035	0.029	0.033	0.037
0.50	0.065	0.073	0.084	0.066	0.074	0.084	0.068	0.075	0.086	0.071	0.079	0.090
0.75	0.104	0.115	0.130	0.104	0.115	0.130	0.106	0.115	0.130	0.109	0.188	0.133
1.0	0.135	0.148	0.165	0.134	0.145	0.162	0.133	0.142	0.158	0.134	0.141	0.157
1.5	0.165	0.175	0.187	0.158	0.164	0.174	0.145	0.146	0.155			
2.0	0.146	0.145	0.142	0.129	0.121	0.117	0.099	0.083	0.077			
2.5	0.077	0.054	0.031	0.047	0.015	-0.010						
3.0	-0.044	-0.099	-0.146	-0.090	-0.158	-0.206						
3.5	-0.219	-0.317	-0.390	-0.283	-0.398	-0.471						
4.0	-0.448	-0.602	-0.701	-0.533	-0.709	-0.807						

factor. A similar problem for a cylindrical shell with an axial crack was considered in [8].

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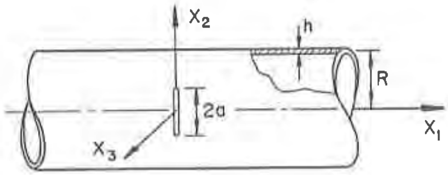

Fig. 1 Notation for cylindrical shell

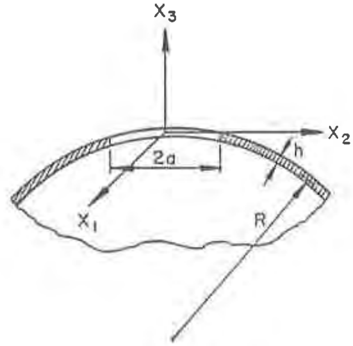

Fig. 2 Notation for spherical shell.

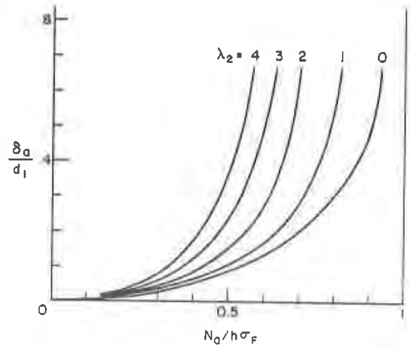

Fig. 3 Crack opening stretch at the crack tip in an isotropic cylindrical shell with a circumferential through crack, subjected to uniform membrane loading, $a/h=2$, $d_1=a\sigma_F/E$.

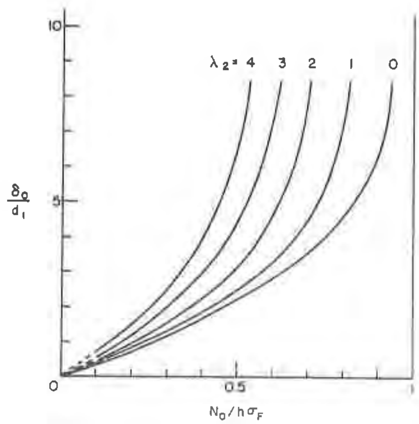

Fig. 4 Crack opening displacement at the center of a circumferential through crack in a cylindrical shell under uniform membrane loading, $a/h=2$, $d_1=a\sigma_F/E$.

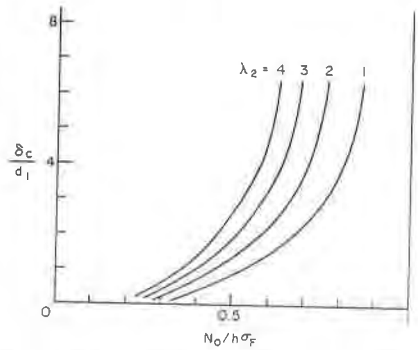

Fig. 5 Crack opening stretch at the midpoint of the leading edge of an external part-through circumferential crack in a cylindrical shell under uniform membrane loading, $a/h=2$, $d/h=0.8$, (d being the crack depth), $d_1=a\sigma_F/E$.