APPROXIMATIONS FOR DYNAMIC MODELING

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Summary

To perform dynamic analysis a complicated system is frequently replaced with an appropriate equivalent simplified member. The most common way of finding such an equivalent member is to place lumped mass and mass moment of inertia at the corresponding floor level of a beam such that the significant frequencies of this beam would match with those of the original structural system. The effects due to the linked members between wall and column as well as the unsymmetrical geometric effects can be taken into consideration by making a 3-D static analysis. Nevertheless, the rotational stiffness is in general difficult to be determined. The simplest way of finding such a floor characteristics could be the least square method or the collocation method.

The collocation approach is proposed to use the multipoint constraint technique. Linear relations to specify the nodal displacements can be easily accomplished. i.e. $a_i u_i=0$, where a a_i are the selected constant coefficients for the static 3-D analysis; u_i are the displacements for the node points i. For symmetrical structure with 2-D in nature, results from collocation method should be close to those obtained from the least square method. Numerical results were manipulated for a three span frame with shear-wall at the center span by varying the wall thickness and the span lengths. The rotational stiffness values are very close for the corresponding cases using different methods. For a problem with 3-D in nature, numerical comparisons are also provided for selected structural layouts by varying the locations and the dimensions of the shearwalls and the columns. In general, a "unique" solution is difficult to be obtained because of the following reasons. (I) The complicated geometry layout makes the dynamic behaviors difficult to be predicted. (II) The floor characteristics cannot be described with one significant rotational mode and one translational mode. Although there is no general principles can be concluded in this study, certain rules are recommended on how each structural members should be considered in evaluating their effective stiffness. These rules will be helpful in making approximations to the irregular floor conditions such as floor openings, T-beam effective flanges, etc.

Material non-linearity may be a problem during heavy load conditions. It is very expensive for each time step to evaluate the structural behaviors. Discussions are also provided in the paper on the effective-strain method to be applied to the cracking of concrete.

1. Introduction

The reliability of analytical results of wall-column frame systems under the actions of impulsive loads and earthquake excitations depends upon the accuracy of structural modeling. There are many valuable papers published on this subject recently. e.g. References [1], [4], [5], [7]. Nevertheless, those methods have limited application in practical design due to one of the following reasons: (1) Assumptions made in the theoretical formulation limit the applicability of the methodology. (2) Complicated hand computations are needed in the analysis. (3) Engineering decisions should be made before modeling in order to reduce the computer cost for complicated structures. In engineering practice, a complicated system is frequently replaced with an appropriate equivalent simplified member. A common practice, is to represent a complicated structure by a lumped-mass beam model. The lumped mass and mass moment of inertia are placed at the floor levels. The beam properties are found such that the significant frequencies of this beam model would match with those of the original system. With this approach, three-dimensional static finite element analyses are needed to obtain the translational and rotational stiffnesses for the asymmetric structures. However, the rotational stiffness is in general difficult to be determined due to the irregularity of the structures. This paper is to present simpler ways of approximating such floor characteristics using collocation method and least square method. In the following sections, the analytical methods are firstly presented. Examples and comparisons are given afterwards with the selected framing systems. Conclusions and discussions on these methods and the related factors are provided based on the analytical results of this study.

2. Analysis and Methodology

The dynamic characteristics of a beam are completely defined if its material and sectional properties, shear area (A) and moment of inertia (I), are specified. Thus an equivalent beam can be established when the section properties are found through the following relations:

$$f_1 \ (I, A, l, p) = k_1 \qquad (1)$$

$$f_2 \ (I, A, l, p) = k_2 \qquad (2)$$

where f_1, f_2 are functions depend upon the boundary conditions of the beam.

l : length of the beam.

p : material properties, either Young's modulus (E), or shear modulus (G)

k_1: translation stiffness of a structural system

k_2: rotational stiffness of a structural system

As described earlier, k_1 is an unique value in general and can be found quite straight forward. But to find K_2 approximations will have to be built in. To approximate a set of known displacement functions ϕ_n (X_j) to an arbitrary function H (X_j) the best approximation can be represented as:

$$H(X_j) - G[\phi_n(X_j)] = \min. \quad (3)$$

where H, G : certain functions.

X_j : coordinate systems

$\phi(X_j)$: displacement functions

If G is selected to be a linear combinations of known function $_n$ and $_n$ an error quantity, then eq. (3) becomes

$$H(X_j) - \Sigma a_i \phi_j = \epsilon_n \quad (4)$$

If $H(X_j)$ is set as linear function, eq. (4) can be written as:

$$\Sigma a_j \phi_j = 0 \quad (5)$$

If least square method is to be used, the constants will be determined by [3] :

$$1/2 \frac{\partial \|\epsilon_n\|^2}{\partial a_j} = \int_R \epsilon_n W \phi_j = o \quad (6)$$

where W is the weighting function. If W is set equal to 1 and R is in discrete space, eqs. (b) become

$$1/2 \frac{\partial \|\epsilon_n\|^2}{\partial a_j} = \sum_j \epsilon_n \phi_j = o \quad (6a)$$

Eqs. (5) and (6a) are the bases for these methods.

3. Sample problems.

Two structure systems are selected to illustrate the approximations described above. the 1st example is a plane frame structure, and the second one is an asymmetric wall-column floor system. For either cases, couples were applied on the top of the frames. Static analyses were made by utilzing the computer code, NASTRAN [8] and the rotations were then calculated with eqs. (5) and (6a).

(A) Sample problem 1 - plane frame structure

The geometry dimensions and the member sizes are shown in figure 1. The shearwall is at central span. Material was assumed to be concrete with Young's modulus E= 3×10^6 psi and poisson's ratio - .17. The rotational stiffness was found to be 2.86×10^8 ft-$^{k/}$rad by equations (5) and 3.09×10^8 with equations (6 a). The deviation between these methods is within 10%. To vary the wall thickness from 8" to 12", the results are close to each other within the same range.

(B) Sample problem 2 - an asymmetric wall-column system

The geometric dimensions and the member sizes are shown in figure 2a. Material was assumed the same as example 1. The couples are uniformly applied on the floor. Equations (5) and (6a) were applied separately for each plane frame from grid 1 to grid 4. The entire rotational stiffness was found to be 3.2×10^9 ft-^{K}/rad from collocation method and approximate 3.0×10^9 ft^K/rad from least square method. The difference is about 7%. However, if the geometric dimensions are of shown in fig. 2b, the difference between the solutions from those methods would be about 12% .

4. Conclusions and Discussions

This study presents approximation methods to find the stiffness of a structural system with no limitations on the methodology. As can be seen from the given examples, the solution of the floor stiffness is not unique because of the following reasons: (1) The complicated geometric layouts makes the dynamic behavior difficult to predict and, (2) the floor characteristics cannot be described with one rotational mode. i.e., the accuracy of the solution is geometric dependent. In general, the rotational stiffness should not affect the significant frequencies of the entire system for heavy shearwall structures. But for relative flexible structures or structures with irregular floor conditions such as floor openings, etc., the procedure with least square approach can give better "feelings" to the engineer and construct an upper bound if needed.

In case the materials fall into a non-linear range due to high dynamic stresses , the section properties found from the above solutions will have to be changed. [2] To avoid the expensive iterative process, the equivalent linear approach such as the effective strain method [6] should be the most reasonable approach. Naturally, experimental data will have to be based on to determine those related factors and establish the degrees of approximation.

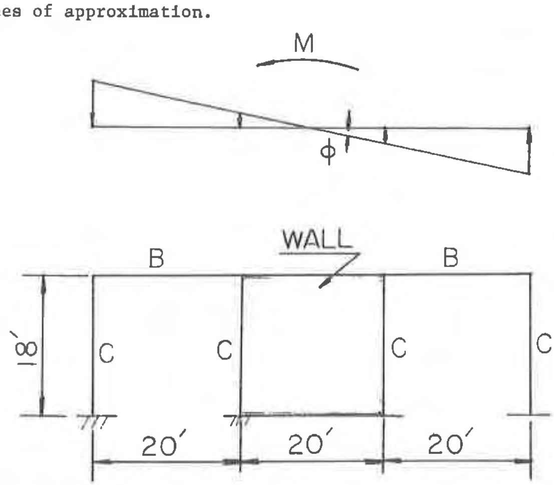

figure 1. Elevated Geometric dimensions of a plane frame -- for Example 1. couples are applied on top of the frame as shown. All the "bars" (C and B), are with dimensions A= 720 in ^{2} and I = 72000 in^4.

References

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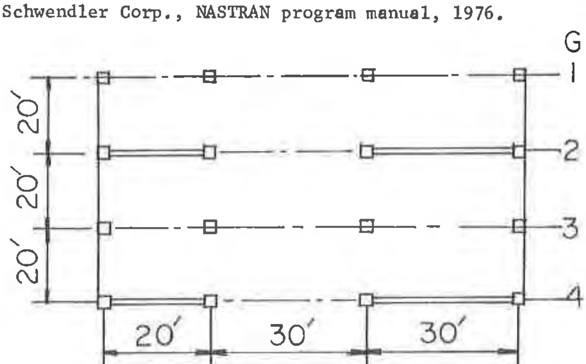

FIG 2A

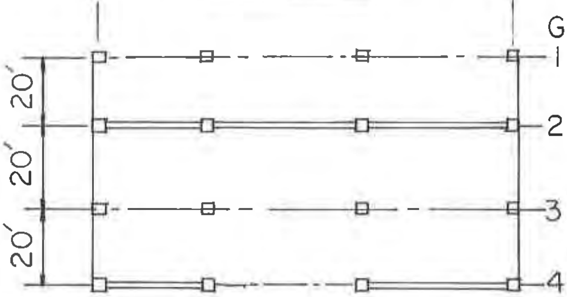

FIG 2B

figure 2. (2A & 2B) Plan Geometric dimensions of a 3-D wall-column system -- for Example 2. The thickness of wall and column is 1 ft. The bar sizes are all with 1.5 ft. by 1.5 ft. The floor height is assumed to be 20 ft.