Contact with friction modeling for the study of a bolted junction

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1 INTRODUCTION

Many structural analysis problems are concerned by contact phenomena. A good knowledge of the contact displacements and the contact forces between the different parts of the structure is generally essential in structure assembling. The special boundary behaviour has a strong influence on the distribution of the stresses in the whole structure and on his total fiability. The contact behaviour is strongly non linear because of the non penetration conditions on the one hand, and because of the friction on the other. On such problems the real contact zone and the contact forces are unknown "a priori" and have to be determined during the resolution. The non penetration will be caracterized by unilateral conditions and the friction will be described by a constitutive law (the Coulomb friction law in this paper).

The application presented here concerns the assembling of the three parts of a bolted junction using a pressing ring. There are three contact zones in this problem. A good description of the contact phenomena is essential to ensure tightness.

From a theoretical point of view, this class of problems leads to variational or quasi-variational inequations containing undifferentiable terms (Raous-Latil). For frictionless problems, formulation and mathematical results have been given by Lions-Stampacchia, Stampacchia,... For problems including friction, one can refer to Duvaut-Lions, Duvaut, Campos-Oden-Kikuchi, Cocu,...

From a numerical point of view, different classes of algorithms are used: penalisation (Campos-Oden-Kikuchi,...), non linear programming methods (Karbling,...), Newton-Raphson (Curnier,...), contact finite elements (Bathe,...). Our methods are based on projection techniques coupled with overrelaxed Gauss-Seidel methods including condensation procedures (reduction of the number of variables). Non linear programming methods and iterative procedures on special boundary conditions are however also used.

2 SIGNORINI PROBLEM INCLUDING FRICTION

The problem of the contact of several solids is a generalization of the contact between a solid and a rigid obstacle, which in turn if one takes friction into account is an extension of the Signorini problem.

2.1 Formulation

Under the small deformation hypothesis the kinematic relations are written :

$$(1)\quad e_{ij} = \tfrac{1}{2}\,(u_{i,j} + u_{j,i})$$

where e_{ij} denotes the strain tensor components and u_i the displacement components. For elastic material, we set :

$$(2)\quad \sigma_{ij} = K_{ijkl}\, e_{kl}$$

where σ_{ij} are the stress tensor components and K_{ijkl} the elasticity tensor. The equilibrium equations are written :

(3) $\sigma_{ij,j} = - \phi^1_i$ (ϕ^1_i are the components of the volumic forces)

(4) $\sigma_{ij}\, n_j = \phi^2_i$ (ϕ^2_i are the components of the surfacic forces)

On the contact boundary we use a local coordinate referential $(\vec{n},\vec{t})$ where $\vec{n}$ is the exterior normal vector to the surface. We denote u^n , u^t , F^n , F^t respectively the normal and tangential components of the displacements and of the forces on the contact aera.The unilateral conditions are written :

$$(5)\qquad \begin{aligned} u^n &\leq 0 \\ F^n &\leq 0 \\ u^n .\, F^n &= 0 \end{aligned}$$

The Coulomb friction law on the tangential components can be written (see Duvaut-Lions) :

(6) $| F^t | \leq \mu\, | F^n |$ (μ is the friction coefficient)

with if $| F^t | < \mu\, | F^n |$ then $u^t = 0$

if $| F^t | = \mu\, | F^n |$ then $u^t = -\lambda\, F^t$, $\lambda > 0$ if $| F^n | \neq 0$

The variational form of the equilibrium equations leads in this case to a quasi variational inequation. We formulate now the problem as a sequence of Tresca problems (where the sliding limit g is given) associated with a fixed point method on this sliding limit $g_{k+1} = \mu\, | F^n (u_k) |$ (see Raous-Latil).

The Tresca problem can be formulated as a variational inequation containing undifferentiable terms. Because of the symetry of the elasticity mapping, this problem is equivalent to the minimization under constraints of the following functional :

$$(7)\quad J(v) = \tfrac{1}{2}\, a(v,v) - (f,v) + \int g\, | v^t |\, dl$$

where : - $a(v,v)$ is a coercive and symetrical bilinear form associated to the elasticity mapping

- (f,v) is a linear form associated to the given loads
- the last term is the work of the friction force in the tangential displacement v^t.

2.2 Numerical methods

We have extended the Cryer-Christopherson method to the friction case including the undifferentiable term. It is an overrelaxed Gauss-Seidel method with projection (see Raous-Latil). Using a condensation procedure (see Taallah), we reduce the problem to the only variables concerned by the contact non linearities. A preliminary preparation for the reduced problem has to be done once and only once : it is a partial inversion of the system. This is specially efficient in the case of several loading cases. Others ameliorations and performances are presented in Raous-Latil.

We also use a direct non linear programming method (Lemke-Cottle-Dantzig algorithm) for frictionless problems. It is very efficient on the condensed problem but it cannot be extended to the friction case.

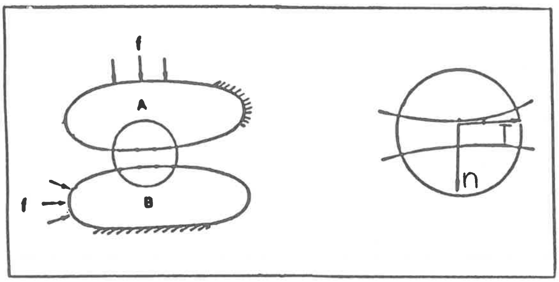

Figure 1 : Contact of two bodies

3 EXTENSION TO A TWO BODY CONTACT CASE

Denoting with the indices A et B the variables concerning respectively the solids A and B, we can write the unilateral conditions and the friction law under the following forms :

$$(8)\quad \begin{cases} u_A^n + u_B^n \leqq 0 \\ F_A^n = F_B^n \\ F_A^n \leqq 0 \\ (u_A^n + u_B^n)\, F_A^n = 0 \end{cases}$$

n_A and n_B are the outside normal to the contact aeras Γ_A^C and Γ_B^C.

$$(9)\quad \begin{cases} F_A^t = F_B^t \\ |F_A^t| \leqq \mu\, |F_A^n| \\ \text{if } |F_A^t| < \mu\, |F_A^n| \text{ then } u_A^t + u_B^t = 0 \\ \text{if } |F_A^t| = \mu\, |F_A^n| \text{ then } u_A^t + u_B^t = -\lambda\, F_A^t\ ,\ \lambda > 0,\ \text{if } F_A^n \neq 0 \end{cases}$$

Using the change of variables $d^n = u_A^n + u_B^n$ and $d^t = u_A^t + u_B^t$, we give a new formulation of (7) with the variables u_A^t and d^t. The variable d^n is used in the definition of the convex K which is the space of the contraints concerning the minimization. This is essential for the projection procedure.

For frictionless problem, the Lemke-Cottle-Dantzig has been modified and extended to this case : it it still efficient.

For the problem including friction, we use the Cryer-Christopherson method associated again with the condensation procedure.

4 APPLICATION

4.1 Contact with a rigid obstacle

We present here an example used as a validation test in a workshop of the GRECO "Grandes Déformations et Endommagement". Computations on this example have been done by different laboratories in this group. It concerns the compression of a long bar on a plane (plane strain problem). The geometry is given on figure 2 :

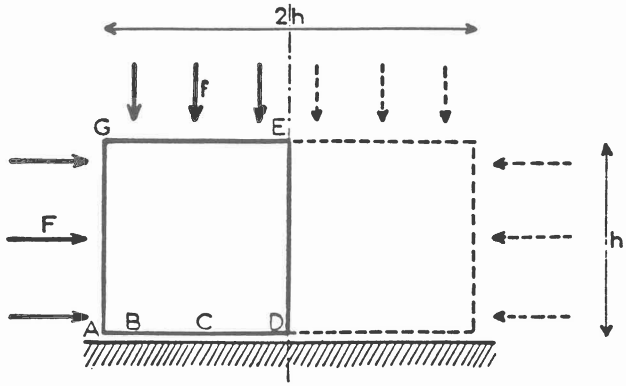

Figure 2 : Compression of a bar on a plane

The values of the parameters are : h = 40 mm, E = 13000 daN/mm^2, ν = 0.2, F = 10 daN/mm, f = -5 dan/mm, μ = 1.

Table 1. Displacements and forces on the contact aera.

NODE	u^n	u^t	F^n	F^t
1	-0.571E-03	0.147E-01	0.333E-03	-0.214E-03
2	-0.271E-03	0.137E-01	0.195E-03	-0.595E-03
3	-0.487E-04	0.128E-01	0.138E-03	-0.504E-03
4	0.000E+00	0.118E-01	-1.16	-1.16
5	0.000E+00	0.107E-01	-2.16	-2.16
6	0.000E+00	0.968E-02	-2.84	-2.83
7	0.000E+00	0.865E-02	-3.40	-3.40
8	0.000E+00	0.765E-02	-3.87	-3.87
9	0.000E+00	0.669E-02	-4.29	-4.27
10	0.000E+00	0.576E-02	-4.66	-4.66
11	0.000E+00	0.487E-02	-5.01	-5.01
12	0.000E+00	0.403E-02	-5.33	-5.33
13	0.000E+00	0.324E-02	-5.64	-5.64
14	0.000E+00	0.250E-02	-5.95	-5.95
15	0.000E+00	0.182E-02	-6.27	-6.27
16	0.000E+00	0.120E-02	-6.61	-6.61
17	0.000E+00	0.667E-03	-6.99	-6.99
18	0.000E+00	0.244E-03	-7.50	-7.50
19	0.000E+00	-0.527E-25	-8.16	-7.61
20	0.000E+00	0.345E-26	-8.59	-5.07
21	0.000E+00	0.113E-26	-8.68	-4.33
22	0.000E+00	-0.738E-28	-8.75	-3.73
23	0.000E+00	-0.122E-27	-8.80	-3.23
24	0.000E+00	0.181E-28	-8.87	-2.79
25	0.000E+00	-0.701E-28	-8.90	-2.41
26	0.000E+00	-0.529E-28	-8.94	-2.06
27	0.000E+00	0.653E-28	-8.96	-1.73
28	0.000E+00	0.262E-29	-8.99	-1.41
29	0.000E+00	-0.156E-27	-9.01	-1.11
30	0.000E+00	0.596E-28	-9.04	-0.827
31	0.000E+00	0.278E-27	-9.04	-0.542
32	0.000E+00	0.158E-27	-9.07	-0.260

We have a unilateral contact with friction on the part AD. The displacements and the forces on the contact zone are given in the table 1.
We observe that the relations (5) and (6) are verified with a very good precision on the three different zones of the contact boundary:

- a non contact zone (nodes 1 to 3): $u^n < 0$ and $F^n = F^t = 0$,
- a sliding zone (nodes 4 to 18): $u^n = 0$, $F^n < 0$ (contact), $u^t \neq 0$ and $|F^t| = |F^n|$ (the friction coefficient $\mu = 1.$),
- a sticked zone (nodes 19 to 33): $u^n = 0$, $F^n < 0$ (contact), $u^t = 0$ and $|F^t| < |F^n|$.

4.2 Study of a bolted junction

The structure is the assembling of the three parts of bolted junction by the use of a pressing ring. Results given on figure 3 concern the application of the closing pressure on the ring before introducing the pressure. The figure 3 gives the deformation of the structure (with an amplification coefficient equal to 100.) and the level curves of the component σ_{zz} of the stress tensor.

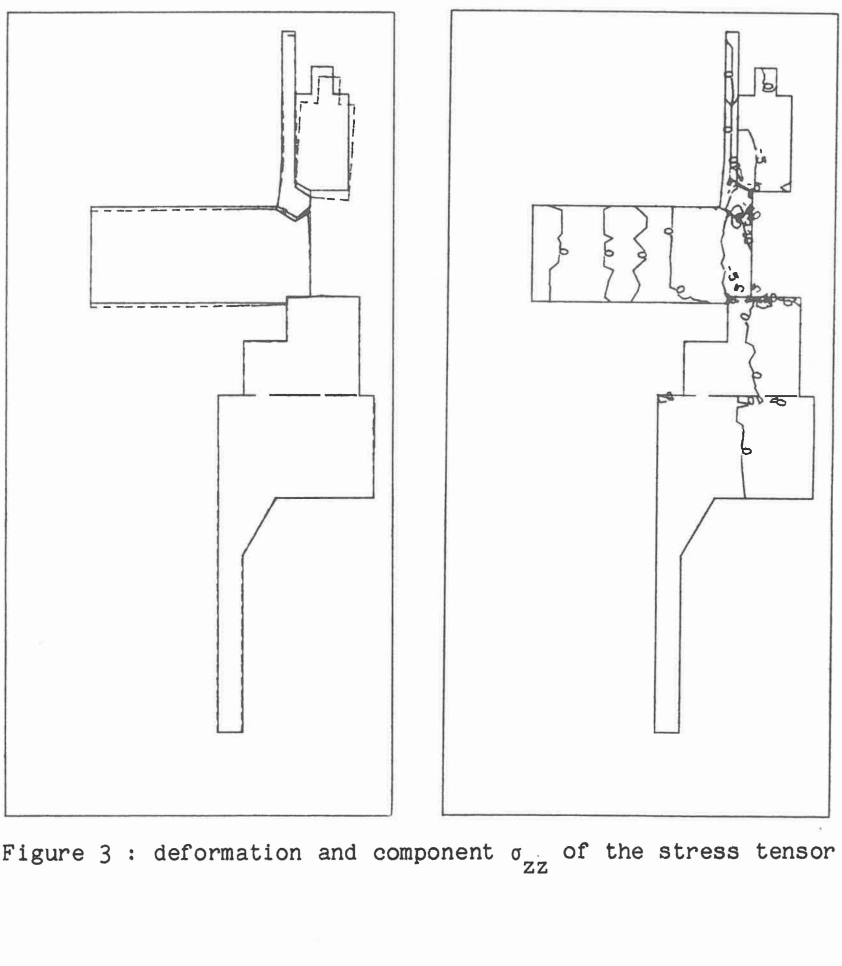

Figure 3 : deformation and component σ_{zz} of the stress tensor

A complete study with different loading cases and different friction coefficients has been done in the context of a contract with Technicatome.

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