B 8/2*

DESCRIPTION OF INTERNAL FLOW PROBLEMS BY A BOUNDARY INTEGRAL METHOD WITH DIPOLE PANELS

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Summary

In reactor safety studies the failure of single components is postulated or sudden accident loadings are assumed and the consequences are investigated. Often as a first consequence highly transient three dimensional flow problems occur. Examples are blowdown flows inside a reactor vessel, a fluid container under seismic accelerations, or the fluid motion during steam condensation in the pressure suppression system of a boiling water reactor.

In contrast to classical flow problems, in most of the above cases the fluid velocities are relatively small whereas the accelerations assume high values. As a consequence both, viscosity effects and dynamic pressures which are proportional to the square of the fluid velocities are usually negligible. For cases, where the excitation times are considerably longer than the times necessary for a wave to traverse characteristic regions of the fluid field, also the fluid compressibility is negligible.

Under these conditions boundary integral methods - also called singularity methods or panel methods - are an appropriate tool to deal with the problem. Flow singularities are distributed over the fluid boundaries in such a way that pressure and velocity fields are obtained which satisfy the boundary conditions. In order to facilitate the numerical treatment the fluid boundaries are approximated by a finite number of panels with uniform singularity distributions on each of them. Consequently the pressure and velocity field of the given problem may be obtained by superposition of the corresponding fields due to these panels with their singularity intensities as unknown factors. Then satisfying the boundary conditions in so many boundary points as panels have been introduced, yields a system of linear equations which in general allows for a unique determination of the unknown intensities.

In aerodynamics, where panel methods are well known, usually source distributions are used as singularities. However, for internal flow problems, which are typical in reactor safety investigations, source panels have some disadvantages such as high leakages at boundary corners and a tendency for ill-conditioned equation systems.

In order to overcome this problem a boundary integral method (program SINGI) has been developed which is based on so-called dipole panels. They may be generated by two source panels having opposite signs and a distance which approaches zero. So far, closed form analytical solutions could be obtained only for the flow field around a rectangular dipole panel. Such solutions are necessary for superposition. It turns out that the pressure field is continous through the whole space except the panels itself, where a pressure step occurs across it. Therefore, dipole panels are especially able to model pressure differences across walls. Furthermore, free fluid surfaces can easily be treated.

For demonstration two different transient three-dimensional flow problems are investigated. The later one concerns the fluid motion due to steam bubble collapse in the pressure suppression system of a boiling water reactor type 1969.

The method represents an excellent tool to treat the added fluid mass effect in coupled fluid-structural dynamics. Applications to the pressure suppression system including this coupling effect are reported in paper B8/4. The structural dynamics of the pressure suppression system without fluid coupling is dealt with in paper B8/3.

1. Definition of the Problem

The flow problems which will be investigated here may be characterized as follows:

- Highly transient flow fields.
- Almost arbitrary three-dimensional flow fields, especially internal flow problems with thin walls wetted on both sides.
- Boundary conditions with prescribed normal accelerations (Neumann type) or with prescribed pressures (free fluid surface, Dirichlet type).

Furthermore, the solution procedure which will be developed here, should be useful for problems in coupled fluid structural dynamics dealt with in paper B8/4.

In order to facilitate the solution of these problems, some restrictions have been introduced:

- Displacements of the fluid boundaries must be small in comparison to characteristic dimensions of the fluid field.
  Corresponding assumptions are widely used in structural dynamics. Consequently, for fluid boundaries directly formed by structural members this assumption represents seldom an additional restriction.
- The dynamic pressures due to the fluid velocity at fluid boundaries with prescribed static pressures must be small in comparison to characteristic pressures of the system. (The dynamic pressure is defined as the square of the velocity multiplied by half of the fluid density.)
  This is always true for incipient flows. For later times, if necessary, an iterative approximation in order to take into account the above dynamic pressure is possible.
- Body forces, for instance gravity forces, are not taken into account.
  However, by applying techniques of fluid structural coupling an approximate description of surface waves is possible.
- The fluid viscosity must be negligible.
  This again is always true for incipient flows, since viscosity effects are proportional to the square of the fluid velocity. It is also acceptable for later times, if high velocity zones do not occur, or if such zones can be treated separately.
- The fluid compressibility must be negligible and the fluid density must be constant.
  This is true as long as characteristic motions of the system (eigen oscillations) take considerably longer times than a wave to traverse characteristic regions of the fluid field.
- Fluid rotations must be negligible.
  With negligible viscosity this is automatically satisfied, if the rotations vanish for a reference time, e.g. at the beginning.

2. Fluid Dynamic Equations and Elementary Solutions

The field equations which govern an inviscid, incompressible, irrotational fluid without sources and body forces are

$$\operatorname{div} \bar{a} = 0 \; ; \qquad \bar{a} + \frac{1}{\rho} \operatorname{grad} P = 0 \tag{2.1}$$

where

$\bar{a}$ = acceleration vector, defined as the partial time derivative of the velocity vector. (In highly transient flow problems the fluid accelerations, rather than the velocities are of primary interest. Therefore the fluid acceleration is introduced here, in order to describe the fluid motion.)

ρ = density

P = total pressure, defined as the sum of the static and dynamic pressure.

As boundary conditions either the acceleration vectors or the static pressures are prescribed. According to section 1 the prescribed static pressure can be approximated by the total pressure. Consequently the acceleration vector $\bar{a}$ and the total pressure P appear linearly in the field equations 2.1 as well as in the boundary conditions. Therefore, the principle of superposition is applicable with respect to both, $\bar{a}$ and P.

This fact can be used to facilitate the solution of the problem: Solutions for complex problems, i.e. solutions of 2.1 under any irregular boundary conditions, can be constructed by superposition of elementary solutions, i.e. by solutions of eqs. 2.1 under very simple boundary conditions. Some of such elementary solutions are given below:

2.1 Pressure and acceleration field due to a source S located at the origin of a spherical coordinate system.

With the spherical coordinates r, ψ, ϕ and the components a^r, a^ψ, a^ϕ of the acceleration vector $\bar{a}$, shown in fig. 1 the solution reads:

$$P = \rho \frac{\partial S/\partial\tau}{4\pi} \frac{1}{r} ; \quad a^r = \frac{\partial S/\partial\tau}{4\pi} \frac{1}{r^2} ; \quad a^\psi = 0 ; \quad a^\phi = 0 \qquad (2.1.1)$$

$\partial S/\partial\tau$ is the time derivative of S

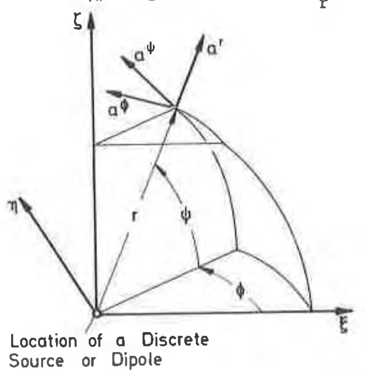

Fig. 1 Spherical coordinate system with a discrete source or dipole

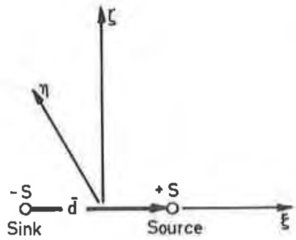

Fig. 2 Definition of the dipole $\bar{T}$

2.2 Pressure and acceleration field due to a dipole (doublet) $\bar{T}$ located at the origin of a spherical coordinate system.

As shown in fig. 2 a source and a sink, both of intensity S, are separated by distance vector $\bar{d}$. Then the dipole $\bar{T}$ is defined as

$$\bar{T} = \lim_{\bar{d}\to 0} S\,\bar{d} ;$$ $\partial T/\partial\tau$ is the amount of the time derivative of $\bar{T}$

According to this definition the pressure and acceleration fields of a dipole $\bar{T}$ can be calculated by superimposing the corresponding fields due to both, the source and the sink with the distance $\bar{d}$ approaching zero. The solution reads:

$$P = \rho \frac{\partial T/\partial\tau}{4\pi} \frac{1}{r^2} \cos\psi \cdot \cos\phi \qquad a^\psi = \frac{\partial T/\partial\tau}{4\pi} \frac{1}{r^3} \sin\psi \cdot \cos\phi$$

$$a^r = 2 \frac{\partial T/\partial\tau}{4\pi} \frac{1}{r^3} \cos\psi \cdot \cos\phi \qquad a^\phi = \frac{\partial T/\partial\tau}{4\pi} \frac{1}{r^3} \sin\phi \qquad (2.2.1)$$

2.3 Pressure and acceleration field due to a source panel, i.e. due to sources continously distributed over a surface element.

The solution can be obtained by superposition of the fields 2.1.1 which leads to double integrals over the surface element. Results, which are given by Hess and Smith [1], for instance, are not re-written here, since source elements are not used as elementary solutions in this paper.

2.4 Pressure and acceleration fields due to a dipole panel, i.e. due to dipoles uniformly distributed over a plane rectangular element. The dipole axes are perpendicular to the element.

The element is shown in fig. 3. Field points are described by Cartesian coordinates ξ, η, ζ, points belonging to the plane rectangular dipole panel are described by the coordinates $\tilde{\eta}$, $\tilde{\zeta}$. The symbols a^{ξ}, a^{η}, a^{ζ} denote the corresponding components of the acceleration vector $\bar{a}$. Introducing the dipole density

$$t = \frac{d^2T}{d\tilde{\eta}d\tilde{\zeta}} \text{ ; } \partial t/\partial\tau \text{ is the time derivative of } t$$

and using appropriate formulae for coordinate transformation between the system r, ψ, ϕ, and the system ξ, η, ζ, the principle of superposition yields:

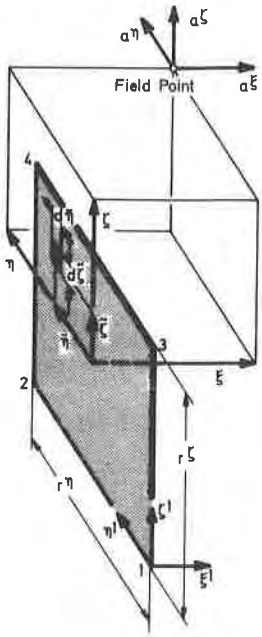

Fig. 3 Cartesian coordinate system with rectangular dipole element

$$P = \rho\,\frac{\partial t/\partial\tau}{4\pi}\int_{-r^{\zeta}/2}^{+r^{\zeta}/2}\int_{-r^{\eta}/2}^{+r^{\eta}/2}\frac{\xi}{\left[\xi^2+(\eta-\tilde{\eta})^2+(\zeta-\tilde{\zeta})^2\right]^{3/2}}\,d\tilde{\eta}d\tilde{\zeta}$$

$$a^{\xi} = \frac{\partial t/\partial\tau}{4\pi}\int_{-r^{\zeta}/2}^{+r^{\zeta}/2}\int_{-r^{\eta}/2}^{+r^{\eta}/2}\frac{2\xi^2-(\eta-\tilde{\eta})^2-(\zeta-\tilde{\zeta})^2}{\left[\xi^2+(\eta-\tilde{\eta})^2+(\zeta-\tilde{\zeta})^2\right]^{5/2}}\,d\tilde{\eta}d\tilde{\zeta}$$

$$a^{\eta} = \frac{\partial t/\partial\tau}{4\pi}\int_{-r^{\zeta}/2}^{+r^{\zeta}/2}\int_{-r^{\eta}/2}^{+r^{\eta}/2}\frac{3\xi(\eta-\tilde{\eta})}{\left[\xi^2+(\eta-\tilde{\eta})^2+(\zeta-\tilde{\zeta})^2\right]^{5/2}}\,d\tilde{\eta}d\tilde{\zeta} \qquad (2.4.1)$$

$$a^{\zeta} = \frac{\partial t/\partial\tau}{4\pi}\int_{-r^{\zeta}/2}^{+r^{\zeta}/2}\int_{-r^{\eta}/2}^{+r^{\eta}/2}\frac{3\xi(\zeta-\tilde{\zeta})}{\left[\xi^2+(\eta-\tilde{\eta})^2+(\zeta-\tilde{\zeta})^2\right]^{5/2}}\,d\tilde{\eta}d\tilde{\zeta}$$

After lengthy calculations the following, relatively simply, closed form solutions for the pressure and acceleration fields due to the rectangular dipole element can be found:

$$P = \rho\,\frac{\partial t/\partial\tau}{4\pi}\left[f^P(\xi,\eta^1,\zeta^1) - f^P(\xi,\eta^2,\zeta^2) - f^P(\xi,\eta^3,\zeta^3) + f^P(\xi,\eta^4,\zeta^4)\right]$$

$$a^{\xi} = \frac{\partial t/\partial\tau}{4\pi}\left[f^{\xi}(\xi,\eta^1,\zeta^1) - f^{\xi}(\xi,\eta^2,\zeta^2) - f^{\xi}(\xi,\eta^3,\zeta^3) - f^{\xi}(\xi,\eta^4,\zeta^4)\right]$$

$$a^{\eta} = \frac{\partial t/\partial\tau}{4\pi}\left[f^{\eta}(\xi,\eta^1,\zeta^1) - f^{\eta}(\xi,\eta^2,\zeta^2) - f^{\eta}(\xi,\eta^3,\zeta^3) - f^{\eta}(\xi,\eta^4,\zeta^4)\right] \qquad (2.4.2)$$

$$a^{\zeta} = \frac{\partial t/\partial\tau}{4\pi}\left[f^{\zeta}(\xi,\eta^1,\zeta^1) - f^{\zeta}(\xi,\eta^2,\zeta^2) - f^{\zeta}(\xi,\eta^3,\zeta^3) - f^{\zeta}(\xi,\eta^4,\zeta^4)\right]$$

where the arguments ξ, η^1, ζ^1, ξ, η^2, ζ^2, etc. relate to coordinate systems which are translated into the corner points 1, 2,... etc. of the dipole element:

$$\eta^1 = \eta + r^\eta/2 \qquad \eta^2 = \eta - r^\eta/2 \qquad \eta^3 = \eta + r^\eta/2 \qquad \eta^4 = \eta - r^\eta/2$$

$$\zeta^1 = \zeta + r^\zeta/2 \qquad \zeta^2 = \zeta + t^\zeta/2 \qquad \zeta^3 = \zeta - r^\zeta/2 \qquad \zeta^4 = \zeta - r^\zeta/2$$

The functions f^P, f^ξ, f^η, f^ζ are defined as follows:

$$f^P\,(x,y,z) = \arctan \frac{yz}{x(x^2+y^2+z^2)^{1/2}} \quad \text{(principal value)}$$

$$f^\xi\,(x,y,z) = \frac{yz(2x^2+y^2+z^2)}{(x^2+y^2)\,(x^2+z^2)\,(x^2+y^2+z^2)^{1/2}} \tag{2.4.3}$$

$$f^\eta\,(x,y,z) = \frac{-xz}{(x^2+y^2)\,(x^2+y^2+z^2)^{1/2}}$$

$$f^\zeta\,(x,y,z) = \frac{-xy}{(x^2+z^2)\,(x^2+y^2+z^2)^{1/2}}$$

A more detailed treatment of the rectangular dipole panel can be found in /¯2, 3_7.

3. Superposition of Elementary Solutions

Before superimposing the elementary solutions in order to describe the given problem one has to take into account, that the field equations 2.1 are not satisfied at those points or panels, where sources or dipoles are located. The solutions 2.1.1÷2.4.2 are singular in these points. Consequently the sources and dipoles, or more general, the singularities must not be located inside the flow field of the given problem. Since the occurance of singularities is characteristic for the elementary solutions, their superposition in order to describe a given problem is often called as "singularity method".

If singularities, concentrated in discrete points are used for superposition, even a certain distance from the fluid boundary is necessary. Therefore these elementary solutions are not able to describe fluid boundaries due to thin walls wetted on both sides. Furthermore the number of singularities which surround the fluid boundary like a starry sky must be relatively high in order to allow for a good approximation of the boundary conditions. As a consequence, considerable computer effort is required. In spite of these drawbacks, singularity methods with sources concentrated in discrete points have been used by Höller /¯4_7, Landweber /¯5_7 and the author /¯6_7. The advantage here is the extremely simple structure of the elementary solution 2.1.1.

However, in most applications of singularity methods source panels are used. They can be located along the fluid boundary, i.e. they can be used to form the boundary. Here, some of the first papers have been published by Hess and Smith /¯1_7. Other contributions are due to Johnson and Rubbert /¯7_7 and Medan /¯8_7, for instance. All of these papers deal with external steady state flow problems, such as flows around wings. By applications to internal flow problems which are under investigations here, Renken found considerable leakages at boundary edges /¯9_7. Also fluid boundaries due to thin walls wetted on both sides cannot be treated, since source panels are not able to model the pressure differences occuring accross these boundaries.

These shortcomings do not occur, if the recently developed dipole panels are used /¯2, 3, 10_7. For field points approaching the edges of the dipole panel the formulae 2.4.1 and

and 2.4.2 become very simple: the amount of the acceleration vector $\bar{a}$ is

$$|\bar{a}| = \frac{1}{\pi}\frac{\partial t}{\partial \tau}\frac{1}{|\bar{e}|} \tag{3.1}$$

The direction of the acceleration vector is obtained by a 90^{o}-turn of the distance vector $\bar{e}$ as shown in fig. 4. From eq. 3.1 it follows that in the edge regions of a dipole panel the acceleration perpendicular to the element plane has a vanishing mean value. Consequently, when a fluid boundary is formed by several dipole panels with vanishing normal accelerations at the center points of the panels, no leakages will occur. Moreover, from the formulae 2.4.1

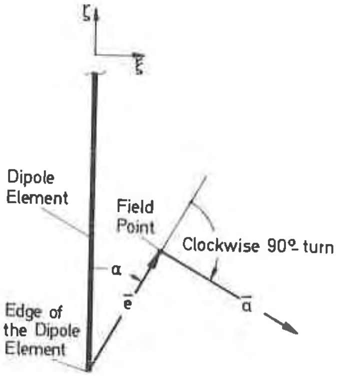

Fig. 4 Coordinates of a field point close to the edge of a dipole element

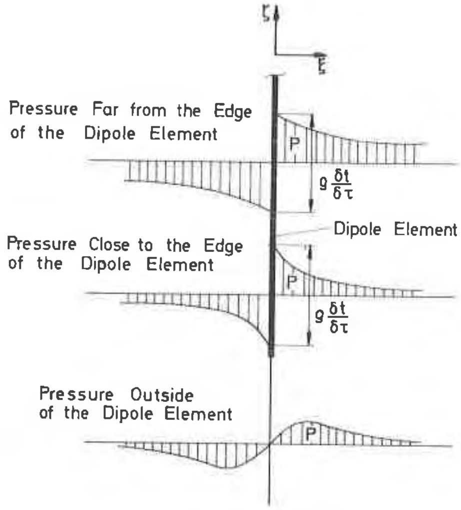

Fig. 5 Pressure distribution perpendicular to the plane of the dipole element

and 2.4.2 it follows that across the dipole panels a pressure step

$$\Delta P = \rho \frac{\partial t}{\partial \tau} \tag{3.2}$$

shown in fig. 5 occurs which may be used to simulate pressure differences across thin walls wetted on both sides.

Therefore the flow problems under investigation here, will be solved primarily by elementary solutions for dipole panels (section 2.4, eqs. 2.4.1 and 2.4.2) which are usually located along the fluid boundary. However, as a special option, the panels can also be located in a certain distance from the fluid boundary (submerged panels). For special problems besides dipole panels also sources concentrated in discrete points (section 2.1, eq. 2.1.1) can be used as singularities.

If the intensities of all singularities (dipole panels and source points) are described by the unknowns

$$X_k, \quad k = 1, 2, \ldots, K$$

then the superimposed pressure P and acceleration vector $\bar{a}$ (components a^x, a^y, a^z) in any field point ℓ can be determined by

$$P = \sum_{k=1}^{K} C^{P}_{\ell k} X_k ; \quad a^x = \sum_{k=1}^{K} C^{x}_{\ell k} X_k ; \quad a^y = \sum_{k=1}^{K} C^{y}_{\ell k} X_k ; \quad a^z = \sum_{k=1}^{K} C^{z}_{\ell k} X_k \tag{3.3}$$

The coefficients $C^{P}_{\ell k}$,... describe the pressures and accelerations due to the particular elementary solutions 2.2.1 or 2.4.1 and 2.4.2. However, in order to apply these formulae the position of the field point ℓ must be expressed with respect to all local coordinate systems ξ, η, ζ attached to each singularity as shown in fig. 1 and 3. Then the calculated acceleration vectors are related to the same local coordinate systems. They must be retransformed to the global coordinate system with the components a^x, a^y, a^z.

In the superposition formulae 3.3 the quantities X_k which describe the intensities of the particular singularities are still unknown. They will be determined such that the boundary conditions of the given problem are satisfied approximately.

Eqs. 3.3 are true not only for any given field point ℓ, but also for boundary points j. However, in these boundary points either

the accelerations normal to the boundary a^b_j (Neumann type), or the pressures P^b_j (Dirichlet) are prescribed. Consequently the following linear equations are obtained:

$$\sum_{k=1}^{K} C^{\xi}_{jk} X_k = a^b_j \quad \text{or} \quad \sum_{k=1}^{K} C^{P}_{jk} X_k = P^b_j \tag{3.4}$$

(In C^{ξ}_j the superscript ξ indicates that the normal direction of the boundary is used as a basis.) If exactly so many boundary points are selected as unknowns X_k have been introduced, namely k = 1,2,...,K, eqs. 3.4 allow for unique determination of X_k. In the selected boundary points which usually are identical with the center points of the panels the boundary conditions are satisfied exactly. In all other boundary points the given boundary conditions undergo slight changes. Now, substituting the calculated values of X_k into eqs. 3.3, acceleration vector and pressure can be determined for any field point; in other words, the flow problem has been solved approximately.

Moreover, if the calculated boundary values which in general are slightly different from the prescribed values are interpreted as "given boundary conditions" then the obtained results represent even a quasi exact solution.

Finally, it should be emphasized, that an unknown X_k does not necessarily correspond to the intensity of a singularity (source point or dipole panel). Rather, as an option, the number of dipole panels, for instance, can be much higher than the number K of the unknowns X_k. Consequently, the discretization of the boundary can be refined by increasing the number of dipole panels without increasing the dimensions of the linear equation system 3.4 which must be solved.

4. Computer code SING1 and Applications

In order to carry through the above calculations the computer program SING1 has been written. Two different examples have been solved with SING1. In both cases the singularity distribution was provided by dipole elements located at the fluid boundary.

The first example is the incipient flow in a T-joint with rectangular surfaces and cross sections. The modelling of the fluid boundary by rectangular dipole elements is shown in fig. 6. The problem is symmetric about the plane z=0 which is automatically taken into account by the code SING1. Furthermore the problem is symmetric about the plane y=0, so independent values X_k for the singularity distribution are specified only for those dipole elements which are located in the region y>0. In this way only 63 unknowns were necessary although the number of dipole elements used was 126. Cross section 1 with a prescribed pressure of 3 bar and cross section 2 with vanishing pressure, form the openings of the T-joint. The remaining fluid boundaries are rigid walls. The pressure distribution over these walls and the normal accelerations at cross sections 1 and 2 have been calculated with SING1. The result immediately after flow begin is shown in fig. 7, where the lengths of the horizontal lines are proportional to the pressure and the lengths of the vertical lines with arrows are proportional to the normal acceleration at the particular surfaces. Despite the sharp edges and the relatively small number of dipole elements and unknowns the results are satisfactory. The ratio of the accelerations in cross-sections 1 and 2 calculated with SING1 differs by less than 0.5 % from the exact value. Therefore it may be concluded that essential fluid losses due to leakages at edges, which had been observed in calculations with other singularity methods /9/, do not occur in SING1.

The second example is the oscillating flow in the water pool of the pressure suppression system of a boiling water reactor. Under certain operating or emergency conditions steam is blown into the water pool through downcomer tubes. At the end of these tubes highly transient condensation processes may take place causing pulsating flows and pressures in the whole system. The fluctuating pressure distributions at the walls are significant for the structural integrity of the system. Fig. 8 shows a 60 degree section of the water pool. The transient condensation process is represented by a fluid source with a given intensity gradient. At all surfaces the normal acceleration vanishes except at the free water surface. Here the pressure vanishes. This means that the walls of the water pool are assumed to be rigid. The discretization of the fluid boundary by a total of 696 dipole elements is also shown in fig 8. The number K of unknowns X_k amounts to 247 only. The results obtained with SING1 are shown in figs. 9 and 10. As in the first example the lengths of the horizontal lines are proportional to the pressures and the lengths of the vertical lines with arrows are proportional to the accelerations at the centers of the dipole elements. Acceleration lines smaller than the length of the arrow are omitted. Since satisfaction of the boundary conditions was not required at the center of each dipole element small accelerations perpendicular to the walls occur representing the actual boundary conditions which are slightly different from the given boundary conditions. In more detailed investigations calculations have been carried through with different numbers K of unknowns X_k. They indicate that local errors in the boundary conditions have very little influence on the pressure distribution.

5. Conclusions

With the computer code SING1 an effective tool for transient, three-dimensional potential flow problems with incompressible fluid is available. Special advantages are:

- Only the fluid boundary, but not the whole three-dimensional fluid domain must be discretized for numerical treatment
- Tailoring the discretization for the fluid boundary allows for an optimal description of the boundary conditions
- Exact solutions are obtained for boundary conditions which differ slightly from the given conditions. The mean values of the deviation, taken over subregions of the boundary, vanish.
- Since the fluid dynamic unknowns are related only to the fluid boundary, and this boundary largely coincides with the surface of the surrounding structures, the development of solution techniques in coupled fluid-structural dynamics is facilitated.

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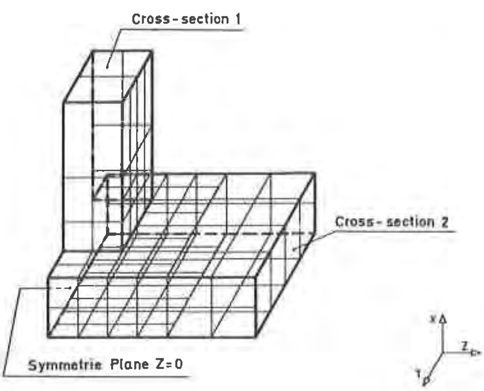

Fig. 6 Boundary discretization of a T-joint having different rectangular cross-sections

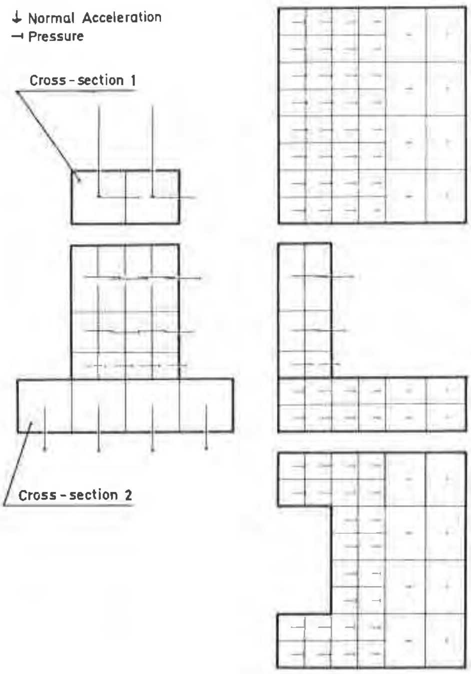

Fig. 7 Pressure and acceleration distribution over the surfaces and cross-sections of a T-joint

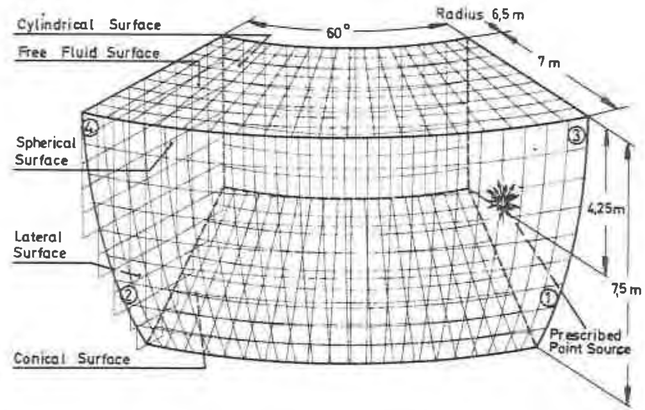

Fig. 8 Boundary discretization of a 60°-section of the pressure suppression system

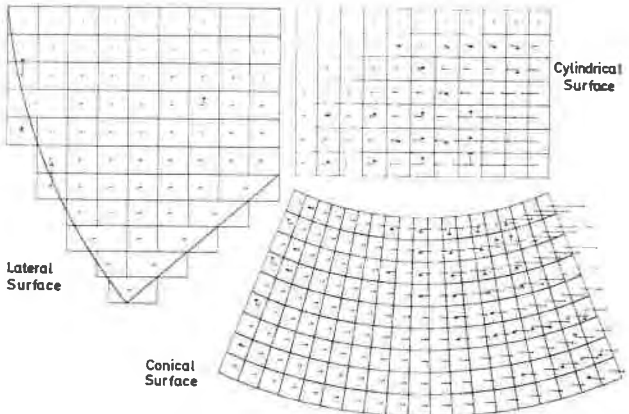

Fig. 9 Pressure distributions over the cylindrical, lateral and conical surfaces

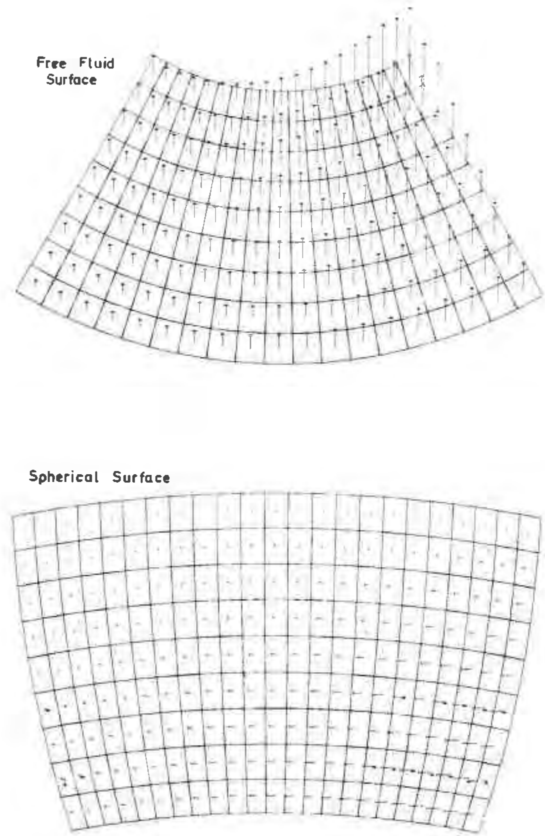

Fig. 10 Pressure and acceleration distributions over the free fluid surface and the spherical surfaces