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LINEAR METHODS OF STRUCTURAL ANALYSIS

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After a brief discussion of the foundations of the finite element methods in linear problems as well as the state-of-the-art of their application to the analysis of complex structures, some new developments are presented. These pertain to thin shells, where new results are obtained using the general curved shell element SHEBA. Also thick plates and shells are treated, where the effects of transverse shear cannot be neglected. The structural analysis of a complex three-dimensional reactor pressure vessel illustrates the versatility of the method.

Extensions of the finite element method to problems other than structural are indicated. A unified approach to the determination of steady and unsteady temperature distribution in structures as well as the associated deformation and stress analysis is given and illustrated by examples. Dynamic response of deformable structures under shock-type loading is treated by finite elements in space and time. Some features of the development of associated software are discussed.

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DISCUSSION

Q D. COSTES, France

I would like to know if Dr. Argyris' Group performed calculations of fast, dynamic, large deformations, for instance in case of a containment structure submitted to an explosion.

A J. H. ARGYRIS, U. K. /Germany

Yes, we have undertaken computation at Imperial College including dynamic problems with large deformations and elastoplastic behaviour. This effectively solves e. g. the elastoplastic wave propagation.

Q A. K. RAO, India

1. The finite element formulation, by virtue of the underlying principles has a tendency to yield accurate results for global or gross quantities. But there are situations where accurate predictions are needed for local values. What techniques do we now have to control and assess the accuracy in the matter of local values ?
2. We have had an excellent review of analysis by the FEM's. Being aware of Prof. Argyris intentions from the earliest years, that the matrix methods should lead to automated design procedures, may I ask him, if he would make any comments on the state of development of automated design methods as they were not covered in the morning's reviews ?

A K. E. BUCK, Germany

1. As regards local values, the displacement method of course yields accurate local displacements. In our experience, a suitable idealisation using not-too-primitive elements, i. e. quadratic or higher-order displacement interpolation, yields a good approximation of the stresses as well. Criteria by which to judge the accuracy of results are e. g. : smoothness of stresses, at least in regions where there are no geometrical or load discontinuities; satisfaction of natural boundary conditions; furthermore, conditioning information on the stiffness matrix allows to judge the numerical reliability of the solution.
2. I think one has to state that we are only at the beginning of a long path leading up to automated design procedures. At the present, stress analysis of complex structures taxes the possibilities even of large computers. Progress in this field will have to reply on availability of efficient interactive graphics devices, which allow to introduce the creative ideas of the engineer into an iterative process leading to an optimum design.

May I take this opportunity to thank you, Professor Rao, for your discussion.

Q R. J. OWEN, U. K.

I would like to ask Dr. Buck to comment further on his statement that isoparam-

etric elements are incapable of reproducing a constant strain condition. In particular I would like to know which element type he is referring to.
Such a requirement is essential for guaranteeing convergence to the exact solution with increasing fineness of subdivision, and all the "serendipity" type elements used and developed at Swansea satisfy this condition.
Furthermore, extensive applications in thermal stress analysis using these elements show them to be extremely efficient and accurate.

A K. E. BUCK, Germany

Dr. Owen, my statement referred to the inability of curved isoparametric elements to describe exactly linear strain distributions. This may lead to calculated stresses due to thermal strains where there ought to be none. It seems this effect may be the more pronounced the more distorted element shapes are used. This point will be discussed in more details at the 3rd WPAFB Conference by Dr. Willam.

Q R. W. CLOUGH, U. S. A.

I would like more details on the dynamic response analysis methods mentioned by Dr. Buck, with the names "Newmark" and "Wilson". The Newmark method includes many different variations, the most prominent being designated as $\beta = 1/4$ and $\dot{\varepsilon}\beta = 1/6$. There also are many methods which have been proposed by Wilson. In order for the results shown to be meaningful, they must be associated with specific forms of the mentioned methods, and I would like Dr. Buck to make clear which were used.

A K. E. BUCK, Germany

Professor Clough, regarding the response results about which we also are not at all happy, a detailed discussion will follow after we find out exactly which algorithms we used.

Q A. V. KRISHNA MURTY, India

This question is concerned with convergence problems in finite element analysis. Are there any methods for getting an indication of error at any given state of computation and to control these errors ?

A K. E. BUCK, Germany

Professor Krishna Murty, this question of course is closely related to the previous one on choice of element model. A displacement solution converges monotonically. Therefore two idealisations of different degree of fineness allow an estimate of how far away from the "exact" solution we are (however, in practice, usually only one idealisation is carried

out). Quality of idealisation may be checked by observing the accuracy to which the natural boundary conditions are satisfied. Also, smoothness of calculated stresses, where appropriate, is a measure of quality of solution. Quality of numerical solution may be checked by observing the conditioning information; e. g. , our program ASKA provides condition numbers.
For errors due to idealisation, I see no other possibility than to change the idealisation where necessary. In case of relative ill-conditioning, double-precision calculations may be necessary; these means are also provided in ASKA, e. g. .

Q A. V. KRISHNA MURTY, India

Recently finite element methods based on approximate methods such as Galerkin, least square methods and their hybrids are being used for the analysis of structural problems. May I request you to tell the potentialities and the state of art of these methods ?

A K. E. BUCK, Germany

Professor Krishna Murty, it is the author's view that for a general procedure, the displacement models or, where feasible, pure equilibrium models are to be preferred to hybrid and mixed models. The former yield bounds, and also the positive-definiteness of the resulting coefficient matrix allows assessment of numerical behaviour of the solution. No doubt there are cases where the hybrid or the mixed models yield more accurate results than a displacement analysis; for a general purpose implementation, we prefer the "pure" models for their reliability and consistency.

Q G. D. STEFANOU, U. K.

1. Is there any way of making best choice of elements for a given problem ?
2. Has the author any suggestions to make on the improvement of solutions in regions of stress concentration in a structure ?

A K. E. BUCK, Germany

1. Professor Stefanou, I feel unable to give a general recipe for an optimum choice of element. Engineering sense in combination with a consistent element formulation have to be applied.
2. The general procedures are mesh refinements. Apart from such (theoretical !) stress singularities as appear at the corners of simply supported skew plates, in our experience the same elements may be used at stress concentrations as in the rest of the structure.