DYNAMIC ANALYSIS OF VITAL PIPING SYSTEMS SUBJECTED TO SEISMIC MOTION

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ABSTRACT

The linear dynamic analysis of the three dimensional piping system of a nuclear power plant is based on a lumped parameter model. Both time history input and design spectrum input are used and discussed. Also both dynamic stiffness matrix and flexibility matrix are employed in the eigenvalue problem and in calculating internal stresses and support reactions. The interaction of piping with the structure can be treated either by floor design spectrum or by combining them into one model.

1. INTRODUCTION

The early development of dynamic piping analysis was carried out by Naval engineers [1]. To assure public safety, the designers of the nuclear power plants in such countries as the U.S.A. and Japan have also been required to perform dynamic analysis of piping subjected to seismic motion [2, 3, 4]. Since the state-of-the-art of the subject has constantly been improved, it is the purpose of this study to summarize the improved analytical procedures which emphasize economy of computation time and accuracy of responses.

The interaction of the piping system with the structure plays an important role in the response of the system. Several methods are available to treat this problem. The conservatism of the result depends on how each method is properly applied instead of on which method is applied.

2. EQUATIONS OF MOTION AND SYSTEM RESPONSE

The equations of motion based on a lumped parameter model of a three dimensional piping system subjected to seismic input may be written as:

$$[M]\ \{\ddot{x}\} + [C]\ \{\dot{x}\} + [K]\ \{x\} = -[M]\ \{\ddot{y}\} \qquad (1)$$

where the system is assumed to be linear elastic. The symmetric, positive definite mass matrix [M] is diagonal for lumped mass approach but non-diagonal for consistent mass approach [5]. The positive definite stiffness matrix [K] and the positive semidefinite damping matrix [C] are both symmetric. $\{x\}$ is the relative displacement vector. The dots over the variables indicate time derivatives. $\{\ddot{y}\}$ is the input acceleration vector at the support. The piping response is affected by the mathematical model used, and some modelling consideration was discussed by Harrington and Vorus [6].

Either the time history of the support acceleration or the floor design spectrum can be used as input. In view of the fact that hundreds of piping systems are analyzed dynamically in a typical PWR plant, and that the time history analysis is time consuming, it is more practical to use floor design spectrum as input. Of course time history analysis has the advantage of providing resposes as a function of time on condition that the mathematical model is correct and that the assumed material properties and time history input are exact.

If the time history is used as input, eq. (1) can be solved either by direct time integration [7, 8, 9], or by superposition of normal modes. With either the time history method by superposition of normal modes or the design spectrum method, the eigenvalues and eigenvectors of the system have to be solved first. They are obtained by solving the free vibrational equations.

$$\{\ddot{x}\} + [M]^{-1} [K] \{x\} = \{0\} \tag{2}$$

or

$$[A] [M] \{\ddot{x}\} + \{x\} = \{0\} \tag{3}$$

where [A] is the symmetric, positive definite flexibility matrix of the system. Methods of deriving stiffness matrix or flexibility matrix were discussed by Przemieniecki [10]. It is well known that the stiffness matrix requires much less time to derive. Various techniques of solving eqs. (2) and (3) are available [11, 12]. The eigenvalues of $[M]^{-1}[K]$ are the squares of the system circular frequencies, whereas those of [A] [M] are the inverses of the squares of the system circular frequencies.

Rayleigh [13] showed that the sufficient condition for a damped system to prossess classical normal modes is that the damping matrix is a linear combination of mass matrix and the stiffness matrix. Caughey [14] pointed out that the necessary and sufficient conditions for the existence of classical normal modes are that the damping matrix be diagonalized by the same transformation that uncouples the undamped system. So after obtaining the normalized eigenvectors $[\phi]$ of eqs. (2) or (3), we can apply the orthogonal transformation

$$\{x\} = [\phi] \{\eta\} \tag{4}$$

to eqs. (1), and premultiply both sides of the equations by the transpose of $[\phi]$. Hence the uncoupled equations are

$$\{\ddot{\eta}\} + (\xi + \alpha[\,\lceil \omega^2 \rfloor\,]) \{\dot{\eta}\} + [\,\lceil \omega^2 \rfloor\,] \{\eta\} = -[\phi]^T [M] \{\ddot{y}\} \tag{5}$$

where we made use of the orthonormal conditions

$$[\phi]^T [M] [\phi] = [\,\backslash I \backslash\,], \tag{6}$$

$$[\phi]^T [K] [\phi] = [\,\backslash \omega^2 \backslash\,], \tag{7}$$

and the proportional damping relations

$$[C] = \xi [M] + \alpha [K]. \tag{8}$$

The i^{th} component of eqs. (5) is

$$\ddot{\eta}_i + (\xi + \alpha\omega_i)\, \dot{\eta}_i + \omega_i^2 \eta_i = -\{\phi_i\}^T [M] \{\ddot{y}\} \tag{9}$$

where $\{\phi_i\}^T$ is the transpose of the i^{th} column of $[\phi]$. If we define the percentage of critical damping β as

$$\beta = \frac{\xi + \alpha\omega_i^2}{2\omega_i}, \tag{10}$$

then eq. (9) becomes

$$\ddot{\eta}_i + 2\beta\omega_i \dot{\eta}_i + \omega_i^2 \eta_i = -\{\phi_i\}^T [M] \{\ddot{y}\} \tag{11}$$

If time history of support acceleration is chosen as input, eq. (11) can be applied directly to obtain the modal response as a function of time. If design spectrum is chosen as input, we can replace the support acceleration vector as

$$\{\ddot{y}\} = \ddot{y}_o \{d\} f(t) \tag{12}$$

where $\ddot{y}_o$ is the maximum support acceleration, $\{d\}$ is the earthquake direction vector and $f(t)$ is the time function of support acceleration. Here we assume that all supports have the same time function. Supports of different time functions can also be handled [15, 16]. Furthermore define the participation factor, γ_i as

$$\gamma_i = \{\phi_i\}^T [M] \{d\} \tag{13}$$

which is a measure of the extent to which the i^{th} normal mode participates in synthesizing the total loads on the system [17]. With the initial conditions

$$\dot{\eta}(0) = \eta(0) = 0, \tag{14}$$

the solution of eq. (11) is [17]

$$\eta_i(t) = \frac{\gamma_i \ddot{y}_o}{\omega_i} \int_o^t e^{-\beta\omega_i(t-\tau)} f(\tau) \sin\omega_i(t-\tau)\, d\tau \tag{15}$$

where we assumed small damping, $\sqrt{1-\beta^2} \to 1$. Let the acceleration response spectrum value be [18, 19]

$$Sa(\omega) = \omega\, \ddot{y}_o \, [\int_o^t e^{-\beta\omega(t-\tau)} f(\tau) \mathrm{Sin}\omega(t-\tau)\, d\tau]_{max.} \tag{16}$$

Then the maximum modal response is

$$(\eta_i)max = \frac{\gamma_i\ Sa}{\omega_i^2} \ . \tag{17}$$

The maximum displacements at each degree of freedom in i^{th} mode are

$$\{x_i\}_{max} = (\eta_i)_{max}\ \{\phi_i\} \tag{18}$$

If one is interested in the maximum equivalent forces applied at each degree of freedom the maximum absolute modal acceleration will be derived first. It is [20]

$$(\ddot{\eta}_i + \gamma_i \ddot{y}_o f(t))_{max} = \gamma_i Sa \tag{19}$$

Then the maximum forces in i^{th} mode are

$$\{F_i\} = \gamma_i Sa[M]\ \{\phi_i\} \ . \tag{20}$$

This set of forces may also be obtained from maximum displacements and system stiffness matrix,

$$\{F_i\} = (\eta_i)_{max}\ [K]\ \{\phi_i\}. \tag{21}$$

The internal stresses and support reactions for each mode are obtained by applying statically either the set of displacements of eqs. (18) or the set of forces of eqs. (20) or (21) to the system. The most probable response of the system is the square root of the sum of squares (SRSS) of all the contributing modes. It had been a general practice in the past to apply the SRSS of the displacements or the equivalent forces to obtain the internal stresses and support reactions. By doing this, the computation time can be saved, but it will not yield the most probable system response. This is due to the fact that the internal stresses and support reactions are not only a function of the magnitudes but also the signs of the applied displacements or forces, and the signs of the SRSS of the displacements or forces are lost.

The SRSS method was first used for structural analysis [21]. It give satisfactory results for most cases when the frequencies of the modes are well separated. But it happens quite often that some modes of a three dimensional piping system have frequencies close to each other. The maximum response of these modes may occur at same time. To accomodate this, the absolute sum should be applied to close frequency modes, and then take SRSS with the rest of the modes. Since some modes are insignificant in comparison with others, it is desirable to choose the contributing ones. Following the derivation, we can see that the modal acceleration as defined in eq. (19) is a natural basis for modal selection.

The final maximum stress at a point is then compared with the allowable one. In case of overstress, perturbation technique can be applied to choose the design changes [22]. A general approach without analysis is to put rigid restraints, e.g. snubbers, at location of maximum deflection; hopefully this can drive the fundamental frequency toward the higher frequency side of the peak area of the design spectrum. Of course this does not promise to be an economical redesign.

3. COMPARISON OF STIFFNESS AND FLEXIBILITY MATRIX METHOD

For stiffness matrix method, the overall stiffness matrix is obtained by combining the individual branch stiffness matrices as done in displacement finite element method [23]. This matrix includes elements corresponding to branch points which are not assigned as mass points. These unwanted elements can be eliminated by condensation scheme as follows. Let the overall stiffness matrix be partitioned as

$$[\bar{K}] = \begin{bmatrix} \bar{K}_{ii} & \bar{K}_{ij} \\ \bar{K}_{ji} & \bar{K}_{jj} \end{bmatrix} \qquad (22)$$

where the subscript i indicates elements corresponding to mass points. The modified matrix will be

$$[\tilde{K}] = [\bar{K}_{ii}] - [\bar{K}_{ij}]\,[\bar{K}_{jj}]^{-1}\,[\bar{K}_{ji}]. \qquad (23)$$

When the branch stiffness matrix is generated, each point is assigned with six degrees of freedom, three translational and three rotational. It is a general practice in dynamic analysis to consider three translational degrees only [24]. Under this case we can apply the condensation scheme again to obtain the dynamic stiffness matrix.

$$[K] = [\tilde{K}_{ii}] - [\tilde{K}_{ij}]\,[\tilde{K}_{jj}]^{-1}\,[\tilde{K}_{ji}] \qquad (24)$$

This matrix [K] is the one used in eqs. (2). These maximum displacements obtained in eqs. (18) correspond to translational degrees of freedom of mass points only. The rotations of mass points are obtain as

$$\{x_j\} = -[\tilde{K}_{jj}]^{-1}\,[\tilde{K}_{ji}]\,\{x_i\} \qquad (25)$$

Let

$$\{\tilde{x}_i\} = \begin{Bmatrix} x_i \\ x_j \end{Bmatrix} \qquad (26)$$

The translations and rotations of branch points not assigned as mass points are obtained as

$$\{\tilde{x}_j\} = -[K_{jj}]^{-1} [\bar{K}_{ji}] \{\tilde{x}_i\} \qquad (27)$$

The total displacement vector is

$$\{\bar{x}\} = \begin{Bmatrix} \tilde{x}_i \\ \tilde{x}_j \end{Bmatrix} \qquad (28)$$

Applying these displacements $\{\bar{x}\}$ to individual branch stiffness matrix, the internal stresses and support reaction can be calculated accordingly.

If one wants to use flexibility matrix method, the flexibility matrix [A] in eqs. (3) can be obtained either by taking the inverse of [K] in eq. (24) or by applying unit load method. When unit load method is used, we will solve the set of simultaneous equations

$$[\bar{K}] \{\bar{x}\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \qquad (29)$$

as many times as there are number of dynamic degrees of freedom. Since the only thing which changes is the force vector on the right hand side of eqs. (29), we can triangularize $[\bar{K}]$ once and store it. The displacement vectors due to different force vectors will be obtained by back substitutions. Various efficient methods [11, 25, 26] are available for solving eqs. (29). After the maximum displacements in eqs. (18) or maximum forces in eqs. (20) are derived, one can obtain the internal stresses and the support reactions either by the same way as described in stiffness matrix method or by applying these forces to the system to solve for the overall displacements $\{\bar{x}\}$ or multiplying these forces by those influence coefficients obtained in eq. (29). Although it is well known that the stiffness matrix method is faster in obtaining the internal stresses and support reactions, some programs still use forces defined in eqs. (20) or (21) in calculation. The possible reasons are two. Firstly engineers working on dynamics are more familiar with the concept of equivalent static load method. Another reason is that engineers perform the dynamic piping analysis by modifying the static or thermal piping stress program available, and the simplest way of modification is to use unit load method.

4. PIPING STRUCTURE INTERACTION

For piping with small mass in relation to the structure, it is a general practice to use floor design spectrum as input. The spectrum can be derived either by time history method [2], or by design spectra method [15, 16, 27]. The conservatism of the spectrum obtained depends on how each method is properly performed instead of which method is used. With the design spectrum method, special care should be exercised to obtain the amplification curves and to combine the modal responses. With the time history method, special care should be exercised to obtain the proper time history and to perform parametric study. Due to the abrupt changes of the unsmoothed response spectrum obtained from the actual strong motion earthquake records, the general trend is to use simulated earthquake [28] as input such that the unsmoothed response spectrum derived from it will simulate closely the design spectrum.

For primary coolant loop of a PWR plant, the mass is not small comparing with the supporting structure. The response will usually be overestimated if the floor design spectrum is used as input. Under this case, the loop and the structure can be combined into one model and analyzed using ground design spectrum as input [29]. The other alternative is to perform component mode analysis [30].

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DISCUSSION

Q

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Do you calculate stresses of piping systems due to earthquake loading in your code through either forces or moments ?
Is there compatibility of your computer code with piping code, or USAS B31.1 and B31.7 which include flexibility factors and stress indices ?

A

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In B31.1 code the stresses are calculated by moments. The piping program complies with all the requirements specified in the code.