A MODEL FOR SOIL BEHAVIOR UNDER MONOTONIC AND CYCLIC LOADING CONDITIONS

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SUMMARY

A mathematical model capable of describing the soil behavior under any loading conditions, monotonic or cyclic, is presented within the framework of critical state soil mechanics.

The soil is considered as an elasto-plastic material without a purely elastic range. Therefore, the concept of the yield surface is completely abandoned and instead the concept of the bounding surface in stress space is introduced, within or on which the stress state always lies. For a given stress state plastic strains occur always for any stress rate, except for neutral loading. The bounding surface provides the basic elements for the plastic constitutive relations as follows: 1) The direction of loading and that of the plastic strain rate are defined by the unit normal to the bounding surface at points properly defined by the given stress state and stress rate direction. 2) The value of the plastic modulus is a function of the distance, in stress space, between the stress state and the corresponding point on the bounding surface which defines the direction of loading by means of its unit normal. The change of the size, position, and shape of the bounding surface is determined by the change of the plastic void ratio. In the course of continuous plastic loading along a fixed direction the stress state eventually reaches the bounding surface and remains on it until failure occurs at a critical void ratio.

The concept of bounding surface has been introduced earlier by Dafalias and Popov in conjunction with an enclosed yield surface for the cyclic behavior of metals. Here the novel feature is the adaptation of a direct bounding surface plasticity formulation (without a yield surface) to account for unique characteristics of soil behavior. The model allows the qualitative, at this stage, description of the following phenomena: 1) the densification of lightly overconsolidated soils with increasing deviatoric stress until failure at the critical state, 2) the dilatation and unstable behavior of heavily overconsolidated soils with increasing deviatoric stress until critical failure, 3) the decrease of the mean normal effective stress and increase of pore water pressure under undrained cyclic loading of consolidated or overconsolidated soils leading to liquefaction. Note that this last phenomenon for overconsolidated soils cannot be accounted for by classical yield surface soil plasticity because it does not allow for plastic strains inside the yield surface. The present formulation introduces only one additional soil parameter to the ones of critical state soil mechanics, related to the soil response under cyclic loading.

1. Introduction

From the simplest curve fitting analytical expressions to the more sophisticated elasto-plastic constitutive laws for soils the common weakness is one: they are pertinent to either monotonic or cyclic loading of a very specific nature. Existing constitutive relations, proper for monotonic loading, cannot describe the cyclic soil behavior. On the other hand, studies of cyclic loading response deal primarily with gross overall soil behavior under specifically chosen cyclic conditions which are useless for monotonic loading or interchange of monotonic and cyclic loading or different conditions of cyclic loading. It is believed that the constitutive relations employed for soil characterization must be of fundamental nature, i.e., must be equally applicable to monotonic or cyclic, drained or undrained, or any other form of loading conditions in order to be of value for the analysis of an earth structure under complex loading.

Within the framework of the last statement, it is proposed to extend and properly adapt a recently developed "bounding surface" plasticity theory by Dafalias [1] and Dafalias and Popov [2,3,4] in order to describe the specific characteristics of the inelastic behavior of granular and cohesive soils. The elastic response can be treated separately as usual. This new plasticity theory is capable of describing the material response under any kind of loading conditions, monotonic and/or cyclic, drained or undrained, by a single constitutive model expressed in terms of effective stress and basic soil state variable (e.g., void ratio, etc.). Although the "bounding surface" plasticity offers many novel features especially fitted to describe soil behavior, it still remains within the framework of well established concepts of soil plasticity and critical state soil mechanics [5] such as critical state, critical stress ratio, yield, failure, etc. In this sense is different from Valanis' endochronic theory [6] as applied to soils by Bazant and Krizek [7]. Two points are worth mentioning with respect to the bounding surface model: 1) its simplicity renders possible an easy incorporation into existing numerical analysis procedures, 2) there is only one new material parameter introduced in addition to the commonly used parameters in critical state soil plasticity, which is associated with the cyclic response.

2. Classical Yield Surface Soil Plasticity and its Shortcomings

In order to pave the road for the subsequent introduction of the "bounding surface" concept in soil plasticity, it is instructive to briefly discuss the common features of classical yield surface soil plasticity models and emphasize their basic inadequacies with respect to the description of soil plastic response.

The rate-independent plastic constitutive theory can be summarized in two equations, that of the yield surface

$$f(\sigma_{ij}, q_n) = 0 \tag{1}$$

and that of the plastic strain rate-stress rate relationship

$$\dot{\varepsilon}_{ij}^{p} = \frac{1}{K} < \dot{\sigma}_{k\ell}\, n_{k\ell} > \rho_{ij} \tag{2}$$

where the Macauley brackets $<>$ define the operation $<x> = xH(x)$, H being the step function and σ_{ij} is the stress, ε_{ij}^{p} is the plastic strain (a dot stands for the rate, i.e., increment per unit time), q_n are plastic internal variables (such as plastic strain, plastic work, plastic change of void ratio, etc.), $n_{k\ell}$ is the normal to the yield surface defining the direction of loading-unloading in stress space, ρ_{ij} is the direction of the plastic strain rate $\dot{\varepsilon}_{ij}^{p}$ and K is the plastic modulus. In general, $\rho_{ij} \neq n_{ij}$ (non-associative flow rule). Equation (2) shows that the basic elements for a rate independent plastic constitutive relation are the n_{ij} (and the yield surface), ρ_{ij} and K.

The determination of these three elements fully specifies the plastic response. Such a formulation has the following shortcomings when applied to soils:

1) No purely elastic range exists for soils. Although small, plastic irreversible deformation begins at the outset of loading simultaneously with elastic one. The sharpest criticism of the assumption of a purely elastic range for soil comes from the well established phenomenon of liquefaction during an undrained cyclic triaxial experiment: the classical theory simply cannot predict the pore-water pressure built up because the cyclic stress remains (after the first semicycle) continuously inside the yield surface, therefore no plastic volume change occurs which is necessary to lead the undrained specimen to liquefaction [5]. Also, the direction of loading n_{ij} (normal to the yield surface) is fixed and independent of the direction of the stress rate direction $\dot{\sigma}_{ij}$; this may be true for metals, but there is much concern whether this is valid or not for soils, as recently discussed by Hardin [8] in the context of two kinds of loading.

2) The plastic strain rate direction ρ_{ij} is assumed to be fixed (normal to a plastic potential) and independent of the direction of the stress rate direction $\dot{\sigma}_{ij}$. The validity of this assumption is also questioned and again discussed by Hardin [8].

3) It can be said that the determination of the plastic modulus K is the Achilles' heel (Homer's Illiad, 900 B.C.) of classical plasticity theory when it comes to describing cyclic loading response, or more general, response under stress reversal events. The traditional approach of determining K through the consistency equation $\dot{f} = 0$ lacks the necessary flexibility for the proper description. A recent extension of Mroz's model of nested surfaces [9] by Prevost [10] exhibits this flexibility, but the numerous nested surfaces necessary for such a description render the model highly impractical for use in large-scale numerical computations.

The "bounding surface" plasticity theory has none of the above shortcomings as explained in the following section.

3. Adaptation of Bounding Surface Plasticity to Soils

The concept of a bounding surface in plasticity has been originally introduced by Dafalias [1], Dafalias and Popov [2,3,4], and independently by Krieg [11] to describe cyclic metal plasticity in connection with a yield surface. The details can be found at the above references and here only a brief outline of the general model will be presented, emphasizing subsequently its adaptation to soil plasticity with complete elimination of the yield surface concept.

Generalizing observations on uniaxial cyclic experiments to a multiaxial stress space, the concept of the "bounding surface" is introduced which always encloses the yield surface and moves and changes simultaneously with it. The proximity of the two surfaces in the course of plastic loading, measured by the distance δ between the stress state point a and a corresponding point $\bar{a}$ on the bounding surface (such that the unit normals at a and $\bar{a}$ are parallel), determines the value of the plastic modulus K. In addition, the initial value δ_{in} of δ is used as a discrete memory parameter associated with events of unloading-reloading.

The next step, which tremendously simplifies the model and is pertinent to soil behavior, is to eliminate the yield surface. In other words, the yield surface shrinks to zero, i.e., becomes the stress point itself, and the bounding surface now can be used to determine n_{ij}, ρ_{ij}, and K. This version has been studied by Dafalias and Popov [4] for artificial graphite which is a metal with zero purely elastic range. A number of different approaches are possible because the concept of a bounding surface lends itself to a very broad and versatile interpretation. For the sake of brevity and definiteness, only one approach will be presented within the framework of critical state soil mechanics.

The bounding surface is schematically shown as an ellipse in the space of the negative first stress invariant $-I_1$ (compressive stress is negative) and the square root of the second deviatoric stress invariant J_2, Figure 1. All stresses are effective. The critical state lines OA (compression) and OB (extension) intersect the ellipse at point A,B where the normal is parallel to the J_2 axis. The only plastic internal variable q_n is the plastic component e^p of the void ratio (the total e can be easily decomposed into an elastic and plastic component). The e^p and its change defines the hardening rule for the bounding surface (isotropic and/or kinematic). The analytical expression of the bounding surface can be written as

$$F(\bar{\sigma}_{ij}, e^p) = 0 \tag{3}$$

where a bar over the stress σ_{ij} implies stress states on the bounding surface. The current stress state σ_{ij} lies inside the bounding surface and a stress rate $\dot{\sigma}_{ij}$ defines a corresponding point $\bar{\sigma}_{ij}$ as the intersection of the direction of $\dot{\sigma}_{ij}$ with $F = 0$. The unit normal n_{ij} erected at $\bar{\sigma}_{ij}$ defines the loading direction, n_{ij}, and ρ_{ij} is taken identical to it for simplicity. Observe that n_{ij}, ρ_{ij} depend now on the direction of $\dot{\sigma}_{ij}$. Different ways of defining $\bar{\sigma}_{ij}$, n_{ij}, and ρ_{ij} are possible satisfying certain continuity conditions and depending on the kind of soil, state conditions, etc. (the fixed n_{ij} of classical plasticity is included as special case) but will not be discussed here.

It is now possible to write an equation similar to eq. (2) as follows:

$$\dot{\varepsilon}^p_{ij} = \langle \frac{1}{K} \dot{\sigma}_{k\ell} n_{k\ell} \rangle n_{ij} = \langle \frac{1}{K_b} \dot{\bar{\sigma}}_{k\ell} n_{k\ell} \rangle n_{ij} \tag{4}$$

where K is the plastic modulus associated with the stress state σ_{ij} and stress rate $\dot{\sigma}_{ij}$, and K_b is the value of the plastic modulus associated with the fictitious stress state $\bar{\sigma}_{ij}$ and stress rate $\dot{\bar{\sigma}}_{ij}$ on the bounding surface. Observe that the possible unloading implied when $(1/K)\dot{\sigma}_{k\ell} n_{k\ell}$ is negative (stress reversal) is only instantaneous, immediately followed by reloading associated with the relocation of $\bar{\sigma}_{ij}$ on the bounding surface when $\dot{\sigma}_{ij}$ changes direction. Observe also that the inclusion of K and K_b in the brackets is aimed at treating unstable material response where both K and $\dot{\sigma}_{k\ell} n_{k\ell}$ are negative but, nevertheless, their product remains positive (falling stress-strain curve of dense samples after the peak stress).

The plastic volumetric strain rate $\dot{\varepsilon}^p_{ii}$ (trace of the plastic strain rate tensor) is related to the rate of the plastic void ratio $\dot{e}^p$ by

$$\dot{e}^p = (1 + e_o)\, \dot{\varepsilon}^p_{ii} \tag{5}$$

where e_o is the total void ratio of the reference configuration with respect to which strains are measured (if natural or logarithmic strains are used, then e_o becomes the current void ratio e). Compressive strains are negative; the opposite convention would introduce a minus sign in eq. (5). Use of eqs. (3), (4, second part), and (5) into the consistency equation $\dot{F} = 0$ yields

$$K_b = -\frac{\partial F}{\partial e^p}\,\frac{1+e_o}{\left[\frac{\partial F}{\partial \bar{\sigma}_{ij}}\,\frac{\partial F}{\partial \bar{\sigma}_{ij}}\right]}\,\frac{\partial F}{\partial \bar{\sigma}_{kk}} \tag{6}$$

which defines K_b at $\bar{\sigma}_{ij}$. Subsequently K can be related to K_b by means of the distance $\delta = (tr((\bar{\sigma}_{ij} - \sigma_{ij})^2))^{1/2}$ between $\bar{\sigma}_{ij}$ and σ_{ij} and its initial value δ_{in} at the initiation of a loading process. Many such relations are possible, and one which was proved to be very successful [1,3] is

$$K = K_b + h\left(\frac{\delta}{\delta_{in}-\delta}\right) \tag{7}$$

where h is a shape hardening parameter, <u>the only new parameter</u> introduced by the present theory. The set of equations (3), (4, first part), (5), (6), and (7) fully determines the material plastic response. Observe that when $\delta = 0$, i.e., point σ_{ij} lies on $F = 0$, then $K = K_b$ from eq. (7). When $\delta = \delta_{in}$, $K = \infty$ indicating the initiation of a loading process. At any stress reversal a new $\bar{\sigma}_{ij}$ is located by the changing $\dot{\sigma}_{ij}$, a new δ_{in} is defined and a new process begins, during which the material keeps in memory the δ_{in} as δ changes. This change of δ_{in} conveniently describes the irreversible plastic response upon stress reversal. If $h \to \infty$ then $K \to \infty$ except when $\delta = 0$ in which case the bounding surface acts as a yield surface since with $K = \infty$ there is no plastic strain for stress states inside it.

Equation (6) reveals the very important fact that K_b changes on the bounding surface as $\partial F/\partial \bar{\sigma}_{kk}$ changes on it. (Note that $\bar{\sigma}_{kk} = I_1$). With the representation of eq. (3) and assuming that $(\partial F/\partial e^p) > 0$ it is easily seen that in the $-I_1, \sqrt{J_2}$ space (Figure 1) there exists a consolidation region OACBO where $K_b > 0$, a dilatancy region OADBO where $K_b < 0$, and the critical states A and B where $K_b = 0$, (unlimited distortional flow without volumetric change, according to eq. (4), since the normal n_{ij} is parallel to $\sqrt{J_2}$ axis). These changes of K_b reflect into changes of K through eq. (7), showing stable or unstable behavior according to the degree of overconsolidation, as discussed in detail in the following section. For the consolidation region the bounding surface expands while for the dilatancy region it contracts. For either case the stress state is eventually brought at a critical state (A or B) where failure occurs for a critical void ratio under given stress. Pore water pressure build-up (positive or negative) for normally consolidated or overconsolidated soil subjected to undrained cyclic stress conditions can be predicted very easily; this is because plastic volumetric strain can occur for stress and stress rates within the bounding surface (overconsolidation), something which is impossible according to the classical yield surface plasticity models.

4. Particular Example

A particular form of the general model is presented here for conditions of triaxial compression, and the qualitative behavior of the model is studied numerically. The stress variables now are the effective mean normal pressure $p = (1/3)(\sigma_1 + 2\sigma_3)$ and the deviatoric stress $q = (\sigma_1 - \sigma_3)$. The corresponding plastic strain measures are the plastic volumetric strain $\varepsilon_p = (\varepsilon_1{}^p + 2\varepsilon_3{}^p)$ and the plastic deviatoric strain $\varepsilon_q = (2/3)(\varepsilon_1{}^p - \varepsilon_3{}^p)$. Compressive stresses and strains are taken as positive (opposite convention to the one adopted earlier).

In the p, q space, the bounding surface is a function of $\bar{p}$, $\bar{q}$, and e^p. The equation of the bounding surface corresponding to eq. (3) is assumed to be that of an ellipse passing through the origin with a ratio of principal axes equal to M for simplicity, where $q = Mp$ is the critical line. If $\bar{a}$ is the ellipse semiaxis along the p axis and q is normalized by M the equation for F becomes

$$F = \bar{p}^2 + \bar{q}^2 - 2\bar{p}\,\bar{a}\,(e^P) = 0 \tag{8}$$

i.e., the equation of a circle of radius $\bar{a}$ passing through the origin as shown in Figure 3. In the following development it is to be remembered that q (real) = Mq and ε_q (real) = ε_q/M. For a given stress point p, q and a stress increment $\dot{p}$, $\dot{q}$ such that $(\dot{q}/\dot{p}) = r$, the corresponding point $\bar{p}$, $\bar{q}$ on the bounding surface can be found as the intersection of the vector $\dot{q}$, $\dot{p}$ with $F = 0$, hence it allows that:

$$\bar{p} = \frac{(pr - q)\,r + \bar{a} + [\bar{a}^2 + q^2 + 2q(p-\bar{a})\,r - p(p - 2\bar{a})\,r^2]^{1/2}}{1 + r^2} \tag{9a}$$

$$\bar{q} = \sqrt{\bar{p}(2\bar{a} - \bar{p})} \tag{9b}$$

The distance δ is now obtained from $\delta = [(\bar{p} - p)^2 + (\bar{q} - q)^2]^{1/2}$. The derivative of $\bar{a}$ with respect to e^P is given by

$$d\bar{a}/de^P = -\bar{a}/(\lambda - \kappa) \tag{10}$$

where λ and κ denote the slopes of the normal consolidation line and the rebound line in a typical $e - \ell np$ plot, Fig. 2.

For the present particular case, following the procedure of the general development and using eqs. (8) and (10), the following set of eqs. correspond to eqs. (4), (5), (6), and (7) respectively

$$\dot{\varepsilon}_p = \langle \frac{1}{K}(\dot{p}n_p + \dot{q}n_q)\rangle\, n_p = \langle \frac{1}{K_b}(\dot{\bar{p}}n_p + \dot{\bar{q}}n_q)\rangle\, n_p \tag{11a}$$

$$\dot{\varepsilon}_q = \langle \frac{1}{K}(\dot{p}n_p + \dot{q}n_q)\rangle\, n_q = \langle \frac{1}{K_b}(\dot{\bar{p}}n_p + \dot{\bar{q}}n_q)\rangle\, n_q \tag{11b}$$

$$\dot{e}^P = -(1 + e_o)\dot{\varepsilon}_p \tag{12}$$

$$K_b = \frac{1 + e_o}{\lambda - \kappa}\left[\frac{\bar{p}\,(\bar{p} - \bar{a})}{\bar{a}}\right] \tag{13}$$

$$K = \frac{1 + e_o}{\lambda - \kappa}\left[\frac{\bar{p}\,(\bar{p} - \bar{a})}{\bar{a}} + h\left(\frac{\delta}{\delta_{in} - \delta}\right)\right] \tag{14}$$

where n_p, n_q are the components of the unit normal n on the bounding surface at $\bar{p}$, $\bar{q}$, Fig. 3, given by $n_p = (\bar{p} - \bar{a})/\bar{a}$, $n_q = \bar{q}/\bar{a}$, and eq. (14) is slightly modified compared to eq. (7) (note that the first term in eq. (14) is equal to K_b). Observe that K_b changes values along the bounding surface. It is positive on the right side ($\bar{p} - \bar{a} > 0$, consolidation), negative on the left ($\bar{p} - \bar{a} < 0$, dilatation), and zero at the point of intersection with the $q = Mp$ line (failure at critical state).

For undrained conditions, the rate of pore water pressure build-up $\dot{u}$ can be found by equating the plastic volumetric compression to an elastic extension, i.e.,

$$\dot{u} = \frac{p}{\kappa}(1 + e_o)\,\dot{\varepsilon}_p \qquad (15)$$

The set of equations, (8) - (15), allows us to predict the material response under any conditions once the parameters M, λ, κ, and h are known.

In the following the qualitative behavior of the model is studied numerically. For San Francisco Bay Mud Clay, values of $\lambda = .85$ and $\kappa = .23$ were taken from an $e - \log_{10}p$ plot; hence, for an $e - \ell n p$ plot, we have $\lambda = (.85/\ell n 10) = .37$ and $= (.23/\ell n 10) = .10$. The critical ratio M equals 1.60 and, as reference state, the point $p = .4$ kg/cm^2, $e_o = 2.15$, on the normal consolidation line is chosen (see point a in Figure 2). The only new parameter introduced here is the shape hardening parameter h, which is taken equal to 1.72, so that $(1+e_o)h/\lambda-\kappa = 20$ in eq. (14). The selection of h is the only arbitrary step taken in this example. It is possible to have h a function of the ratio q/p or other appropriate quantities [1,3].

The following loading histories are considered: 1) Path abb', Fig. 2 (preconsolidation at $p_o = 1.18$ kg/cm^2, point b), subsequently q increases up to failure, keeping p constant, point B, Fig. 3. 2) Path acc', Fig. 2 (preconsolidation at $p_o = 4$ kg/cm^2, point c), and failure by increasing q, point B, Fig. 3. 3) Path abb', Fig. 2; cyclic change of q between $0 \leq q \leq .64$ kg/cm^2 under undrained conditions. The response of the model is calculated from eqs. (11) - (15).

In Figures 3, 4, and 5 the results of the first loading are shown by the curves AB (loose sample, OCR = 1.18, expansion of bounding surface) and those of the second loading by the curves ACB (dense sample, OCR = 4, contraction of bounding surface). Failure occurs at the same final critical void ratio $e \simeq 1.62$, Fig. 5. the δ_{in} are shown in Fig. 3. It is interesting to study the curves ACB (second loading) in connection with eqs. (13) and (14). Since $\bar{p} \leq \bar{a}$, the K_b eq. (13) is always negative, hence the continuous contraction of the bounding surface. On the other hand, K from eq. (14) is positive when δ is large enough until point C is reached. At C, K = 0 since the negative first term in the bracket, eq. (14), cancels the positive second term and subsequently K remains negative (falling $q - \varepsilon_1$ curve) until point B is reached where $\bar{p} - \bar{a} = 0$, $\delta = 0$, and K becomes zero again. During the stable-unstable behavior the unit normal to the changing bounding surface has a component n_p along the negative p direction, while $(1/K)\dot{q}n_q$ remains positive, eq. (11a), since K and $\dot{q}n_q$ change sign simultaneously (recall $\dot{p} = 0$ during the cyclic loading). Thus, $\dot{\varepsilon}_p$ remains positive along n_p indicating the continuous dilatation.

In Fig. 6 the predicted cyclic behavior of the model during the third loading is shown. The bounding surface expands progressively at each cycle while the effective p reduces due to pore water pressure build-up. The δ, δ_{in} for the first semicycle are shown. For stress reversals towards the negative q, the corresponding point $\bar{p}$, $\bar{q}$ is taken on the other half of the bounding surface not shown in Fig. 6. Observe that if a yield surface were used instead of the bounding surface, the cyclic q would remain inside the elastic region due to the preconsolidation and consequently, no volumetric plastic strain occurs and no pore water pressure increase can be predicted contrary to the real behavior.

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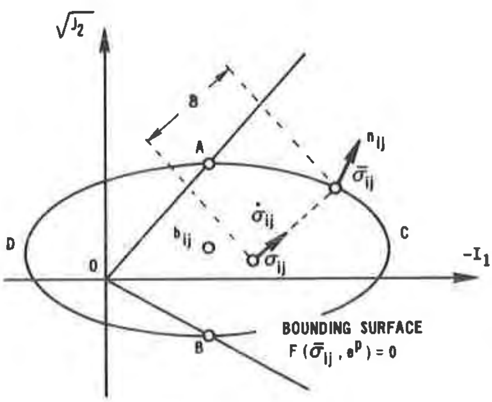

Fig. 1 Schematic Illustration of the Bounding Surface in $-I_1$, J_2 Space for Soil Materials Without a Yield Surface

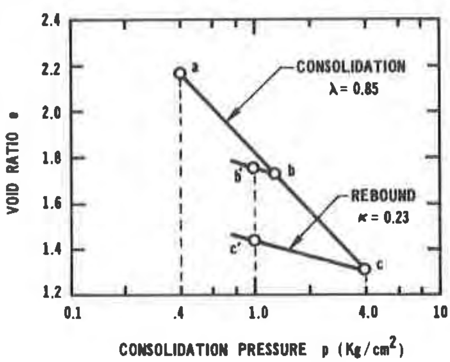

Fig. 2 Loading Histories Before the Application of the Deviatoric Stress q

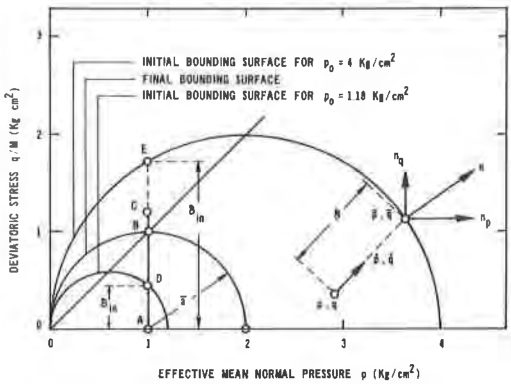

Fig. 3 Bounding Surface and Its Change for First and Second Loadings

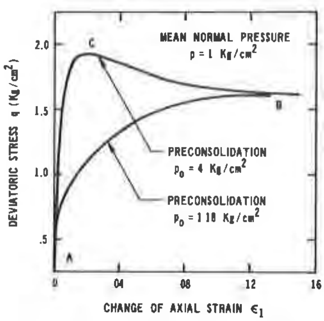

Fig. 4 Change of q versus ε_1 for First and Second Loadings

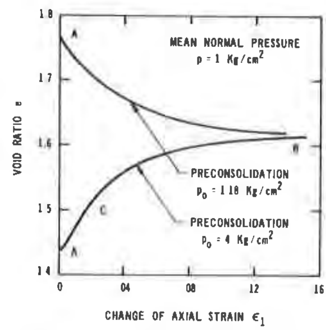

Fig. 5 Change of e versus ε_1 for First and Second Loadings

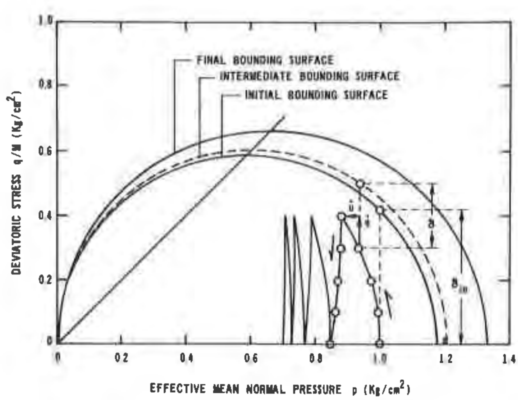

Fig. 6 Undrained Cyclic q Loading of Overconsolidated Sample (Third Loading) showing Pore Water Pressure Build-Up and Reduction of p.