A Simplified Method for Elastic Follow-up Analysis of Elevated Temperature Piping Systems

T. Teramae, J. Hamanaka
Ishikawajima-Harima Heavy Industries Co., Ltd., 1-15, Toyosu 3-chome, Koto-ku, Tokyo 135-91, Japan

T. Kano
Power Reactor and Nuclear Fuel Development Corporation, 9-13, Akasaka 1-chome, Minato-ku, Tokyo 107, Japan

Summary

Estimation of the accumulated deformation and strain caused by elastic follow-up, which is one of the most important problems in the design of elevated temperature piping system, is very difficult even by the finite element method since the elastic follow-up is a phenomenon concerned with the creep deformation of the entire structure. In the ASME Code Case N-47, it is specified that a secondary stress with a large amount of elastic follow-up is to be included in the category of load controlled stress, but this simplification not only makes designing of elevated temperature piping systems very conservative, but leaves designers unable to understand the actual elastic follow-up behavior.

In this paper, the authors propose a simplified method for elastic follow-up analysis of elevated temperature piping systems composed of straight pipes and pipe bends. Here, the elastic follow-up is considered as a relaxation phenomenon of the overall structure, and, for statically determinate structures, relaxation characteristic function is explicitly given for Norton's creep law by an approximation method based on the complementary virtual work theorem.

Next, the authors define a global reference stress (G.R.S.) to obtain a relaxation characteristic function for general creep law other than Norton's, where G.R.S. is expressed as a function of initial section forces and the local reference stress (L.R.S.) which represents fundamental creep responses of the piping components concerned. It is clarified analytically that the degree of concentrated deformation by elastic follow-up depends largely on the ratio L.R.S./G.R.S. To obtain L.R.S. and other coefficients for the proposed method, parametric numerical calculations are performed using ISTRAN/3D, a general purpose three-dimensional structure analysis program, on pipe bends subjected to in-plane and out-of-plane bending moments.

Thirdly, a computer program for the present simplified elastic follow-up analysis is developed for piping systems comprising pipe bends with the pipe factor λ ranging from 0.05 to 0.57. The results of analyses performed on two-and three-dimensional piping models agree well with the numerical analyses by ISTRAN/3D or by SINAP-II, a program which takes flexibility factors into account.

Since the proposed method adopts the reference stress method, no restriction is imposed on the creep law of the material and no structural analysis is necessary other than the elastic calculation of the piping system. Therefore, much less computing time is needed than for the detailed finite element analysis, and it is at most 10 percent or so as compared with the flexibility factor method. It may be concluded then that the proposed simplified method provides a handy and effective tool for the basic designs of elevated temperature piping systems.

1. Introduction

Estimation of the accumulated deformation and strain caused by elastic follow-up, which is one of the most important problems in the design of creeping structures, is very difficult even by the finite element method since the elastic follow up is a phenomenon concerned with the creep deformation of the entire structure. To cope with this difficulty, designers have adopted one of several simplifications that have been proposed. In the ASME Code Case N-47, it is specified that a secondary stress with a large amount of elastic follow-up is to be included in the category of load controlled stress. This simplification, however, not only makes designing of structures very conservative, but leaves designers unable to understand the actual behavior of elastic follow-up.

Elevated temperature piping system is one of the most typical structures for which the elastic follow-up should be taken into account in the design, and prescribed thermal deformation is concentrated consistently with elapse of time at pipe bends due to their large flexibility. Therefore, it is important to establish a method evaluating the accumulated deformation and strain caused by elastic follow-up for the safe and reasonable design of elevated temperature piping systems.

In this paper, the authors propose a simplified method for elastic follow-up analysis of elevated temperature piping systems. Here, the elastic follow-up is considered as a relaxation phenomenon of the overall structure, and the complementary virtual work theorem is applied to obtain a fundamental equation. Furthermore, this equation is modified in the form which is suitable for applying the reference stress method. Then, parametric numerical analyses are performed to establish the proposed method on pipe bends subjected to in-plane and out-of-plane bending moments. Finally, the validity of the proposed method is examined through the analyses of two- and three-dimensional piping models.

2. Formulation of the Simplified Method

2.1 Relaxation of Structures

For the structure subjected to prescribed displacements, the complementary virtual work theorem is expressed:

$$\int_V \dot{\varepsilon}_{ij} \delta\sigma_{ij} \, dv = 0 \qquad (1)$$

where ε_{ij} is the strain tensor, $\delta\sigma_{ij}$ the variation of stress tensor σ_{ij}, and ($\dot{}$) the differential symbol with respect to time. When Norton's creep law is used, the constitutive equation for elastic creep deformation is given:

$$\dot{\varepsilon}_{ij} = C_{ijk\ell} \, \dot{\sigma}_{k\ell} + \frac{3}{2} k \, \bar{\sigma}^{\,n-1} S_{ij} \qquad (2)$$

where $C_{ijk\ell}$ is the elastic compliance tensor, $\bar{\sigma}$ the Mises' equivalent stress, S_{ij} the deviatoric stress tensor, k the coefficient of Norton's law, and n the creep exponent of Norton's law. For a structure that is close to being statically determinate, stress relaxation can be approximated:

$$\sigma_{ij}(\mathbf{r},t) = \sigma_{ij}^{s}(\mathbf{r}) \, \phi(t) \qquad (3)$$

where $\mathbf{r}$ is the position vector, t the time, σ_{ij}^{s} the steady stress in equilibrium with the initial deformation-controlled force. Substituting Eqs. (2) and (3) into Eq. (1), $\phi(t)$, to be called "relaxation characteristic function", is obtained:

$$\phi(t) = [1 + (n-1)k\left(\frac{\int_v (\bar{\sigma}^s)^{n+1}\, dv}{\int_v C_{ijk\ell}\, \sigma_{ij}^s\, \sigma_{k\ell}^s\, dv}\right) t]^{\frac{1}{1-n}} \tag{4}$$

This equation can be rewritten using the Young's modulus E and the newly defined stress σ_R^G:

$$\phi(t) = [1 + (n-1)\; E\; k\; (\sigma_R^G)^{n-1}\; t]^{\frac{1}{1-n}} \tag{5}$$

where

$$\sigma_R^G = \left(\frac{\int_v (\bar{\sigma}^s)^{n+1}\, dv}{E\int_v C_{ijk\ell}\, \sigma_{ij}^s\, \sigma_{k\ell}^s\, dv}\right)^{n-1} \tag{6}$$

Equation (5) is the well-known solution of uniaxial relaxation for the initial stress σ_R^G, and σ_R^G itself corresponds to Mackenzie's definition of reference stress [1]. Hence, σ_R^G will be called the "global reference stress" (G.R.S.).

While the foregoing result was obtained for Norton's creep law, it can be extended to a general creep law, for which relaxation characteristic function can be calculated numerically from the G.R.S. It must be noticed here that a suitable creep exponent is to be assigned for the determination of G.R.S. even to creep laws other than the Norton's.

Although the elastic stress can be used in Eq. (3) instead of the steady stress, it results in faster relaxation than the actual behavior, while the steady stress the slower[2].

2.2 Simplified Analysis of Elastic Follow-up of Piping System

In this paragraph, sectional forces and moments of piping system are expressed as generalized forces p_i (i=1, ..., 6). When the piping undergoes elastic follow-up, internal forces can be approximated:

$$P_i\,(\mathbf{r}, t) = P_i^o\,(\mathbf{r})\,\phi(t) \tag{7}$$

where $P_i^o\,(\mathbf{r})$ is the initial force, which can be calculated by an elastic analysis.

Now, when a pipe bend or straight pipe of unit length is subjected to forces P_i^o, the reference stress σ_R^L, which is here called the "local reference stress" (L.R.S.), is expressed in accordance with Mackenzie's definition:

$$\sigma_R^L = \left(\frac{\int_\nu (\bar{\sigma}_n^s)^{n+1}\, d\nu}{\int_\nu (\bar{\sigma}_1^s)^2\, d\nu}\right)^{n-1} \tag{8}$$

where $\bar{\sigma}_1^s$ and $\bar{\sigma}_n^s$ are the steady equivalent stresses corresponding to creep exponents of 1 and n respectively, and $\int_\nu d\nu$ denotes the volume integral for a pipe of unit length. Here, the following substitutions are adopted.

$$\int_\nu (\bar{\sigma}_1^s)^2\, d\nu = B_{ij}\, P_i^o\, P_j^o \tag{9}$$

$$E\int_\nu C_{ijk\ell}\, \sigma_{ij}^s\, \sigma_{k\ell}^s\, d\nu = F_{ij}\, P_i^o\, P_j^o \tag{10}$$

Then, G.R.S. is rewritten from Eqs. (8), (9) and (10):

$$\sigma_R^G = \left(\frac{\int (\sigma_R^L)^{n-1}\, B_{ij}\, P_i^o\, P_j^o\, d\ell}{\int F_{ij}\, P_i^o\, P_j^o\, d\ell}\right)^{\frac{1}{n-1}} \tag{11}$$

where $\int d\ell$ denotes the line integral along a piping. Therefore, if L.R.S., B_{ij} and F_{ij} are obtained in advance, G.R.S. can be calculated from Eq. (11) by an elastic analysis of piping system.

Applying Mackenzie's method, generalized displacements δi (i=1, ..., 6) can be estimated from the initial elastic displacements $\delta_i^{\,o}$ by the equation:

$$\delta_i \,(\mathbf{r}\,,\,t) = \delta_i^{\,o}(\mathbf{r})\phi(t) + \frac{Q_{ij}\delta_j^{\,o}(\mathbf{r})}{\sigma_R^{\,L}/E} \int_o^t \dot{\varepsilon}^c \,(\sigma^L(t))\, dt \tag{12}$$

where Q_{ij} is the interaction coefficients for creep displacements, $\dot{\varepsilon}^c(\sigma^L(t))$ the creep strain rate due to the stress $\sigma^L(t)$, and $\sigma^L(t) = \sigma_R^{\,L}\phi(t)$. For Norton's law, Eq. (12) can be rewritten by performing the time integration explicitly:

$$\delta_i(\mathbf{r}\,,t) = \delta_i^{\,o}(\mathbf{r})\phi(t) + Q_{ij}\delta_j^{\,o}(\mathbf{r})\left(\frac{\sigma_R^{\,L}}{\sigma_R^{\,G}}\right)^{n-1} (1-\phi(t)) \tag{13}$$

Since $\phi(t)$ approaches zero with the time increasing infinitely, the upper bound of accumulated inelastic displacements $\delta_i^{\,u}(\mathbf{r})$ is given by the simple equation:

$$\delta_i^{\,u}(\mathbf{r}) = Q_{ij}\delta_j^{\,o}(\mathbf{r})\left(\frac{\sigma_R^{\,L}}{\sigma_R^{\,G}}\right)^{n-1} \tag{14}$$

Equation (14) shows that the degree of accumulation of displacements depends largely on the ratio L.R.S./G.R.S.

Again, for creep laws other than Norton's, the accumulated displacements can be calculated numerically according to Eq. (12).

3. Analysis for Establishment of the Simplified Method

To establish a simplified method for elastic follow-up analysis of elevated temperature piping systems, analytical and numerical studies are undertaken on straight pipe and pipe bend subjected to two axial bending moments (in-plane and out-of-plane bending moments for the pipe bend), and the L.R.S. as well as the several coefficients defined in the preceding chapter are determined as shown in what follows. The other sectional forces and internal pressure are not considered in this study, since they are almost independent of the oval deformation of pipe bend, which has essential effect on the elastic follow-up of piping system.

3.1 Straight Pipe

The local reference stress of straight pipe under the bending moment M is given according to the definition (Eq. (8)) from the analytical result [3]:

$$\sigma_R^{\,L} = \xi \frac{|M|}{r_m^{\,2}h} \tag{15}$$

where r_m is the mean radius, h the thickness, and the coefficient ξ is:

$$\xi = [\pi\left(\frac{\Gamma((3n+1)/2n)}{2\sqrt{\pi}\ \Gamma((2n+1)/2n)}\right)^n]^{\frac{1}{n-1}}$$

in which Γ denotes the Gamma function. The coefficient ξ ranges from 0.255 to 0.252 for the creep exponent varying from 3 to 10, and the valve 0.253 is used in this study. In the case of two-axial bending moments My and Mz acting, L.R.S. is given from the vector summation of moments:

$$\sigma_R^{\ L} = \sqrt{(\sigma_R^{\ L})_y^2 + (\sigma_R^{\ L})_z^2} \tag{16}$$

where $(\sigma_R^{\ L})_y$ and $(\sigma_R^{\ L})_z$ are the L.R.S.s for the bending moments My and Mz acting separately. The coefficients B_{ij}, F_{ij} and Q_{ij} are also given directly according to the definitions from the analytical result [3]:

$$B_{ij} = \frac{\Delta_{ij}}{I} \tag{17}$$

$$F_{ij} = \frac{\Delta_{ij}}{I}\left(1.29 - \frac{0.71}{n}\left(2 - \frac{3.29}{n}\right)\right) \tag{18}$$

$$Q_{ij} = \Delta_{ij} \tag{19}$$

where the subscripts i and j imply y and z directions for their values of 1 and 2, Δ is the Kronecker delta, and I the second moment of sectional area. Equation (18) has been obtained by interpolating the analytical values for the creep exponents of 3, 5 and 7.

3.2 Pipe Bend

For the pipe bend, numerical analyses have been performed by ISTRAN/3D[4], a finite element computer program for the inelastic analysis of three-dimensional structures. The problem is solved for pipe bend having bend radius R = 838.2 mm, outer pipe diameter Do = 558.8 mm, and thickness h = 4.5, 9.5 and 45 mm which correspond to the pipe factor $\lambda (= 4Rh/(Do-h)^2)$ of 0.049, 0.106 and 0.572. The initial bend angle of the model is $\pi/6$ rad. for the case of in-plane bending moment acting alone, and is reduced to $\pi/24$ rad. for the case of out-of-plane bending moment acting in order to diminish to negligibly small degree the effect of the twisting moment derived from out-of-plane bending moment. The model is divided with the 8-noded isoparametric shell element into 16 elements in circumferential direction. Elastic constants used are Young's modulus of 16,110 kg/mm^2 and Poisson's ratio of 0.303, and Norton's law of the creep exponents of 3, 5 and 7 is employed. Twenty five cases have been calculated where thickness, creep exponent and the ratio of in-plane and out-of-plane bending moments have been changed systematically. Numerical calculations have been made using 2x2x4 - points Gauss integration.

Here, the definitions of L.R.S., B_{ij} and F_{ij} are slightly modified so as to reduce required numerical calculations. Namely, the elastic stress is used instead of the steady stress in the denominator of Eq. (8) and also in Eqs. (9) and (10). These modifications are considered permissible since the definition of the reference stress is not unique, nor theoretical. Then, the calculations have resulted in the coefficients B_{ij} and F_{ij}:

For in-plane bending, $B_{11} = \dfrac{1.54}{I\lambda}$, $F_{11} = \dfrac{1.65}{I\lambda}$ (20-A)

For out-of-plane bending, $B_{22} = \frac{1.54}{I\lambda}$, $F_{22} = \frac{1.67}{I\lambda}$ (20-B)

The coupling terms of these coefficients have been negligibly small, and therefore are ignored as in the case of straight pipe.

Now, the L.R.S. of pipe bend $(\sigma_R^L)_b$ is expressed with the L.R.S. of straight pipe $(\sigma_R^L)_s$:

$$(\sigma_R^L)_b = \alpha\,(\sigma_R^L)_s \tag{21}$$

The calculations have yielded the coefficient α:

For in-plane bending, $\alpha = \frac{1.06}{\lambda^{0.625}}$ (22-A)

For out-of-plane bending, $\alpha = \frac{1.16}{\lambda^{0.642}}$ (22-B)

From the definition of L.R.S. (Eq. (8)), the coefficient α can be expressed:

$$\alpha = \left(\frac{(1-\nu)^2\,Kc}{(1-0.5)^2\,Ke}\right)^{\frac{1}{n-1}} \tag{23}$$

where Ke and Kc are the elastic and creep flexibility factors. Equation (23) yields the clear understanding of the coefficient α. For the combined bending moments, L.R.S. $\bar{\sigma}_R^L$ will be given:

$$\bar{\sigma}_R^L = \sqrt{(\sigma_R^L)_I^2 + (\sigma_R^L)_o^2} \tag{24}$$

where $(\sigma_R^L)_I$ and $(\sigma_R^L)_o$ are the L.R.S.s for the cases of in-plane and out-of-plane bending moments acting separately. According to the numerical analyses for the cases of $(\sigma_R^L)_I/(\sigma_R^L)_o$ nearly equal to $\frac{2}{3}$, 1 and $\frac{3}{2}$, Eq. (24) yields the fairly close value of L.R.S. to that obtained by the definition within the error of ±3%.

3.3 Estimation of Accumulated Deformation and Strain for Pipe Bend

Applying Mackenzie's method under the application of energy theorem which is not discussed here, the rate of creep curvature change $\dot{\kappa}_i^c$ due to constant combined bending moments is expressed in the form:

$$\dot{\kappa}_i^c = \frac{\kappa_i^e\,(M_i)}{(\bar{\sigma}_R^L)/E}\,\dot{\varepsilon}^c\,(\bar{\sigma}_R^L) \tag{25}$$

where $\kappa_i^e\,(M_i)$ denotes elastic curvature change by moment M_i alone. It is noticed that no further coupling effect appears other than the L.R.S. in Eq. (25). Figure 1 shows examples of the curvature changes κ_I and κ_O which are normalized by their initial elastic values. From the numerical results including Fig. 1, the accumulated curvature change caused by the elastic follow-up is expressed:

$$\kappa_i(\mathbf{r},t) = \kappa_i^e(\mathbf{r})\,(0.41+\phi(t)) + \frac{Q_{ij}\,\kappa_j^e(\mathbf{r})}{\sigma_R^L/E}\int_o^t \dot{\varepsilon}^c(\bar{\sigma}^L(t))\,dt \quad (i,j=1,2) \tag{26}$$

where

$$[Q_{ij}] = \begin{bmatrix} 0.81 + 0.02n & 0 \\ 0 & 0.81 + 0.05n \end{bmatrix}$$

, and both of the subscripts i and j imply in-plane and out-of plane bendings for their values of 1 and 2, and $\sigma^L(t) = \sigma^L_R\,\phi(t)$. Equation (26) includes the additional term corresponding to the transitional curvature change due to constant bending moment, which results in the overestimation of curvature change caused by the elastic follow-up.

Figure 2 shows examples of the relation between the curvature change and the maximum strain. The coordinates adopted in this figure are:

$$\tilde{\varepsilon}_{max} = \frac{\bar{\varepsilon}_{max}}{\bar{\varepsilon}^e_{max}}, \quad \tilde{\kappa} = \frac{\bar{\kappa}}{\bar{\kappa}^e} \tag{27}$$

where $\bar{\varepsilon}_{max}$ is the maximum equivalent strain, $\bar{\varepsilon}^e_{max}$ its initial elastic value, $\bar{\kappa}$ and $\bar{\kappa}^e$ are the equivalent curvature change and its initial elastic value given by the equation:

$$\bar{\kappa} = \sqrt{(\kappa_I^2 + \kappa_O^2)}$$

From the numerical results including Fig. 2, the normalized maximum strain can be estimated on the safe side by the equation:

$$\tilde{\varepsilon}_{max} = 2\,\tilde{\kappa} \tag{29}$$

Although Eq.(29) has been derived from the analyses for constant bending moment, it is considered applicable to the estimation of accumulated maximum strain for the pipe bend undergoing elastic follow-up. For the simplified method, $\bar{\varepsilon}^e_{max}$ is calculated analytically by employing the result by Shimizu et al. [5].

To calculate the creep damage during the elastic follow-up, one of the authors has proposed a simplified procedure [6]. That is to perform the relaxation analysis in which creep strain rate is assumed to be $1/\tilde{\varepsilon}_{max}$ of its original value, and then to calculate the creep damage with the use-fraction rule along the stress history obtained. This procedure is also introduced in the simplified method.

4. Examples of Numerical Analysis

To examine the validity of the proposed simplified method, numerical analyses have been performed by a computer program developed for piping systems comprising pipe bends with the pipe factor λ ranging from 0.05 to 0.57, and the results have been compared with those by the finite element method (F.E.M.) and what is called the flexibility factor method (F.F.M.). Material constants are those of SUS316, and what is known as HEDL's equation [7] is employed for creep constitutive equation. For comparison, HEDL's equation is approximated by the Norton equation for the temperature of 510°C:

$$\dot{\varepsilon}^c = k\sigma^n \tag{30}$$

where n=4.57, and $k = 1.397 \times 10^{-13}$ $(1/(kg/mm^2)^{4.57}/hr)$.

4.1 Two-Dimensional Piping Model

Figure 3 shows the two-dimensional piping model subjected to the prescribed displacement at the free edge. Adoption of the original creep properties (Case 1) has resulted in insufficient elastic follow-up, and therefore the original creep strain rates have been multiplied by 100 in the next calculations (Case 2). Figure 4 shows the relaxation characteristics where results by the F.E.M. and F.F.M. represent the change of moment at the clamped edge. For the creep exponents of 3, 4.57 and 7, the present method has yielded the G.R.S.s of 7.11, 7.71 and 8.08 kg/mm^2 respectively. Generally speaking, the G.R.S. approaches the maximum L.R.S. (9.12 kg/mm^2 in Case 2) with increase of the creep exponent. As shown in Fig. 4, the Norton equation has resulted in slower relaxation as compared to HEDL's, since the former does not reflect the primary creep which seems to have the essential effect in the elastic follow-up. It is also shown that the present method and F.F.M. yield almost the same relaxation characteristics, since the both methods are based on the assumption of stationary creep deformation. On the other hand, the F.E.M. gives faster relaxation than those by the other methods, because it follows the actual elastic creep behavior. Figure 5 shows the stress and strain changes at the point of maximum accumulated strain, and the moment change at the clamped edge, which are all normalized by their initial elastic values and expressed as $\tilde{\sigma}$, $\tilde{\varepsilon}$ and $\tilde{M}$. The stress change obtained by the aforementioned simplified procedure is also shown in Fig. 5. Although this calculation has been made by the use of normalized strain of 1.78 at the time of 2.1 x 10^5 hours, whereas the upper bound of the accumulated at infinite time is to be used theoretically, the simplified procedure has yielded a stress relaxation very similar to that obtained by the F.E.M. Figure 6 shows the relation between $\tilde{\varepsilon}_{max}$ and $\tilde{\kappa}$ obtained by the F.E.M. In this figure, Case 3 denotes a calculation in which all data are the same as Case 2 except the length of horizontal part of the straight pipe and the prescribed displacement which are multiplied respectively by factors of 10 and 20 from their original values. This figure proves that Eq. (29) estimates the maximum accumulated strain on the safe side even when a complicated creep constitutive equation such as HEDL's is employed.

4.2 Three-Dimensional Piping Model

Figure 7 illustrates a three-dimensional piping model which is heated from the room temperature (20°C) to the temperature of 510°C or 566°C, and then is left to relax for 2.1 x 10^5 hours. For Norton's equation, the present method has resulted in the relaxation almost the same with that obtained by the F.F.M. as the case of two-dimensional model. In the analyses what follows, the original HEDL's equation and the creep exponent of 5 are employed. As shown in Fig. 8, the relaxation has ceased at a certain stress level following a rapid decrease of stress due to what is called the creep limit of the material for the temperature of 566°C; while the relaxation is very slow for 510°C. Figure 9 shows the stress relaxation obtained by the aforementioned procedure at the point of maximum initial elastic strain, occured in the pipe bend ① , which is considerably slower than that of the uniaxial calculation. Accumulated creep damage Dc has been calculated from this stress history by the procedure given in Code Case N-47, of which the SUS316 creep rupture data also has been used. The numerical results are tabulated in Table 1. The L.R.S.s are greater than the G.R.S. for the pipe bends ① and ② , and smaller for ③ , ④ , ⑤ and ⑥. Therefore, the pipe bends ① and ② have suffered accumulation of creep deformation. On the other hand, in the pipe bends ③ , ④ , ⑤ and ⑥ , the deformation is expected to decrease, but has proved to increase in some cases due to the several conservative assumptions.

5. Conclusions

The authors have proposed a simplified method for the elastic follow-up analysis of elevated temperature piping systems. This method is based on the assumption of piping system being statically determinate, and applies the reference stress method, where the global and local reference stresses have been newly defined. The validity of the proposed method has been shown by comparing the numerical results with those of the flexibility factor method and the finite element method.

Since the present method adopts the reference stress method, it has two advantages: (a) The actual creep properties such as the primary creep and creep limit can be taken into consideration in the analysis. (b) No other structural analysis is necessary than the elastic analysis of the piping system, and much less computing time is needed than for the detailed finite element analysis and it is at most 10 percent or so as compared with the flexibility factor method. It may be concluded that the proposed simplified method provides a handy and effective tool for the basic designs of elevated temperature piping systems.

References

[1] Mackenzie, A.C., "On the Use of a Single Uniaxial Test to Estimate Deformation Rates in Some Structures Undergoing Creep", Int. J. Mech. Sci., Vol. 10, pp. 441-453 (1968)

[2] Spence, J. and Hult, J., "Simple Approximations for Creep Relaxation", Int. J. Mech. Sci., Vol. 15, pp. 741-755 (1973)

[3] Odqvist, F.K.G. and Hult, J., "Kriechfestigkeit Mettalischer Werkstoffe", Springer-Verlag (1962)

[4] Teramae, T. and Ohya, H., "Analysis and Design of Structures and Machinery Employed at High Temperatures - First Report Inelastic Analysis Program of Three-Dimensional Structures and Creep Buckling-", to be appeared in IHI Engineering Review, Vol. 14 (1981)

[5] Shimizu, T., Kano, T., Sakata, S. and Sumikawa, M., "A Simplified Method of Thermal Ratchetting of Plate in Multi-Axial Stress State - Second Report Application to Piping Elbows-", (in Japanese), Preprints of JSME, No. 780-13, pp. 302-304 (1978)

[6] Teramae, T., "A Simplified Method for Elastic Follow-up Analysis", (in Japanese), Preprints of JSME, No. 770-1, pp. 173-175 (1977)

[7] Blackburn, L.D., "Isochronous Stress-Strain Curves for Austenitic Stainless Steels", The Winter Annual Meeting of ASME, Special Publication, ASME, pp. 15-48 (1972)

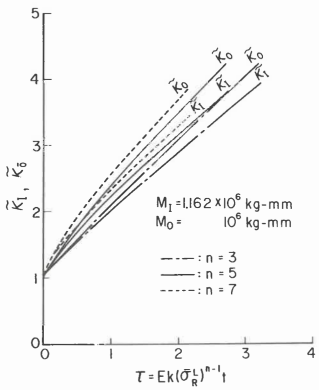

Fig. 1 Changes of normalized curvatures $\tilde{\kappa}_I$ and $\tilde{\kappa}_O$ for pipe bend of thickness h = 9.5 mm (λ = 0.106).

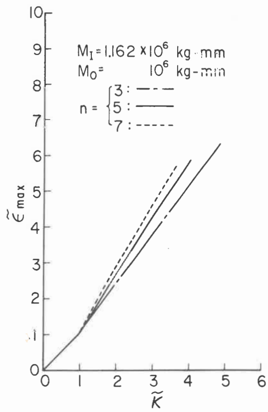

Fig. 2 Relation between normalized curvature $\tilde{\kappa}$ and normalized maximum equivalent strain $\tilde{\varepsilon}_{max}$ for pipe bend of thickness h = 9.5 mm (λ = 0.106).

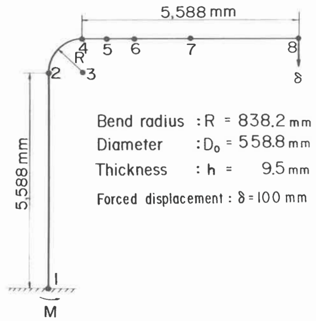

Fig. 3 Two-dimensional piping model.

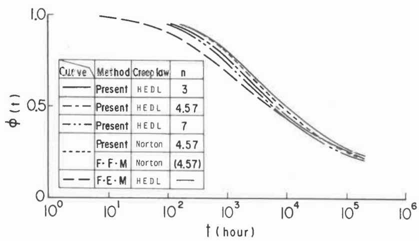

Fig. 4 Relaxation characteristics of two dimensional piping model (Case 2).

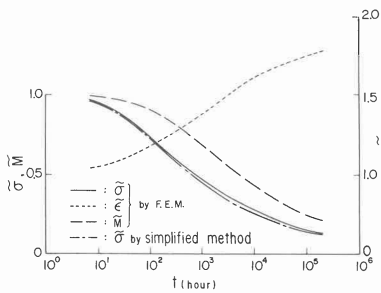

Fig. 5 Changes of stress $\tilde{\sigma}$, strain $\tilde{\varepsilon}$ and clamped moment $\tilde{M}$ of two-dimensional piping model (Case 2).

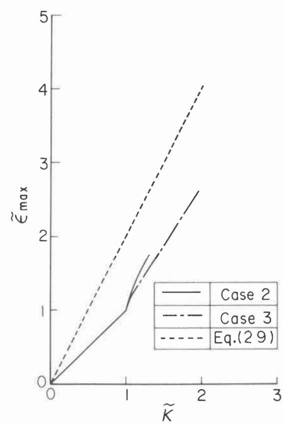

Fig. 6 Relation between normalized curvature $\tilde{\kappa}$ and normalized maximum equivalent strain $\tilde{\varepsilon}_{max}$ of two-dimensional piping model.

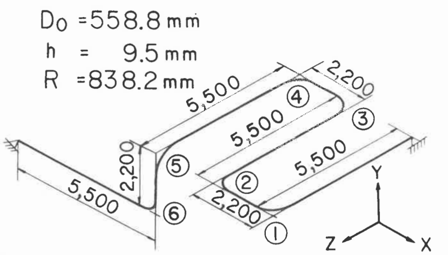

Fig. 7 Three-dimensional piping model.

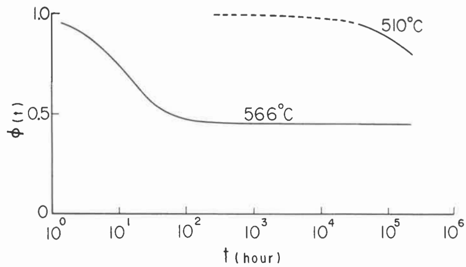

Fig. 8 Relaxation characteristics of three dimensional piping model.

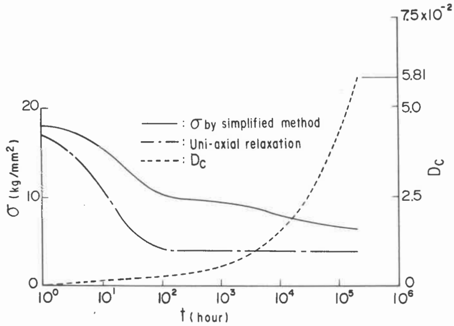

Fig. 9 Changes of stress σ and creep damage Dc at the point of maximum initial elastic strain in pipe bend ① of three-dimensional piping model. (566°C).

Condition	σ_R^G (kg/mm²)	Bend No.	σ_R^L (kg/mm²)	$\bar{K}$		$\bar{\epsilon}_{max}$		$\bar{\sigma}_{max}$ (kg/mm²)	D_c (t 210000 hr)
				t=0 hr	t=210000 hr	t=0 hr	t=210000 hr		
Creep law: HEDL Temp.: 510°C	6.07	(1)	7.10	.729E-5	.109E-4	.125E-2	.375E-2	16.6	.152E-1
		(2)	6.57	.674E-5	.976E-5	.116E-2	.335E-2	15.4	.107E-1
		(3)	2.44	.246E-5	.314E-5	.423E-3	.108E-2	5.48	.166E-4
		(4)	2.86	.288E-5	.367E-5	.493E-3	.126E-2	6.53	.661E-4
		(5)	3.61	.334E-5	.428E-5	.539E-3	.139E-2	6.77	.886E-4
		(6)	4.14	.405E-5	.492E-5	.615E-3	.160E-2	7.61	.213E-3
Creep law: HEDL Temp.: 566°C	6.58	(1)	7.69	.790E-5	.119E-4	.136E-2	.410E-2	17.9	.581E-1
		(2)	7.12	.732E-5	.105E-4	.126E-2	.562E-2	16.7	.520E-1
		(3)	2.65	.267E-5	.219E-5	.459E-3	.802E-3	5.93	.585E-3
		(4)	3.10	.323E-5	.275E-5	.534E-3	.945E-3	7.08	.107E-2
		(5)	3.92	.361E-5	.549E-5	.585E-3	.113E-2	7.34	.174E-2
		(6)	4.48	.410E-5	.452E-5	.667E-3	.140E-2	8.25	.346E-2

Table 1 Numerical results of the elastic follow-up analysis of three-dimensional piping model by the present simplified method.