Behaviour of Reinforced Concrete Beams Subjected to Bending Under any Normal Force

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INTRODUCTION AND OBJECTIVE

One of the main concerns for nuclear power plants is the resistance of containment structures under accidental loadings. For a safety analysis, the design of a whole structure requires an economical methodology using a global approach.

Some models have already been proposed for the case of pure bending, accounting for a constant normal force (Gauvain, 1979), and for the case of a predominant bending load with a relatively weak normal component (Bairrão, 1986).

The main objective of the present paper is the extension of the previous global models in order to allow any value of the normal force component, and to consider an actual behaviour for concrete and steels.

KEY

- N normal force
  N=X%C (or X%T) designates a normal force value equal to X% of the maximum compressive (or tensile) stress of the concrete making up the beam studied, multiplied by its straight cross-section
- M bending moment
- e deformation of membrane
- X curvature
- H.R.M. heavily reinforced in the middle
- L.R.M. lightly reinforced in the middle
- H.R.S. heavily reinforced near the surface
- L.R.S. lightly reinforced near the surface
- B.I.D. bending inelastic deformation
- M.I.D. membrane inelastic deformation

HOMOGENEIZATION METHOD

All the calculations presented here were made with the SAMSON code (Hoffmann, 1977). This program simulates the behaviour of the cross-section of a beam subjected to a bending moment under constant normal force;it performs a local analysis, leading then to a global description in terms of M-X and B.I.D.- M.I.D. laws of the "equivalent homogeneous material". Calculations are made step by step, each step corresponding to an increase of the curvature imposed.

WEAKNESS OF ELASTIC-PLASTIC IDEALIZATIONS

In order to get a broad set of results, we designed three "ideal" elastic-plastic steels (high, medium and light strength) and similarly three "ideal" concretes. The stress-strain laws of the idealized materials were developed from the actual laws, as shown in figure 1. The "actual" laws represent a mean value of a series of tests carried out on materials currently used in nuclear containment structures.

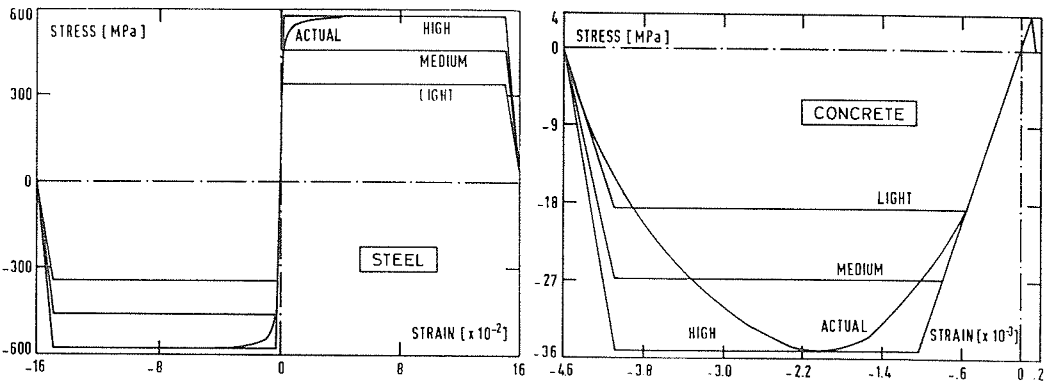

Figure 1

The actual case is a combination of actual steel and concrete. A total of nine combinations of idealized materials is possible which results in nine additional idealized cases referenced from 2 to 10 on table 1 and figure 2.

The geometry considered corresponds to an average based on a series of beams previously tested.

A series of calculations was performed, for each of the ten cases, considering three different values of the normal force: 33%C, 0, 33%T, representing the limits of the load range which we assume to act on the actual structures.

For discussion of the errors, we compared:

- The maximum strength point in the M-X and B.I.D.-M.I.D. laws of the idealized cases and of the actual case;

- The overall shape of the M-X and B.I.D.-M.I.D. laws for the idealized cases, and for the actual case.

In table 1 we present the percent relative errors, associated with the calculation of the ultimate strength point for each of the nine idealized cases, and for the three values of the normal force considered. We conclude that none of the idealized cases shows full agreement throughout the range of normal force (we cannot obtain relative errors systematically smaller then 10%).

CASE			N = 33% C				N = 0				N = 33%T			
REF.	STEEL	CONC.	MOMENT	CURV.	M.I.D.	B.I.D.	MOMENT	CURV.	M.I.D.	B.I.D.	MOMENT	CURV.	M.I.D.	B.I.D.
2	l	l	-36.0	-11.5	-186.2	2.9	-38.6	23.3	23.3	26.6	-44.4	22.7	56.7	60.2
3	m	l	-27.9	-3.3	-171.1	5.3	-20.0	12.4	7.2	14.1	-23.2	13.6	37.1	45.5
4	h	l	-19.6	0.1	-145.7	7.1	-1.6	1.3	-9.2	1.4	-1.8	2.3	16.1	28.7
5	l	m	-13.8	18.1	-37.0	29.4	-35.5	37.2	43.8	41.0	-41.2	33.4	77.9	76.4
6	m	m	-3.5	17.5	-37.4	24.9	-16.7	26.2	27.6	28.5	-19.5	22.2	56.7	59.5
7	h	m	6.7	19.9	-36.8	24.5	2.0	15.1	11.4	15.8	2.0	17.2	43.5	50.0
8	l	h	-3.5	42.2	116.2	58.2	-32.7	48.0	60.1	52.3	-38.1	41.0	93.5	88.9
9	m	h	6.8	40.3	112.6	52.1	-13.8	37.3	44.2	40.0	-16.5	34.4	79.7	78.6
10	h	h	15.6	47.3	79.7	58.5	5.0	26.9	28.9	28.0	5.3	21.2	55.6	58.0

Table 1

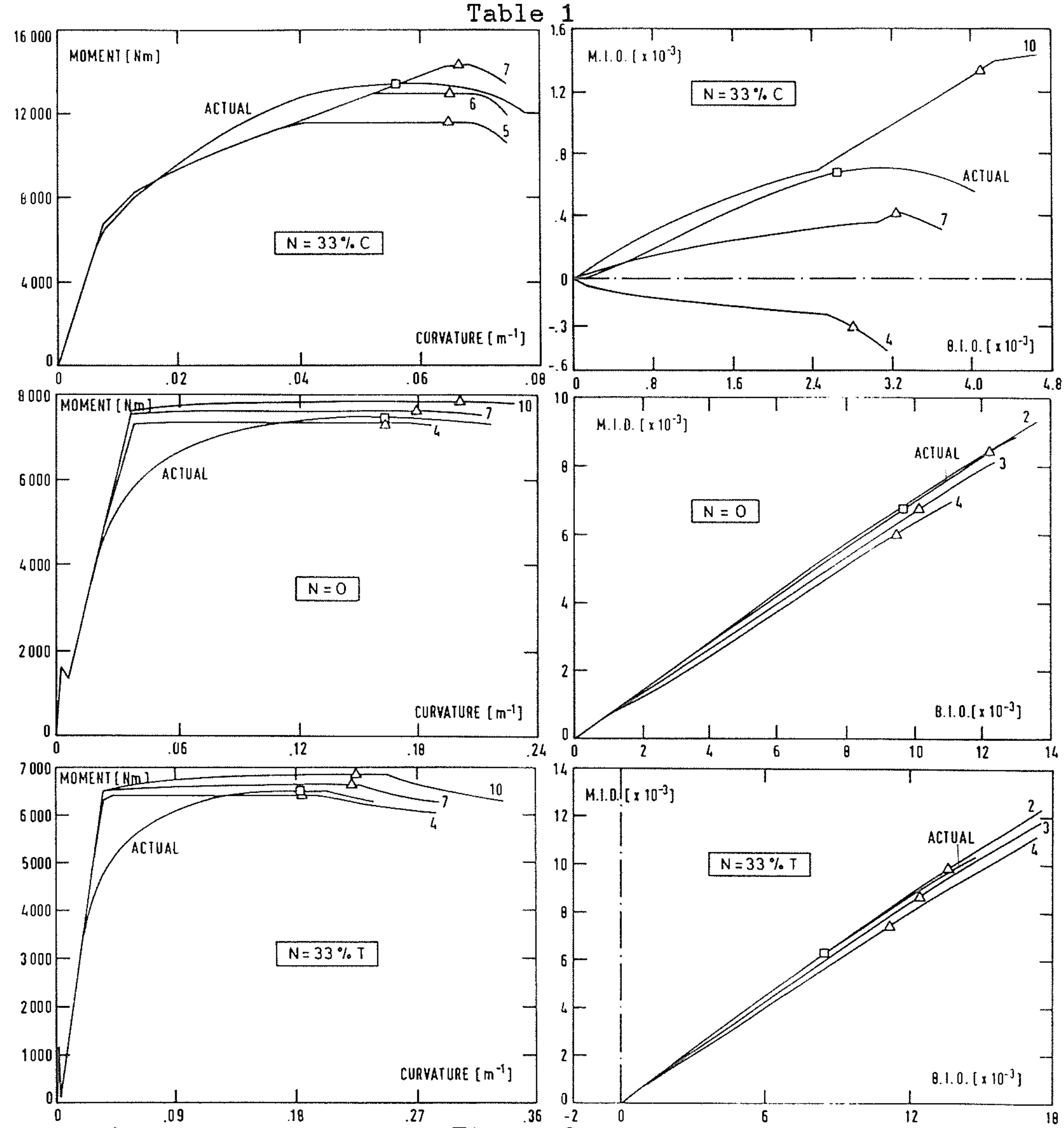

Figure 2

The overall shape of the M-X and B.I.D.-M.I.D. curves corresponding to the idealized cases is not fully acceptable either. Figure 2 represents some of the best idealizations and shows maximum values.

We can conclude that the elastic-plastic idealization of the behaviour for the materials leads to an unacceptable amount of error, therefore no further use will be made of such idealization.

MODELIZATION OF LIMIT SURFACES AND PLASTIC FLOW-RULE

We considered four different geometries (figure 3): HRS, HRM, LRS, LRM. For each geometry, a series of calculations was performed covering the whole range of normal forces: From 100%C to 0 (by steps of 10%C) and from 0 to 100%T (by steps of 10%T).

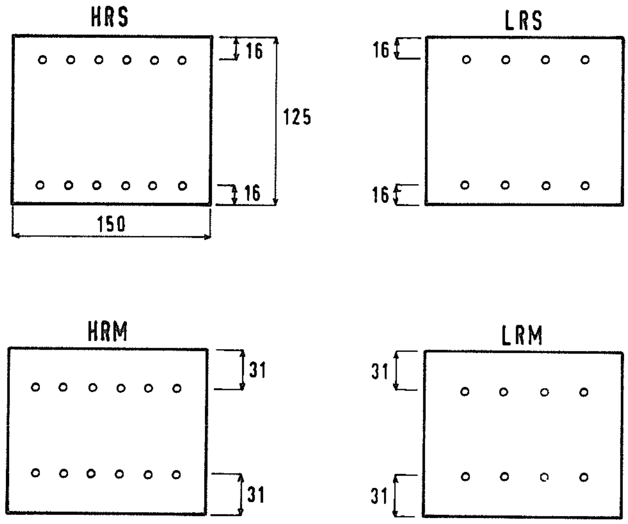

Figure 3

Figure 4 shows M-X and B.I.D.- M.I.D. laws for a H.R.M. geometry under a constant 20%C normal force.

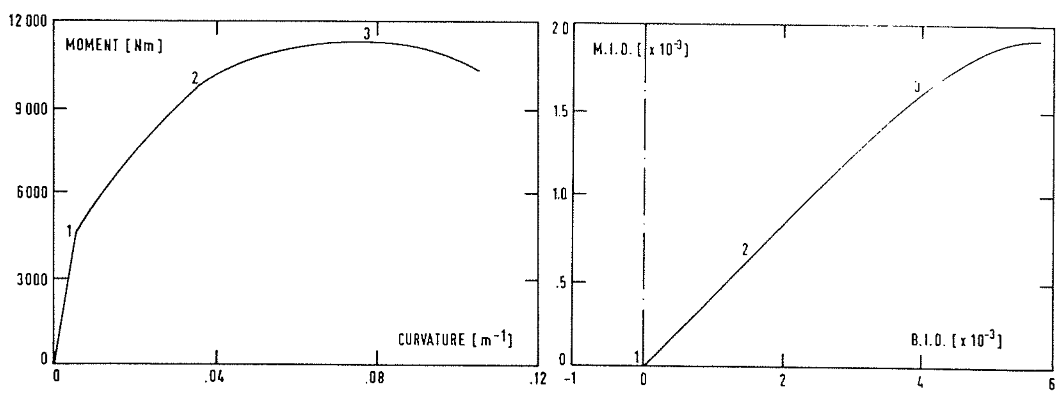

Figure 4

Close inspection of this curves allows the identification of three transition points:

- Elastic limit of concrete (point 1), limiting the beam's linear elastic domain;
- Elastic limit of steel (point 2), limiting the beam's non-linear elastic domain;
- Maximum strength point (point 3), representing the ultimate strength of the beam.

For a given geometry, and for every value of the normal force, we obtain a series of 1, 2 and 3 points. Plotting them in the M-N plane leads to the definition of three limit surfaces:

- Concrete yield surface (set of points 1);
- Steel yield surface (set of points 2);
- Ultimate strength surface (set of points 3).

Concrete yield surface is adequatly modelized by two straight lines. Modelization of steel yield surface and ultimate strength surface is best accomplished by two parabola.

In order to minimise the error we found that control points best suited were those corresponding to N=0, N=50%C and N=90%C. Figure 5 shows these surfaces for the H.R.M. case.

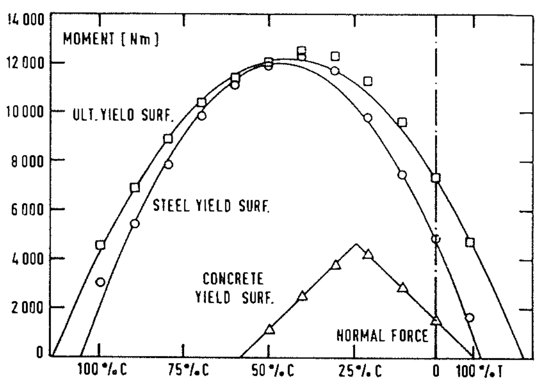

Figure 5

If we consider the evolution of the B.I.D.-M.I.D. curve between points 2 and 3, we can determine the plastic flow-rule for a given geometry.

In order to modelize a plasticity flow-rule using a third degree curve the best control points are N=100%T, N=0, N=50%C and N=90%C. Figure 6 shows that function for the same geometry.

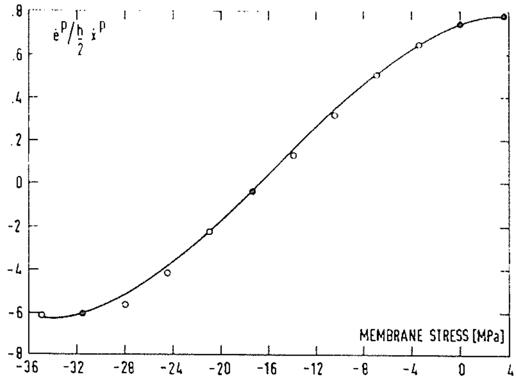

Figure 6

CONCLUSIONS

The extension of the previous models to any normal force value shows the non-linearity of the flow-rule to be considered and the need for new internal variables.

The extension to non-simplified behaviours for the materials shows the emergence of a steel yield surface, no longer coincident with the ultimate yield surface, as opposed to what happens when considering perfect elastic-plastic materials.

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