

Three-Dimensional Dynamic Response of Underground Vertical Well by Using AS-BE Coupling Method

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INTRODUCTION

The dynamic interaction problems of buried circular pipeline have been of great interest to many researchers and engineers in recent years. Most of previous publications are limited to the 2-D dynamic problems (Crawford et al 1974 and Datta et al 1984). The dynamic response of underground vertical well is a 3-D dynamic interaction problem which is much more difficult than that of buried infinite pipeline treated by 2-D theory. In mineral engineering, hardened structural engineering and nuclear engineering, there are many underground vertical wells whose dynamic characteristics must be taken into account. Few papers can be found in 3-D dynamic analysis of these underground structures.

In this paper, an analytical solution-boundary element coupling method (ASBECM) is outlined and used to study the transient problem shown in Fig.1. Here, the vertical well is treated as a finite thick cylindrical shell. The dynamic response of thick cylindrical shell with arbitrary time-dependent boundary conditions is analyzed by a general analytical method-space variable transform method (Ni, 1987) and (Ni et al, 1988). The 3-D wave propagation problems in surrounded semi-infinite media are studied by a semi-analytical boundary element method (Ni et al, 1987). Continuous conditions of stresses and displacements are satisfied on the interface of the well and media, and a set of coupling equations is established and solved. A numerical example of plane shock wave acting on the underground vertical well is given in this paper. Some significant conclusions are obtained here.

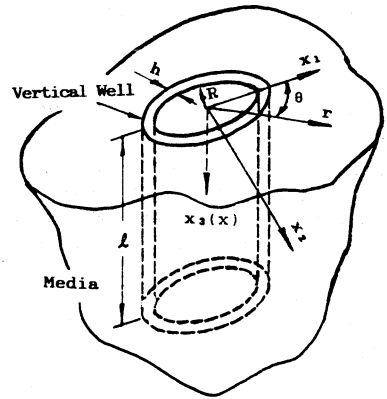


Fig.1 Geometry of Underground Vertical Well

DYNAMIC RESPONSE OF THICK CYLINDRICAL SHELL

Consider a circular, cylindrical shell lying vertical to free plane surface in semi-infinite elastic media, seeing Fig. 1. It is considered that the boundary conditions of thick cylindrical shell are time-dependent. In this paper, Mirsky-Herrmann motion equations of elastic thick cylindrical shell with non-axially symmetric deformation (Mirsky, 1957) are employed. Kirchhoff's assumption is discarded and effect of rotatory inertia is considered. The basic motion equations with displacement type are written in differential operator form

$$[L_{ij}]\{u \ \beta_x \ v \ \beta_o \ w\} = 0 \quad (1)$$

Here, L_{ij} are differential operators, u , v and w are axial, circumferential and radial displacements on the middle surface of shell respectively, β_x and β_o are the axial and circumferential angle function of rotation respectively.

The dynamic response functions of thick cylindrical shells with arbitrary time-dependent boundary conditions can be written in the form

$$u(x, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} U_{mn}(x, \theta) T_{mn}(t) + \sum_{i=1}^{10} \sum_{n=0}^{\infty} g_{iun}(x) f_{in}(t) \cos(n\theta) \quad (2a)$$

$$\beta_x(x, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} B_{xmn}(x, \theta) T_{mn}(t) + \sum_{i=1}^{10} \sum_{n=0}^{\infty} g_{ibxn}(x) f_{in}(t) \cos(n\theta) \quad (2b)$$

$$v(x, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} V_{mn}(x, \theta) T_{mn}(t) + \sum_{i=1}^{10} \sum_{n=0}^{\infty} g_{ivn}(x) f_{in}(t) \sin(n\theta) \quad (2c)$$

$$\beta_o(x, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} B_{omn}(x, \theta) T_{mn}(t) + \sum_{i=1}^{10} \sum_{n=0}^{\infty} g_{ibon}(x) f_{in}(t) \sin(n\theta) \quad (2d)$$

$$w(x, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} W_{mn}(x, \theta) T_{mn}(t) + \sum_{i=1}^{10} \sum_{n=0}^{\infty} g_{iwn}(x) f_{in}(t) \cos(n\theta) \quad (2e)$$

Here, U_{mn} , B_{xmn} , V_{mn} , B_{omn} , W_{mn} are modal functions and $T_{mn}(t)$ is the generalized coordinate. $f_{in}(t)$ is the component of boundary disturbing function $f_i(\theta, t)$.

$$f_i(\theta, t) = \sum_{n=0}^{\infty} f_{in}(t) \cos(n\theta) \quad \text{or} \quad f_i(\theta, t) = \sum_{n=1}^{\infty} f_{in}(t) \sin(n\theta) \quad (3)$$

$g_{iun}(x)$, $g_{ivn}(x)$, $g_{ibxn}(x)$, $g_{ibon}(x)$ and $g_{iwn}(x)$ are space variable transform functions which are expressed in the form

$$g_{iun}(x) = a_{iun}x^2 + b_{iun}x + c_{iun} \quad (4)$$

and the boundary differential operator is used to determine the coefficients a_{iun} , b_{iun} , c_{iun} , ...

The second terms of Exp. 2 stand for the quasi-static solution satisfying time-dependent boundary conditions.

SEMI-ANALYTICAL BEM

In this paper, the previous BEM is improved by reducing 3-D problem to 1-D discrete calculation. A new semi-analytical boundary element method (SBEM), basing on the analytical solutions denoted in cylindrical coordinates in elastodynamics, is given for analyzing 3-D wave motion. The authors use complete orthogonal functions to expand the solution of wave motion in one direction, and in other direction discrete solution is employed.

The 3-D problem here corresponds to the small displacement theory for layered homogeneous isotropic linear elastic materials. Under these assumptions the 3-D boundary integral equations for elastodynamics can be obtained as following

$$c(\xi) u_k(\xi, t) = \int_B [U_{ik} * t_{(n)i}(x, t) - T_{(n)i} * u_1(x, t)] ds(x) + \beta \int_R V_{ik} f_i(x, t) dv(x) + \rho \int_R (v_{oi} U_{ik} + u_{oi} V_{ik}) dv(x)$$

Where, $k = 1, 2, 3$

$$\begin{aligned} c(\xi) &= 1 && \text{if } \xi \in R \\ c(\xi) &= 1/2 && \text{if } \xi \in B \\ c(\xi) &= 0 && \text{if } \xi \text{ is outside } R+B \end{aligned} \quad (5)$$

$T_{(n)ik}$ and U_{ik} are the fundamental solutions of stress and displacement, respectively. Function U_{ik} can be written as following

$$\begin{aligned} U_{ik} &= \frac{1}{4\pi\rho} \left\{ \frac{t}{r^2} \left(\frac{3r_i r_k}{r^3} - \frac{\delta_{ik}}{r} \right) \left[H\left(t - \frac{r}{C_1}\right) - H\left(t - \frac{r}{C_2}\right) \right] \right. \\ &\quad \left. + \frac{r_i r_k}{r^3} \left[\frac{1}{C_1^2} \delta\left(t - \frac{r}{C_1}\right) - \frac{1}{C_2^2} \delta\left(t - \frac{r}{C_2}\right) \right] + \frac{\delta_{ik}}{r C_2^2} \delta\left(t - \frac{r}{C_2}\right) \right\} \end{aligned} \quad (6)$$

Where, $H(\dots)$ is the Heaveside function and $\delta(\dots)$ is the Dirac function, $r=|x-\xi|$ and $r_k=(x_k-\xi_k)$, δ_{ik} is the Kronecker delta, ρ is the mass density, C_1 and C_2 are the propagation speeds of dilatational and distortional waves. Function $T_{(n)ik}$ can be obtained by differentiating U_{ik} with Known formula.

In order to solve a system of boundary integral equations given by Eq. 5, an approximation of the unknown quantities at boundary has to be employed. The general procedure is to divide the boundary to several elements including a finite number of nodal point, and to confine the unknowns to these points. Unlike the previous work, the generalized unknown quantities are employed here. Basing on time marching scheme which is expressed by linear discretion, the present paper uses a set of complete orthogonal functions to write the quantities as following

$$u_i(r, \theta, x, t) = \sum_{j=1}^J \sum_{n=0}^N \sum_{k=1}^S [u_{cij}^{(k)} N_k(r, x_3) \cos(n\theta) + u_{sij}^{(k)} N_k(r, x_3) \sin(n\theta)]$$

$$t_i(r, \theta, x, t) = \sum_{j=1}^J \sum_{n=0}^N \sum_{k=1}^S [t_{cij}^{(k)} N_k(r, x_3) \cos(n\theta) + t_{sij}^{(k)} N_k(r, x_3) \sin(n\theta)] \quad (7)$$

Where, u_i and t_i are displacement and stress ($i=1,2,3$), respectively, $N_k(r, x_3)$ is the interpolation function, here $s=2$. It can be seen that Exp. 7 is most suitable to the cylindrical problems. In Exp. 7, the number of unknown quantities is $3J(2N+1)$, J being the number of discrete nodes and N being the numbers of expansion function terms. Substituting Exp. 7 into Eq. 5, a system of algebra equations is yielded by performing integral versus time and treating Cauchy principal integral in which singularities of the type of $1/r^2$ and $1/r$ when $r \rightarrow 0$ can occur. When $t=m\Delta t$, the system of boundary integral equations

$$\begin{aligned} & \sum_{n=0}^N \left\{ \sum_{q=1}^{J_r} (A_{ikjqcn}^{mm} u_{iqcn}^m + B_{ikjqsn}^{mm} u_{iqsn}^m) + \sum_{q=J_r+1}^J (C_{ikjqcn}^{mm} u_{iqcn}^m + D_{ikjqsn}^{mm} u_{iqsn}^m) \right\} \\ & = \sum_{n=0}^N \left\{ \sum_{q=1}^{J_r} (E_{ikjqcn}^{mm} t_{iqcn}^m + F_{ikjqsn}^{mm} t_{iqsn}^m) + \sum_{q=J_r+1}^J (G_{ikjqcn}^{mm} t_{iqcn}^m + H_{ikjqsn}^{mm} t_{iqsn}^m) \right\} \\ & + \sum_{n=0}^N \sum_{l=1}^{m-1} \left\{ \sum_{q=1}^{J_r} [E_{ikjqcn}^{ml} t_{iqcn}^l + F_{ikjqsn}^{ml} t_{iqsn}^l - (A_{ikjqcn}^{ml} u_{iqcn}^l + B_{ikjqsn}^{ml} u_{iqsn}^l)] \right. \\ & \left. + \sum_{q=J_r+1}^J [G_{ikjqcn}^{ml} t_{iqcn}^l + H_{ikjqsn}^{ml} t_{iqsn}^l - (C_{ikjqcn}^{ml} u_{iqcn}^l + D_{ikjqsn}^{ml} u_{iqsn}^l)] \right\} \\ & + F_{kjp}^m + U_{okjp}^m \\ & k = 1, 2, 3; \quad m = 1, 2, \dots, M; \quad j = 1, 2, \dots, J; \quad p = 0, 1, 2, \dots, 2N \end{aligned} \quad (8)$$

Where, $J=J_r+J_x$, J_r and J_x are radial and axial elements number, respectively, M is the number of time steps, m and l denote m th and l th time step ($l < m$), and p is the matching reference point in circumferential direction. Δt is the interval of time step. The coefficients of Eq. 8 are calculated (Ni, 1987). u_{iqcn}^m and u_{iqsn}^m are generalized displacements, t_{iqcn}^m and t_{iqsn}^m are generalized stresses, respectively.

DYNAMIC INTERACTION BETWEEN VERTICAL WELL AND MEDIA

Through the above analytical procedure, the system of coupling linear algebra equations for 3-D dynamic interaction between thick cylindrical shell and soil media can be established considering the compatible conditions on the interface between shell and media.

1. A set of algebra equations for motion of shell:

$$\{u_s\} = [B] \{t_s\} + \{C\} \quad (9)$$

Here, $\{u_s\}$ and $\{t_s\}$ are corresponding generalized displacements and stresses of shell on interface, respectively. $[B]$ will be treated complicatedly and is related to response functions of shell and coupling load on interface. $\{C\}$ is the corresponding displacement of shell under known dynamic loading.

2. A set of boundary integral equations of medium (at $t=n\Delta t$)

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} \{ u_m^s \}_1 \\ \{ u_m^s \}_2 \end{Bmatrix} = \begin{bmatrix} G^{11} & G^{12} \\ G^{21} & G^{22} \end{bmatrix} \begin{Bmatrix} \{ t_m^s \}_1 \\ \{ t_m^s \}_2 \end{Bmatrix} + \begin{Bmatrix} \{ F_1 \} \\ \{ F_2 \} \end{Bmatrix} \quad (10)$$

Here, $\{u_m^s\}_1$ and $\{t_m^s\}_1$ are displacements and stresses of scattering waves on the surface, respectively, which do not include interface in medium, $\{u_m^s\}_2$ and $\{t_m^s\}_2$ are displacements and stresses of scattering waves on the interface, respectively. $\{F_1\}$ and $\{F_2\}$ include the contributions of known displacements and stresses at time $t \leq (n-1)\Delta t$ to the dynamic responses at $t = n\Delta t$, and they are known at $t = n\Delta t$.

3. Compatible conditions of displacements and stresses on interface:

$$\begin{aligned} \text{stresses} \quad t_k^s &= -t_{mk} = -(t_{mk}^I + t_{mk}^s) \\ \text{displacements} \quad u_k^s &= u_{mk} = u_{mk}^I + u_{mk}^s \end{aligned} \quad (11)$$

Here, u_k and t_k denote the displacements and stresses of shell on the interface, respectively, u_{mk} and t_{mk} are the displacements and stresses of medium, respectively, I denotes incident waves, and s stands for scattering waves.

At nth time step, Eq. 9-11 lead to a finite set of algebra equations for determination of unknown constants. Detailed description can be seen (Ni, 1987).

NUMERICAL EXAMPLE

The application of the presented method to the underground vertical well is shown in Fig. 2.

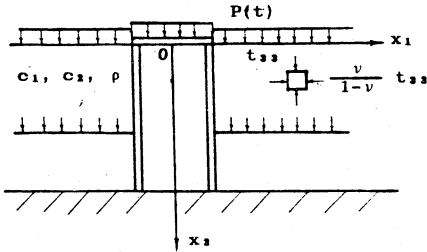


Fig.2 Action of Shock Wave on Vertical Well

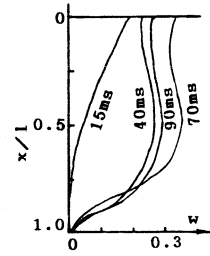


Fig.3 Radial Displacement w (10⁻⁴m)

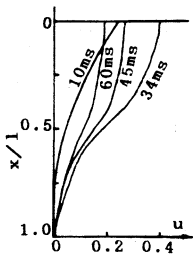


Fig.4 Axial Displacement u (10⁻³m)

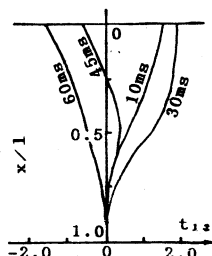


Fig.5 Axial Coupling Shear Stress $t_{13}(9.81 \times 10^4 \text{Pa})$ at $\theta = 0$

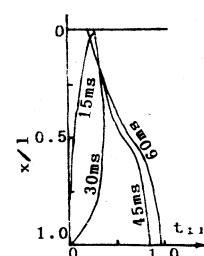


Fig.5 Radial Coupling Norm Stress $t_{11}(9.81 \times 10^4 \text{Pa})$ at $\theta = 0$

The well is built on the stiff rock and is surrounded by other kind of medium. The parameters of the well employed are:

$$l = 45.0\text{m}, h = 1.0\text{m}, R = 7.5\text{m}, E = 27.47\text{GPa}, \nu = 0.1667, \\ \text{and volume density } \gamma = 2500.0\text{kg/m}^3.$$

The basic parameters of medium are:

$$C_1 = 1500.0\text{m/s}, C_2 = 750.0\text{m/s}, \text{ and volume density } \gamma = 2607.0\text{kg/m}^3.$$

The applied pressure of plane shock wave $\Delta p_0 = 9.81 \times 10^4 \text{Pa}$.

Figs. 3-6 are the dynamic response distributions along axial direction at several times given.

Here, the expression of pressure wave is described by the following formula:

$$P(t) = \begin{cases} \Delta P_0 a t / r & 0 \leq t \leq t_r \\ \Delta P_0 (\tau^+ - t) / (\tau^+ - t_r) \text{Esp}[-1.3(t - t_r) / (\tau^+ - t_r)] & t_r \leq t \leq \tau^+ \\ 0 & t > \tau^+ \end{cases} \quad (12)$$

Where $t_r = 20\text{ms}$, $\tau^+ = 200\text{ms}$.

In these figures, it is shown that, because of rapid pressure increase of plane shock wave, upper shear stress becomes greater on the wall of well to resist shear is produced on the wall. Because the plane shock wave is reflected on the stiff rock, the part of well near the bottom will be acted on greater dynamic loading, which affects the axial distribution of displacements (see Figs. 3 and 4). In this paper, all numerical results are obtained on microcomputer IBM/AT.

CONCLUSION

From the presented theoretical analysis, numerical calculation, the following conclusions are summarized:

1. The analytical solution-boundary element coupling method (ASBECM) presented by the authors can be used to solve efficiently the 3-D dynamic interaction problems between thick-walled vertical well and media. Comparing the presented method with FEM and general BEM, the method has the advantages of less discrete elements and higher accuracy.

2. Semi-analytical boundary element method differs from the previous BEM. In circumferential direction, a set of complete orthogonal functions is used to expand the dynamic solutions of medium, and the numerical discrete procedure is reduced to 1-D discrete calculation. Therefore, the number of elements is significantly reduced and very accurate numerical solutions can be obtained. It is seen that SBEM is an efficient method to solve 3-D wave propagation problems in semi-infinite media.

3. From the numerical results, it is shown that the response values of shell in soil medium are less than those of shell in air under the same shock loading, which explains the effects of energy transmission and dispersion of radiating waves or scattering waves because of the shell in medium. Under the shock load presented here, in the upper part if the vertical well there are greater dynamic response values. The load acting on the shell is unsteady and is coupled with response of medium. The coupling load is greater where the dynamic response value, especially displacement is greater. Under plane shock wave shown here, there exists greater axial shear stress opposite to motion direction of the well on the wall, therefore, the reaction at the bottom of well is reduced.

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