APPLICATION OF THREE DIMENSIONAL NORMAL MODE THEORY TO SEISMIC ANALYSIS OF NUCLEAR POWER PLANTS

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SUMMARY

It is the purpose of this paper to present an extension of three dimensional normal mode theory to include arbitrary foundation rotation about any of three orthogonal axes in addition to arbitrary foundation translation. Numerous technical papers have established the significance of soil-structure interaction in the seismic analysis of nuclear power plants. Often equivalent soil springs are evaluated on the basis of dominant vibration modes in various directions. By making use of the proposed theory, the reactions of inertia forces to the ground in each mode of vibration are determined. Therefore, a more exact evaluation of the frequency dependence of the soil response can be obtained than with fixed springs. Since all modes of vibration are coupled, the soil responses in each direction are defined for each mode.

It is shown that an exact analysis of mass point accelerations must include both normal and Coriolis components. By neglecting these values, a linear relation between foundation motion and mass point accelerations can be established. For this case, normal mode methods can be applied to determine modal reactions to the foundation. Structural damping can be included in the modal solution.

By making use of this normal mode approach, the dynamic characteristics of the soil can be separated from the structure. Equivalent soil springs and soil viscous damping can be separated from the structure. Thus, the large viscous damping associated with radiation of energy into the soil can be accurately modeled.

A numerical evaluation of a simple system is presented. Results are compared to computations made using other methods of analysis.

Nomenclature

A dot over a symbol indicates a differentiation with respect to time.

$C(\omega)$	6 x 6 diagonal damping matrix defined by equation (32).
D	scalar defined by equation (12).
F^i	n x 1 matrix of i^{th} direction forces on each of the n masses.
F^r_{ja}	force component acting on m_j in mode a in direction r.
F^r_{fa}	$\sum_{j=1}^{n} F^r_{ja}$
F_a	$\left[F^1_{fa}, F^2_{fa}, F^3_{fa}, T^1_a, T^2_a, T^3_a\right]^T$ defined by equation (34).
$K(\omega)$	6 x 6 diagonal stiffness matrix defined by equation (31).
L^x_i, L^y_i, L^z_i	initial values of $x_i y_i z_i$.
L^j	n x 1 matrix of j^{th} direction initial values of x_i, y_i, or z_i, for each of n masses e.g. $L^1 = \left[L^x_1, L^x_2, \ldots L^x_n\right]^T$.
$\left[L^j\right]$	$\left[L^1, L^2, L^3\right]^T$
m_i	i^{th} mass of lumped structure model.
M^{rs}_a, M^{rs}_{La}	scalars defined by equations (24) and (25).
M	n x n diagonal mass matrix with $m_1, m_2 \ldots m_n$ on the diagonal.
M_f	6 x 6 diagonal mass matrix for foundation defined by equation (29).
P^i_a, P^{ij}_{aL}	participation factors defined by equation (13).
$q_a, q_a(t)$	scalar a^{th} time-dependent modal coordinate defined from equation (9).
$\vec{r}_i$	position vector from XYZ origin to m_i.
$\vec{R}$	position vector from XYZ origin to xyz origin.
t	time
T^r_{ja}	r^{th} component of movement about xyz origin due to force on m_j, in mode a.
T^r_a	$\sum_{j=1}^{n} T^r_{ja}$
u, v, w	components of $\vec{R}$ in XYZ coordinates
u^1, u^2, u^3	n x 1 matrix with all components u, v, or w. e.g. $u^1 = u\left[1, 1, \ldots 1\right]^T$.
$\ddot{U}_g$	6 x 1 free-field acceleration matrix defined by equation (28).
$\ddot{U}_r$	6 x 1 relative acceleration matrix defined by equation (30).
$x_i y_i z_i$	xyz coordinates locating m_i
x^1, x^2, x^3	n x 1 matrices with components x_i, y_i, or z_i. e.g. $x^1 = \left[x_1, x_2, \ldots x_n\right]^T$.

Nomenclature (cont.)

X_i, Y_i, Z_i	XYZ coordinates locating m_i.
X^1, X^2, X^3	n x 1 matrices with components X_i, Y_i, or Z_i. e.g. $X^1 = \left[X_1, X_2 \ldots X_n\right]^T$
$\bar{X}_a^i$	n x 1 matrix representing the $a^{\underline{th}}$ mode shape in the $i^{\underline{th}}$ direction. Defined by equation (8).
$\vec{\rho}_i$	position vector from xyz origin to m_i
ω_a	natural frequency of mode a. Defined by eigenvalues in equation (8).
$\theta_x, \theta_y, \theta_z$	components of Ω in XYZ coordinates.
Ω	angular velocity of the foundation.
δ^{ij}	n x n submatrix of the influence matrix relating n deflections in the $i^{\underline{th}}$ direction to the n forces in $j^{\underline{th}}$ direction. Defined by equation (6).
0	n x n matrix of zeros or above/below diagonal zeros of diagorial matrix depending on context.
$\left[0\right]$	n x 1 matrix of zeros.
$\left[1\right]$	n x 1 matrix of ones.
U_{ra}	6 x 1 relative acceleration matrix caused from mode a.

Introduction

In 1965 Cunniff and O'Hara published a report on three-dimensional normal mode theory which is used as a basis for naval shock design (1). In that report, the transient input motion to the foundation is considered to be translational. An appendix to the report also treats rotational effects from rotational inertia of the individual masses in the lumped structure model relative to a fixed base point. The purpose of this paper is to extend the first part of Cunniff and O'Hara's work to the case which includes rotational transient input to the foundation as well as the translational input under the assumptions that rotation takes place rigidly about the base point in the foundation and that rotational inertia effects of the individual masses in the lumped model can be neglected.

Normal mode methods have been used in seismic analysis for some time (2). Both spectrum methods and time-history analyses (3,4) have been employed. Usually the model is two-dimensional. The proposed method of analysis has the following advantages:

(1) All motions - translation, rotation of the foundation, torsional motion of the system - can be included in a systematic manner.

(2) Interaction equations which separate structural damping from radiation and friction damping at the soil-structure interface can be developed.

(3) Since the structural response to the foundation can be evaluated on a modal basis, the frequency dependence of the soil response can be approximated.

Acceleration of the Mass Points of the Structure Model

The multistoried structure is modeled as a lumped system. A coordinate system xyz fixed to the foundation translates and rotates with the foundation. The location of the concentrated masses m_i of the model relative to origin of the xyz system in the foundation is given by position vector $\vec{\rho}_i$. Another coordinate system XYZ fixed in space locates the masses m_i relative to the origin of the XYZ system by position vector r_i and the origin of the xyz system by $\vec{R}$. Then the position of m_i is given by

$$\vec{r}_i = \vec{R} + \vec{\rho}_i \tag{1}$$

and the acceleration of m_i by

$$\ddot{\vec{r}}_i = \ddot{\vec{R}} + \vec{\Omega} \times (\vec{\Omega} \times \vec{\rho}_i) + \dot{\vec{\Omega}} \times \vec{\rho}_i + \ddot{\vec{\rho}}_i + 2\vec{\Omega} \times \dot{\vec{\rho}}_i \tag{2}$$

where $\vec{\Omega}$ represents the angular velocity of the foundation. (c.f. fig. 1) It will be assumed that the normal acceleration and the Coriolis acceleration can be neglected relative to other terms of equation (2). Although this approximation seems reasonable for the anticipated seismic motion, the validity can be checked by calculating these values at each time increment and estimating the error incurred.

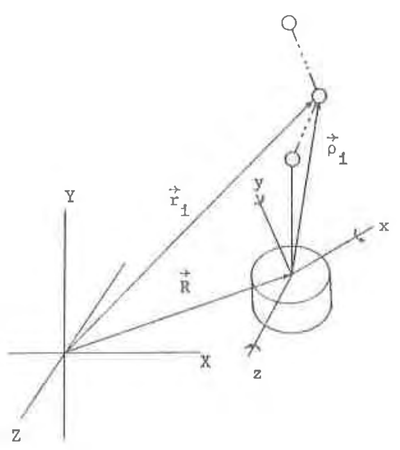

Fig. 1 - XYZ is a coordinate system fixed in space and xyz is a coordinate system fixed to the structure foundation.

In order to adapt normal mode theory to this model by the methods of Cunniff and O'Hara, it is necessary to drop these nonlinear terms. In addition it is assumed that unit vectors along the xyz axes can be approximated by those along XYZ axes in computing $\dot{\vec{\Omega}} \times \vec{\rho}_i$ and that distances between the m_i and the origin of the coordinate system fixed to the foundation can be approximated by their initial values. Under these assumptions, equation (2) can be simplified to

$$\begin{bmatrix} \ddot{X}_i \\ \ddot{Y}_i \\ \ddot{Z}_i \end{bmatrix} = \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \end{bmatrix} + \begin{bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{w} \end{bmatrix} + \begin{bmatrix} \ddot{\theta}_y L_i^z - \ddot{\theta}_z L_i^y \\ \ddot{\theta}_z L_i^x - \ddot{\theta}_x L_i^z \\ \ddot{\theta}_x L_i^y - \ddot{\theta}_y L_i^x \end{bmatrix} \tag{3}$$

The first term on the right side of equation (3) which is the acceleration of m_i relative to the foundation is to be determined by normal mode theory from the second and third terms which represent the translational motion of the foundation and the rotational motion of m_i about the foundation.

Normal Mode Theory

Equations of Motion

In the development of the equations in the sequel, rotary inertia effects of the m_i are neglected but the method of analysis can be extended to include them.

Since F^i represents the forces in the i^{th} direction due to the inertia of all the masses,

$$\begin{bmatrix} F^1 \\ F^2 \\ F^3 \end{bmatrix} = - \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \ddot{X}^1 \\ \ddot{X}^2 \\ \ddot{X}^3 \end{bmatrix} \tag{4}$$

Substitution from equation (3) into equation (4) and modifying the notation in conformance with the nomenclature, yields

$$\begin{bmatrix} F^1 \\ F^2 \\ F^3 \end{bmatrix} = - \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \left\{ \begin{bmatrix} \ddot{x}^1 \\ \ddot{x}^2 \\ \ddot{x}^3 \end{bmatrix} + \begin{bmatrix} \ddot{u}^1 \\ \ddot{u}^2 \\ \ddot{u}^3 \end{bmatrix} + \begin{bmatrix} \ddot{\theta}_y L^3 - \ddot{\theta}_z L^2 \\ \ddot{\theta}_z L^1 - \ddot{\theta}_x L^3 \\ \ddot{\theta}_x L^2 - \ddot{\theta}_y L^1 \end{bmatrix} \right\} \tag{5}$$

The deflection of the mass points relative to the coordinate system xyz fixed to the foundation is related to the forces acting on the masses by the flexibility matrix by equation (6).

$$\begin{bmatrix} x^1 \\ x^2 \\ x^3 \end{bmatrix} = \begin{bmatrix} \delta^{11} & \delta^{12} & \delta^{13} \\ \delta^{21} & \delta^{22} & \delta^{23} \\ \delta^{31} & \delta^{32} & \delta^{33} \end{bmatrix} \begin{bmatrix} F^1 \\ F^2 \\ F^3 \end{bmatrix} \tag{6}$$

where δ^{ij} is an n x n matrix relating the n deflections in the i^{th} direction to the n forces in the j^{th} direction. Multiplication of equation (5) by the flexibility matrix and use of equation (6) yields the equations of motion.

$$\begin{bmatrix} \delta^{11} & \delta^{12} & \delta^{13} \\ \delta^{21} & \delta^{22} & \delta^{23} \\ \delta^{31} & \delta^{32} & \delta^{33} \end{bmatrix} \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \ddot{x}^1 \\ \ddot{x}^2 \\ \ddot{x}^3 \end{bmatrix} + \begin{bmatrix} x^1 \\ x^2 \\ x^3 \end{bmatrix} = - \begin{bmatrix} \delta^{11} & \delta^{12} & \delta^{13} \\ \delta^{21} & \delta^{22} & \delta^{23} \\ \delta^{31} & \delta^{32} & \delta^{33} \end{bmatrix} \cdot$$

$$\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \left\{ \begin{bmatrix} \ddot{u}^1 \\ \ddot{u}^2 \\ \ddot{u}^3 \end{bmatrix} + \begin{bmatrix} \ddot{\theta}_y L^3 - \ddot{\theta}_z L^2 \\ \ddot{\theta}_z L^1 - \ddot{\theta}_x L^3 \\ \ddot{\theta}_x L^2 - \ddot{\theta}_y L^1 \end{bmatrix} \right\} \tag{7}$$

Modal Displacements

$\bar{X}^i_a$ represents the mode shape of the a^{th} mode in the i^{th} direction and satisfies equation (8).

$$\begin{bmatrix} \delta^{11} & \delta^{12} & \delta^{13} \\ \delta^{21} & \delta^{22} & \delta^{23} \\ \delta^{31} & \delta^{32} & \delta^{33} \end{bmatrix} \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix} = \frac{1}{(\omega_a)^2} \begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix} \tag{8}$$

If the 3n eigenvectors from equation (8) are linearly independent, then there exist 3n variables q_a such that

$$\begin{bmatrix} x^1 \\ x^2 \\ x^3 \end{bmatrix} = \sum_{a=1}^{3n} \begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix} q_a \tag{9}$$

The eigenvectors from equation (8) satisfy the orthogonality conditions of equation (10).

$$\begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix}^T \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \bar{X}^1_b \\ \bar{X}^2_b \\ \bar{X}^3_b \end{bmatrix} = 0, \; a \neq b$$

$$\begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix}^T \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \delta^{11} & \delta^{12} & \delta^{13} \\ \delta^{21} & \delta^{22} & \delta^{23} \\ \delta^{31} & \delta^{32} & \delta^{33} \end{bmatrix} \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \bar{X}^1_b \\ \bar{X}^2_b \\ \bar{X}^3_b \end{bmatrix} = 0, \; a \neq b \tag{10}$$

By substituting from equation (9) into equation (7) and using orthogonality equation (10), the equations of motion become

$$\ddot{q}_a + \omega_a^2 q_a = - \frac{\begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix}^T \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \left\{ \begin{bmatrix} \ddot{u}^1 \\ \ddot{u}^2 \\ \ddot{u}^3 \end{bmatrix} + \begin{bmatrix} \ddot{\theta}_y L^3 - \ddot{\theta}_z L^2 \\ \ddot{\theta}_z L^1 - \ddot{\theta}_x L^3 \\ \ddot{\theta}_x L^2 - \ddot{\theta}_y L^1 \end{bmatrix} \right\}}{D} \tag{11}$$

where

$$D = \begin{bmatrix} \bar{X}_a^1 \\ \bar{X}_a^2 \\ \bar{X}_a^3 \end{bmatrix}^T \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} \bar{X}_a^1 \\ \bar{X}_a^2 \\ \bar{X}_a^3 \end{bmatrix} \quad (12)$$

By introducing the nine modal participation factors defined from equation (13)

$$P_a^i = \frac{\bar{X}_a^{i^T} M [1]}{D}, \quad i = 1,2,3 \qquad P_{aL}^{ij} = \frac{\bar{X}_a^{i^T} M [L^j]}{D}, \quad \begin{matrix} i = 1,2,3 \\ j = 1,2,3 \\ i \neq j \end{matrix} \quad (13)$$

equation (11) can be rewritten as:

$$\ddot{q}_a + \omega_a^2 q_a = - P_a^1 \ddot{u} - P_a^2 \ddot{v} - P_a^3 \ddot{w} - P_{aL}^{32} - P_{aL}^{23} \ddot{\theta}_x - P_{aL}^{13} - P_{aL}^{31} \ddot{\theta}_y - P_{aL}^{21} - P_{aL}^{12} \ddot{\theta}_z \quad (14)$$

The solution of equation (14) together with conditions $q_a = \dot{q}_a = 0$ for $t = 0$ is

$$q_a(t) = - \frac{P_a^1}{\omega_a} \int_0^t \ddot{u}(\tau) \sin \omega_a (t-\tau) d\tau - \frac{P_a^2}{\omega_a} \int_0^t \ddot{v}(\tau) \sin\omega_a (t-\tau) d\tau - \frac{P_a^3}{\omega_a} \int_0^t \ddot{w}(\tau) \sin\omega_a (t-\tau) d\tau$$

$$- \left(\frac{P_{aL}^{32} - P_{aL}^{23}}{\omega_a} \right) \int_0^t \ddot{\theta}_x(\tau) \sin\omega_a (t-\tau) d\tau - \left(\frac{P_{aL}^{13} - P_{aL}^{31}}{\omega_a} \right) \int_0^t \ddot{\theta}_y(\tau) \sin\omega_a (t-\tau) d\tau$$

$$- \left(\frac{P_{aL}^{21} - P_{aL}^{12}}{\omega_a} \right) \int_0^t \ddot{\theta}_z(\tau) \sin\omega_a (t-\tau) d\tau \quad (15)$$

Structural damping can be included in the solution by inserting the factor $e^{-\xi(t-\tau)}$ where ξ is the damping coefficient into the integrals of equation (15).

Inertia Forces

Inertia forces are defined by equation (5). Since the $\bar{X}_a^i$ are constant, differentiating equation (9) gives the relative accelerations in terms of the modal coordinates.

$$\left[\ddot{x}^1, \ddot{x}^2, \ddot{x}^3 \right]^T = \sum_{a=1}^{3n} \left[\bar{X}_a^1, \bar{X}_a^2, \bar{X}_a^3 \right]^T \ddot{q}_a \quad (16)$$

By substituting $\ddot{q}_a$ from equation (14) into equation (16),

$$\ddot{x}^1, \ddot{x}^2, \ddot{x}^3 {}^T = \sum_{a=1}^{3n} \left[\bar{X}^1, \bar{X}^2, \bar{X}^3 \right]^T \left(-\omega_a^2 q_a - P_a^1 \ddot{u} - P_a^2 \ddot{v} - P_a^3 \ddot{w} - \left(P_{aL}^{32} - P_{aL}^{23} \right) \ddot{\theta}_x - \left(P_{aL}^{13} - P_{aL}^{31} \right) \ddot{\theta}_y - \left(P_{aL}^{21} - P_{aL}^{12} \right) \ddot{\theta}_z \right) \quad (17)$$

The following identities are used to simplify equation (17). The first of these identities appears in reference (1).

$$\sum_{a=1}^{3n} \bar{X}_{ia}^r P_a^s = \begin{cases} 0, & r \neq s \\ 1, & r = s \end{cases}$$

$$\sum_{a=1}^{3n} \bar{X}_{ia}^r \left(P_{aL}^{32} - P_{aL}^{23} \right) = \begin{cases} 0, & r = 1 \\ -L_i^3, & r = 2 \\ L_i^2, & r = 3 \end{cases} \quad (18)$$

with cyclic permutation of 123 indices yielding two similar equations.

By using the identities of equations (18), equation (17) can be simplified.

$$\left[\ddot{x}^1,\ \ddot{x}^2,\ \ddot{x}^3\right]^T = -\sum_{a=1}^{3n}\left[\bar{X}_a^1,\ \bar{X}_a^2,\ \bar{X}_a^3\right]^T \omega_a^2 q_a - \left[\ddot{u}^1,\ \ddot{u}^2,\ \ddot{u}^3\right]^T -$$

$$\left[\ddot{\theta}_y L^3 - \ddot{\theta}_z L^2,\ \ddot{\theta}_z L^1 - \ddot{\theta}_x L^3,\ \ddot{\theta}_x L^2 - \ddot{\theta}_y L^1\right]^T \tag{19}$$

And as a result of substituting from equation (19) into equation (5), the inertia forces can be written as

$$\left[F^1,\ F^2,\ F^3\right]^T = \left[\delta_{ij} M\right] \sum_{a=1}^{3n} \left[\bar{X}^1,\ \bar{X}^2,\ \bar{X}^3\right]^T \omega_a^2 q_a \tag{20}$$

The force acting on mass m_j in mode a and in direction r is

$$F_{ja}^r = m_j\ \bar{X}_{ja}^r\ \omega_a^2 q_a \tag{21}$$

Reactions to the Foundation

By summation of the inertia forces in mode a, the shear force in direction r to the foundation in mode a is determined

$$F_{fa}{}^r = \sum_j^n F_{ja}{}^r = \sum_j^n m_j\ \bar{X}_{ja}^r\ \omega_a^2 q_a \tag{22}$$

Substitution of q_a from equation (15) yields $F_{fa}{}^r = -M_a^{r1}\omega_a \int_o^t \ddot{u}(\tau)\sin\omega_a(t-\tau)d\tau \ \ldots$

$$- M_{La}^{r1}\ \omega_a \int_o^t \ddot{\theta}_x(\tau)\sin\omega_a(t-\tau)d\tau \ \ldots \tag{23}$$

where the modal masses are defined as (1) $\qquad M_a^{rs} = \sum_{j=1}^n m_j\ \bar{X}_{ja}^r\ P_a^s \qquad (24)$

and the terms associated with the rotation are defined as

$$M_{La}^{r1} = \sum_j m_j\ \bar{X}_{ja}^r \left(P_{aL}^{32} - P_{aL}^{23}\right) \tag{25}$$

with cyclic permutation of 123 indices yielding two similar equations.

The inertia moments to the foundation are determined in a similar manner. From forces acting on m_j, the moment to the foundation is

$$\left[T_{ja}^1, T_{ja}^2, T_{ja}^3\right]^T = \left[L_j^y\ F_{ja}^3 - L_j^z\ F_{ja}^2,\ L_j^z\ F_{ja}^1 - L_j^x\ F_{ja}^3,\ L_j^x\ F_{ja}^2 - L_j^y\ F_{ja}^1\right]^T \tag{26}$$

Thus for the orthogonal directions, the moment reactions to the foundation in mode a

$$T_a^1 = \sum_j^n m_j \left(\bar{X}_{ja}^3 L_j^y - \bar{X}_{ja}^2 L_j^z\right) \omega_a^2 q_a \tag{27}$$

with cyclic permutation of 123 and xyz yielding two more similar equations.

Interaction Equations

From knowledge of the inertia reactions to the foundation, shear forces, participation factors, and moments, interaction equations can be developed. Let the free-field ground acceleration be

$$\ddot{U}_g = \left[\ddot{u}_g,\ \ddot{v}_g,\ \ddot{w}_g,\ o,\ o,\ o\right]^T \tag{28}$$

Free-field ground rotation is assumed to be insignificant. Let the diagonal mass matrix of the foundation be $M_f = \left[\alpha_i \delta_{ij}\right]$ (29) where $\alpha_1 = \alpha_2 = \alpha_3 = m_o$ and $\alpha_4 = I_o^1$, $\alpha_5 = I_o^2$, $\alpha_6 = I_o^3$

The absolute foundation accelerations are related to the free-field accelerations through relative accelerations, $\ddot{U}_r$.

$$\left[\ddot{u},\ddot{v},\ddot{w},\ \ddot{\theta}_x,\ddot{\theta}_y,\ddot{\theta}_z\right]^T = \left[\ddot{u}_r,\ddot{v}_r,\ddot{w}_r,\ \ddot{\theta}_x,\ddot{\theta}_y,\ddot{\theta}_z\right]^T + \left[\ddot{u}_g,\ddot{v}_g,\ddot{w}_g,\ o,o,o\right]^T \qquad (30)$$

The soil response can be a solution to the Lamb problem. Often, equivalent springs are employed. Below, a method of employing frequency-dependent equivalent springs to represent the soil response is discussed. Let the soil springs be defined by a diagonal stiffness matrix. Thus static coupling between rotation and translation of the foundation is neglected. It has been shown by Jennings and Bielak (3) that these terms are not significant.

$$K(\omega) = \left[K_i \delta_{ij}\right] \qquad (31)$$

Damping at the soil-structure interface can also be approximated by a diagonal matrix.

$$C(\omega) = \left[c_i \delta_{ij}\right] \qquad (32)$$

The equations of motion can be written as

$$M_f \ddot{U}_r + C(\omega)\,\dot{U}_r + K(\omega) U_r = \sum_{a=1}^{3n} F_a - M_f \ddot{U}_g \qquad (33)$$

where F_a are the dynamic forces from the n-mass structure acting on the foundation in mode a (i.e. equations (23) and (27)).

$$F_a = \left[F_{fa}^1,\ F_{fa}^2,\ F_{fa}^3,\ T_a^1,\ T_a^2,\ T_a^3\right]^T \qquad (34)$$

The assumption made in this procedure is that the stiffness of the soil response can be approximated by using the relationships developed by Arnold, Bycroft and Warbuton (4,5) evaluated at the fixed-base frequencies of the structure. Thus, relative motion caused by the response from each mode can be calculated.

$$M_f \ddot{U}_{ra} + C(\omega_a)\,\dot{U}_{ra} + K(\omega_a)\,U_{ra} = F_a - \delta_{ab} M_f \ddot{U}_g \qquad (35)$$

The $M_f \ddot{U}_g$ term is applied only to the dominant mode ω_b in the direction of component of the free field acceleration.

Discussion

Inertia reactions to the foundation caused by three dimensional time-dependent translations and rotations of the foundation can be evaluated on a modal basis by employing equations (23) and (27). The modal frequencies of these equations are the fixed-base frequencies of the structure and not the true natural frequencies. As a result, it is the fixed-base frequencies that are meaningful in spectrum curves of the foundation motion.

Interaction equations can be formulated by coupling the structure to the soil by using equivalent soil springs as indicated in equations (28) to (35) or by using solutions of the

Lamb problem. Previously the authors coupled an n-mass system (neglecting rocking) to a plane strain elastic half space in references (6) and (7). This model has also been studied by finite element analyses (8, 9, 10). In order to compare the results of the present analysis to the results of these references, the same three-mass two-mode structural model is examined.

The three masses of the structure are (Fig. (2)):

(1) The base mass ($M_f = 2{,}400{,}000 \frac{\text{lb.sec.}}{\text{ft.}}$)

(2) The containment vessel mass ($M_1 = 475{,}000 \frac{\text{lb. sec.}}{\text{ft.}}$)

(3) The reactor pressure vessel mass ($M_2 = 310{,}000 \frac{\text{lb.sec.}}{\text{ft.}}$)

The fixed-base frequencies of the containment vessel mass and reactor vessel mass are 4 cps and 5 cps respectively. The equivalent foundation radius is taken to be 60 feet.

In the numerical results presented herein, plane motion in the XY plane with rotation about the Z axis is assumed. Ground motion is based on the accelerograph record of the March 22, 1957 San Francisco earthquake recorded at Golden Gate Park. A graph of this input is shown on Figure 3. Using equation (35), the lateral foundation acceleration $\ddot{u}$ and rotational acceleration $\ddot{\theta}_z$ is determined. These time-history motions are plotted on Figures 4 and 5 respectively. The calculated spectrum response of the lateral acceleration is plotted on Figure 6. It should be noted that there is a significant dip in these calculated spectra at the fixed-base structure frequencies (4 cps and 5 cps).

By comparing the results of Figures (4) and (5) it can be observed that the foundation rotation is out-of-phase with the base translation. Thus, the peak acceleration of the containment vessel mass will be reduced by the phase relationships between the foundation translation and foundation rotation. This aspect of the interaction phenomenon has been described in detail in reference (7).

As seen on Figure 6, free-field spectra are exceeded by foundation spectra at low frequencies. These results are consistent with those obtained by Isenberg (8).

A comparison of the lateral spectra of this analysis and other computations is presented on Table I. In the table, the acceleration spectrum at the fixed-base frequencies is divided by free-field acceleration spectra at each of these frequencies. The present analysis indicates a reduction of the spectra by a factor of about 2. These results are in close agreement with those obtained from finite element analysis (8, 9, 10). Calculated interaction effects from using a plane-strain half-space solution are less than those obtained with the present analysis.

Conclusions

By neglecting normal accelerations and the Coriolis components of mass points, normal mode theory is applied to a structure with foundation rotation as well as foundation translation. As a result all reactions to the foundation of a structure, the orthogonal force and moment components are computed on a modal basis. By using these reactions, interaction equations are developed. Damping at the structure-ground interface is separated from structural damping.

A method of analysis is proposed in which the dynamic response of the soil is approximated by a diagonal stiffness matrix and a diagonal damping matrix. These frequency dependent matrix elements are evaluated by using the fixed-base structural frequencies.

The lateral foundation acceleration spectra are compared with previous computations. Reductions in lateral spectra calculated with this model are similar to those obtained with the referenced analyses for equivalent soil stiffness.

Appendix

Derivation of the Second of Equation (18)

In order to relate P^1_a to a particular $q_a(t)$ and therefore to a particular $\left[x^1, x^2, x^3\right]^T$ through equation (9), equation (15) can be specialized by making the following choice of translational and rotational accelerations - let $\ddot{\theta}_x(t)$ be replaced by $\dot{\theta}_1\delta(t)$ where $\dot{\theta}_1$ is constant and $\delta(t)$ is the Dirac delta function and let $\ddot{u}(t) = \ddot{v}(t) = \ddot{w}(t) = \ddot{\theta}_y(t) = \ddot{\theta}_z(t) = 0$. For $t = \Delta t$ and for this choice of the accelerations, equation (15) yields the $q_a(t)$ corresponding to $\left[x^1, x^2, x^3\right]^T = \left[[o], L^3\dot{\theta}_1\Delta t, -L^2\dot{\theta}_1\Delta t\right]^T$. Then equation (9) specializes to

$$\begin{bmatrix} [o] \\ L^3\dot{\theta}_1\Delta t \\ -L^2\dot{\theta}_1\Delta t \end{bmatrix} = \sum_{a=1}^{3n} \begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix} - \frac{\left(P^{32}_{aL} - P^{23}_{aL}\right)}{\omega_a} \int_o^{\Delta t} \dot{\theta}_1 \delta(\tau) \sin\omega_a(\Delta t - \tau)\, d\tau$$

$$= - \sum_{a=1}^{3n} \begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix} \frac{\left(P^{32}_{aL} - P^{23}_{aL}\right) \dot{\theta}_1 \sin\omega_a \Delta t}{\omega_a}$$

After dividing by $\theta_1\Delta t$ and letting $\Delta t \rightarrow o$, this equation gives

$$\begin{bmatrix} [o] \\ -L^3 \\ L^2 \end{bmatrix} = \sum_{a=1}^{3n} \begin{bmatrix} \bar{X}^1_a \\ \bar{X}^2_a \\ \bar{X}^3_a \end{bmatrix} \left(P^{32}_{aL} - P^{23}_{aL}\right)$$

which implies

$$\sum_{a=1}^{3n} \bar{X}^1_{ia} \left(P^{32}_{aL} - P^{23}_{aL}\right) = 0, \qquad \sum_{a=1}^{3n} \bar{X}^2_{ia} \left(P^{32}_{aL} - P^{23}_{aL}\right) = -L^3_i$$

$$\sum_{a=1}^{3n} \bar{X}^3_{ia} \left(P^{32}_{aL} - P^{23}_{aL}\right) = L^2_i$$

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TABLE 1

Comparison of Interaction Computations of a Three-Mass Two-Mode Reactor Model

Method of Analysis	Free-Field Motion	A_b / A_{ff} f_1=4 cps	f_2=5 cps	V_s ft/sec	Remarks
Present Analysis	G.G.P.	1.01	0.62	1000	Surface Structure
Plane Strain Half Space (6,7)	G.G.P.	0.27	0.41	1000	Surface Structure
Finite Element Case III (8,9)	G.G.P.	0.50	0.63	875	Embedded Structure
Finite Element Case V (8,9)	G.G.P.	0.59	0.51	1440	Embedded Structure
Finite Element Model I (10)	Computed*	0.38	0.16	802	Surface Structure
Finite Element Model II (10)	Computed*	0.39	0.32	802	Surface Structure
Finite Element Model III(10)	Computed*	0.48	0.22	802	Embedded Structure

A_b = Acceleration response spectra of the foundation motion at the fixed-base frequencies
A_{ff} = Acceleration response spectra of the free-field motion at either of the fixed-base frequencies
V_s = Shear wave velocity
f_1 = Fundamental fixed-base frequency (4 cps)
f_2 = Second mode fixed-base frequency (5 cps)
G.G.P. = S 80E ground motion recorded at Golden Gate Park during the March 1957 San Francisco earthquake
* = Free-field motion at the foundation site is determined by the program (9, 10)

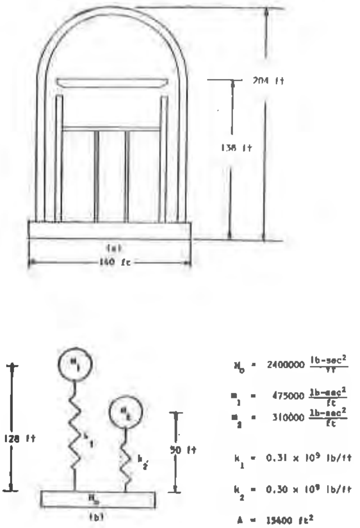

Figure 2 - The nuclear power plant shown in Figure 2a has been idealized as indicated in Figure 2b. The masses M_o, m_1, and m_2 represent the base, containment vessel, and internal structure, respectively.

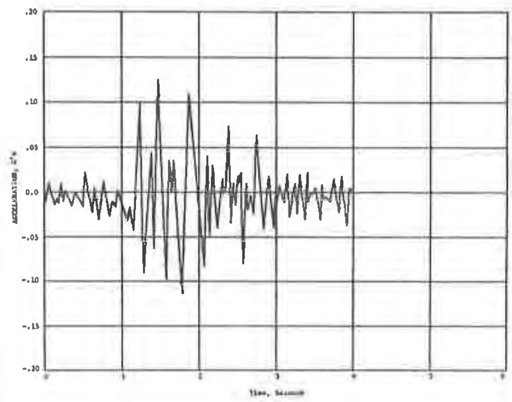

Figure 3 - East-west ground acceleration recorded at Golden Gate Park during the March 1957 San Francisco earthquake.

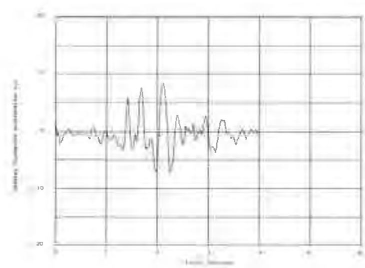

Figure 4 - The lateral foundation acceleration response $\ddot{v}(t)$ to the earthquake input $\ddot{v}_g(t)$ is plotted as a function of time.

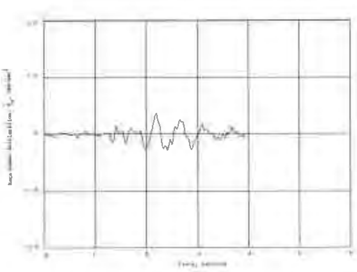

Figure 5 - The rotational foundation acceleration response $\ddot{\theta}_z(t)$ to the earthquake input $\ddot{v}_g(t)$ is plotted as a function of time.

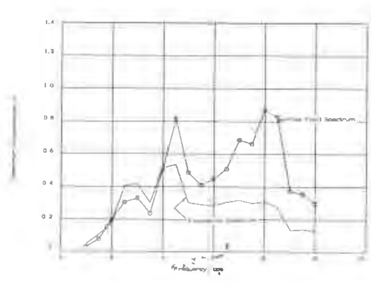

Figure 6 - The acceleration spectrum response of the lateral foundation acceleration $\ddot{v}$ (t) is compared to the acceleration spectrum response of the earthquake input acceleration $\ddot{v}_g(t)$.