

Stress Analysis in Welded Butt Joint

O. S. YAHSI, Saban YAZICI
Middle East Technical University, Ankara, Turkey

ABSTRACT

In this paper the contact problem for two bonded dissimilar homogeneous elastic strips is considered. Two strips are assumed to be perfectly bonded through a finite length. The lower strip is assumed to have two symmetric cracks perpendicular to the interface and subject to uniform tensile stresses far from the contact zone. The problem is formulated by using integral transform techniques and stress intensity factors for different geometries and material combinations are calculated.

1. INTRODUCTION

The problem of great importance is that of the bonding of materials with different elastic properties to one another, thereby forming a joint. The history of this kind of problems began with the work by Williams [1] and was developed by several researchers (e. g. [2, 3]). The problem of a line bond between two layers has been considered by Keer [4], but numerical results are given there only for the case of identical layers. Recently Yahsi and Göçmen [5] have considered a finite contact problem for two perfectly bonded dissimilar infinite strips.

In this study elastic continuum, isotropic, infinite strips of different material properties are considered. Two strips are perfectly bonded to each other through a finite length and the lower strip which contains two symmetric cracks as shown in Fig. 1 is acted on by forces far from the interface.

To solve the problem, the stress and displacement fields of an elastic infinite strip are obtained from the plane elasticity equations for an isotropic material by using Fourier transform techniques and the solution for a pair of point dislocations in infinite media is added to the lower strip solution. Finally by using the appropriate boundary conditions the problem is reduced to a system of two singular integral equations which are then solved numerically.

2. FORMULATION OF THE PROBLEM

The problem to be considered in this paper is illustrated in Fig. 1

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Instead of dislocations if the lower strip contains cracks along $c < x < d$, $y_0 = a = \text{constant}$ as shown in Figure 1 under a given set of surface tractions as stated in the equations (6a, b), by integrating the solution found for dislocations one would obtain the solution for the cracks.

By using the stress and displacement equations for the lower and upper strips given in [6] and the boundary conditions (1-4) and equations (7) and (8a, b) all the unknowns of the problem can be expressed in terms of $f_i(x)$, $G_i(y)$, ($i = 1, 2$). Thus the complete solution of the problem is obtained once the unknown functions f_1 , f_2 , G_1 , and G_2 , are determined. Finally, by substituting the stress and displacement expressions obtained in terms of $f_i(x)$, $G_i(y)$, ($i = 1, 2$), into the continuity conditions (5a, b) and the crack surface boundary conditions (6a, b) one would obtain the system of four singular integral equations given in [6] to determine the functions $f_i(x)$ and $G_i(y)$, ($i = 1, 2$).

By extending the cracks to the upper and lower boundaries of the lower strip (i.e. $c = 0$, $d = h_2$) and increasing the value of "1" the problem can be reduced to a lap joint under the effect of uniform stresses. For this case by introducing the following substitutions.

$$x = 0.5 h_2 (1 + \tau), \quad -1 < \tau < +1, \quad (9a)$$

$$t_2 = 0.5 h_2 (1 + \rho), \quad -1 < \rho < 1, \quad (9b)$$

$$t_1 = 0.5 (f + e) + 0.5 (f - e)\eta, \quad -1 < \xi < 1, \quad (9c)$$

$$y = 0.5 (f + e) + 0.5 (f - e)\eta, \quad -1 < \eta < 1, \quad (9d)$$

$$\phi(\xi) = G_2(\xi) + iG_1(\xi), \quad \psi(\tau) = f_1(\tau) + if_2(\tau), \quad i = \sqrt{-1}, \quad (10a, b)$$

these singular integral equations are reduced into the following form:

$$\begin{aligned} \gamma \phi(\eta) + \frac{1}{\pi i} \int_{-1}^{+1} \frac{\phi(\xi)}{\xi - \eta} d\xi + \int_{-1}^1 \left\{ \psi(\rho) K_{11}(\rho, \eta) + \bar{\psi}(\rho) K_{12}(\rho, \eta) \right\} d\rho \\ + \int_{-1}^1 \left\{ \phi(\xi) K_{13}(\xi, \eta) + \bar{\phi}(\xi) K_{14}(\xi, \eta) \right\} d\xi = h_1(\eta), \quad -1 < \eta < 1, \end{aligned} \quad (11a)$$

$$\begin{aligned} \int_{-1}^{+1} \psi(\rho) K_{2S}(\rho, \tau) d\rho + \int_{-1}^1 \left\{ \psi(\rho) K_{21}(\rho, \tau) + \bar{\psi}(\rho) K_{22}(\rho, \tau) \right\} d\rho \\ + \int_{-1}^1 \left\{ \phi(\xi) K_{23}(\xi, \tau) + \bar{\phi}(\xi) K_{24}(\xi, \tau) \right\} d\xi = h_2(\tau), \quad -1 < \tau < 1, \end{aligned} \quad (11b)$$

where the expressions of kernels K_{ij} ($i = 1, 2$, $j = S, 1, 2, 3, 4$) and known functions $h_1(\eta)$ and $h_2(\tau)$ are given in [6].

3. NUMERICAL RESULTS AND DISCUSSION

The singular integral equations (11a, b) can be solved with the conditions

$$\psi(\pm 1) = 0, \quad \int_{-1}^1 \phi(\xi) d\xi = 2 \frac{p+Q}{f-e} \quad (12a, b, c)$$

by using the method given in [6] where the solution to equations (11a, b) are approximated by

$$\phi(\eta) = F(\eta) (1 - \eta)^{\alpha_1} (1 + \eta)^{\beta_1}, \quad \psi(\tau) = R(\tau) (1 - \tau)^{\alpha_2} (1 + \tau)^{\beta_2} \quad (13a, b)$$

$$\text{where } \alpha_1 = \frac{1}{2} - i\omega, \quad \beta_1 = -\frac{1}{2} + i\omega, \quad \alpha_2 = \beta_2 = -\frac{1}{2}, \quad \omega = \frac{1}{2\pi} \log \left(\frac{1 + \gamma}{1 - \gamma} \right) \quad (14 a, d)$$

and the unknown functions $F(\eta)$ and $R(\tau)$ are approximated by a series expansion of orthogonal polynomials with unknown coefficients.

By using the following definition of stress intensity factor

$$k_1(e) - ik_2(e) = \lim_{t \rightarrow e} (t - f)^{-\alpha_1} (e - t)^{-\beta_1} (G_1 - iG_2) \quad (15)$$

and equation (13 a) the normalized stress intensity factors can be obtained as follows:

$$k_1(e) - ik_2(e) = \frac{i}{2} \frac{F(-1)}{P+Q}, \quad k_1(f) - ik_2(f) = \frac{i}{2} \frac{F(+1)}{P+Q} \quad (16a, b)$$

Figures 2-5 give the stress intensity factors versus e/h_1 ratio for different combinations of Aluminium and Steel. From these figures one can see that the normal traction near the bond edges could also be tensile or compressive for certain combinations of h_2/h_1 and $(f - e)/h_1$.

It can be found that at $y = e$, first mode stress intensity factor increases with the increase in e/h_1 ratio. On the other hand, at $y = f$ end, first mode stress intensity factor decreases with the increase in e/h_1 ratio. It is also found that the material dependence of k_2 is much more stronger than k_1 .

REFERENCES

- Williams, M. L. (1959). The Stresses Around a Fault or Crack in Dissimilar Media, Bulletin of the Seismological Society of America, Vol. 49, pp. 199-204.
- Erdoğan, F. and Aksoğan, O. (1976). Bonded Half-Planes Containing an Arbitrarily Oriented Crack, International Journal of Solids and Structures, Vol. 10, pp. 569-585.
- Arin, K. and Erdoğan, F. (1971) Penny-Shaped Crack in an Elastic Layer Bonded to Dissimilar Half-Space, Vol. 9, pp. 9213-9232.
- Keer, L. M. (1974). Stress Analysis for Bonded Layers, Journal of Applied Mechanics, Vol. 41. pp. 679-683.
- Yahşi, O. S. and Göçmen, A.E. (1987). Contact Problem for Two Perfectly Bonded Dissimilar Infinite Strips, International Journal of Fracture, Vol. 34, pp. 161-177.
- Yazıcı, Ş. (1990). Stress Analysis in Welded Lap Joints, M. Sc. Thesis, Middle East Technical University, Ankara, Turkey.

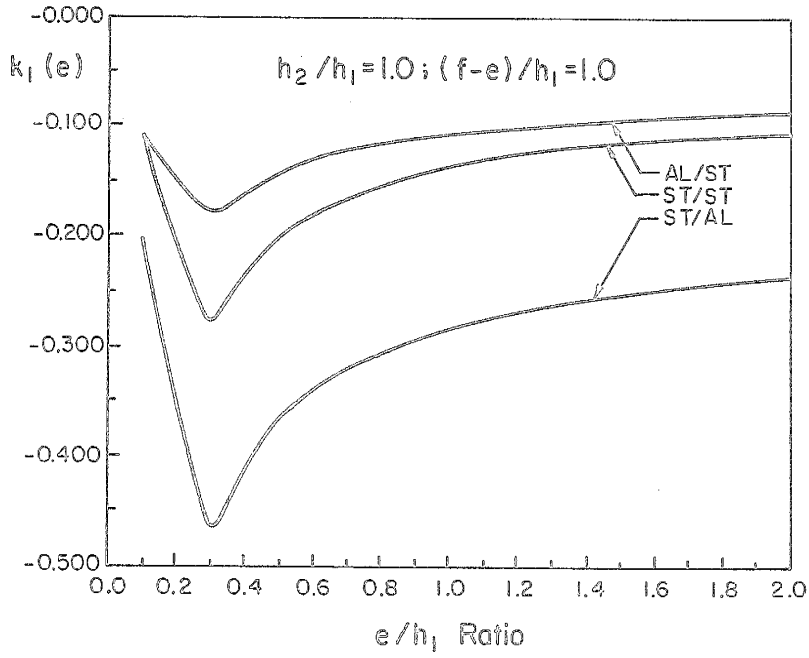


Figure 2. First Mode Stress Intensity Factor Ratios at the Contact End ($y=e$) of Perfectly Bonded Infinite Symmetric Strips.

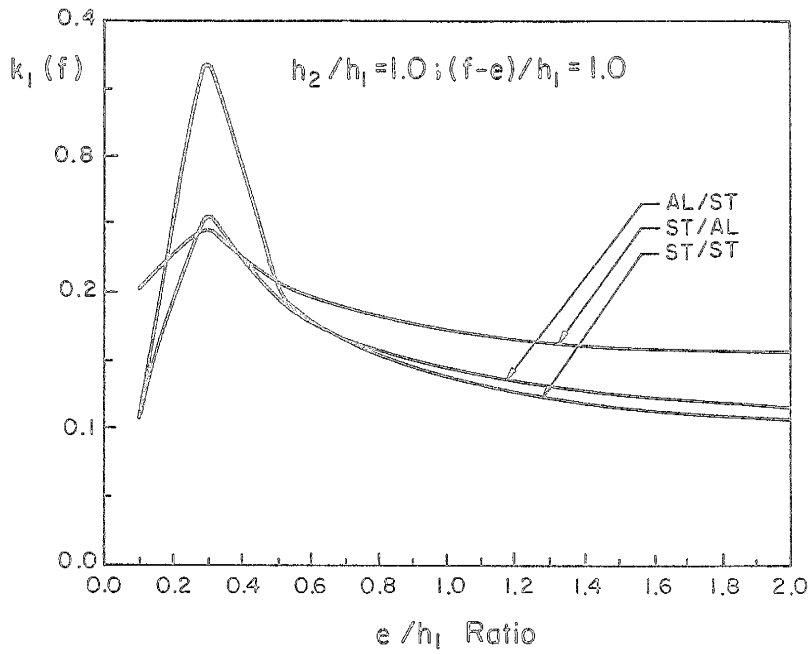


Figure 3. First Mode Stress Intensity Factor Ratios at the Contact End ($y=f$) of Perfectly Bonded Infinite Symmetric Strips.

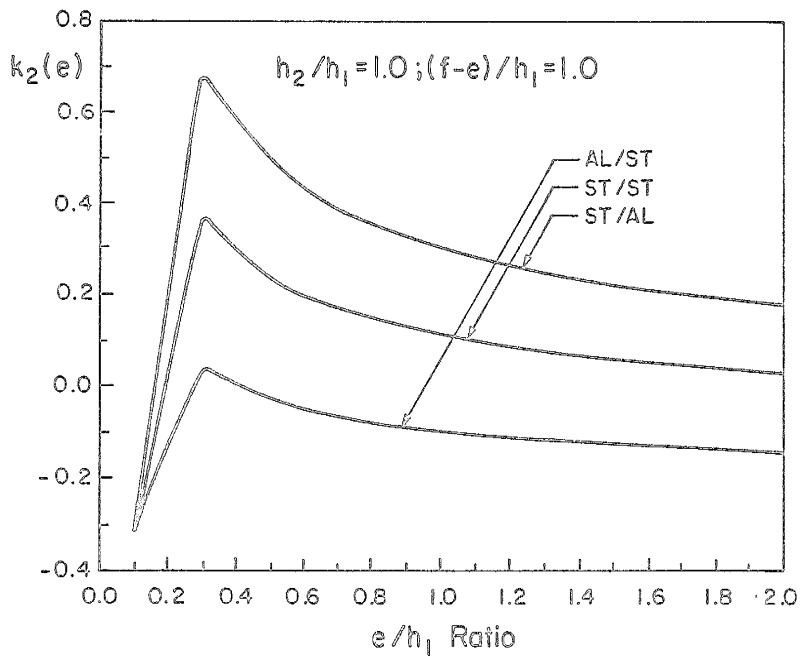


Figure 4. Second Mode Stress Intensity Factor Ratios at the Contact End ($y=e$) of Perfectly Bonded Infinite Symmetric Strips.

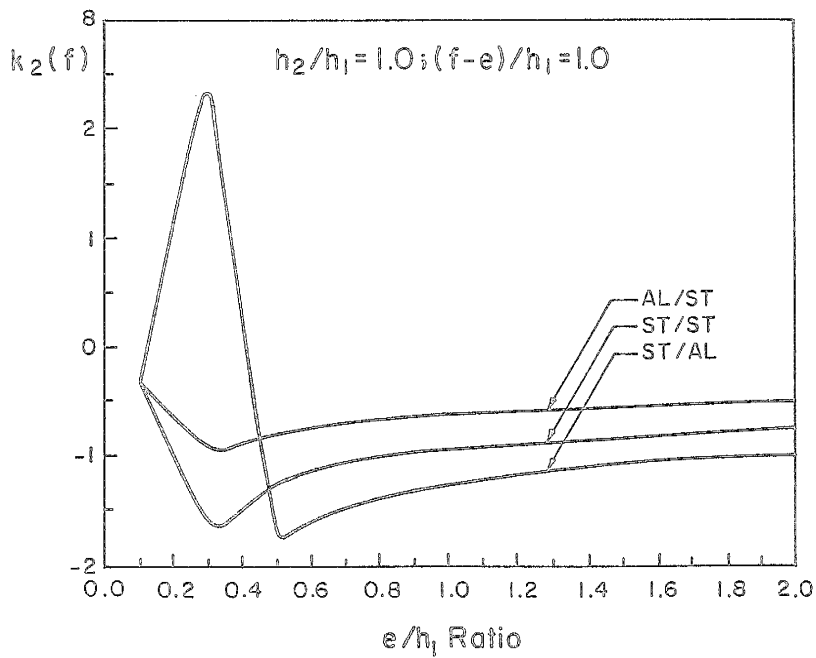


Figure 5. Second Mode Stress Intensity Factor Ratios at the Contact End ($y=f$) of Perfectly Bonded Infinite Symmetric Strips.