Evaluation of Kinematic Interaction of Soil-Foundation Systems by Boundary Element Method

Y. Huh, G. Schmid
Institut für Konstruktiven Ingenieurbau, AG IV, Ruhr-Universität Bochum, Postfach 10 21 48, D-4630 Bochum 1, Germany

M. Ottenstreuer
Ottenstreuer und Beckedahl, Kreuzstr. 45, D-5608 Radevormwald, Germany

SUMMARY

The boundary element method enables solution of the mixed boundary value problem in closed and open domain. The solution includes, in contrast to the finite element method, the radiation boundary condition if the domain extends to infinity.

Unknown displacements or stresses can be calculated on the soil-structure interface, on the surface or in the interior of the subgrade. Since the fundamental solution for the FOURIER transformed wave equation does not have to satisfy the boundary conditions - except at infinity - the method is applicable to imbedded foundations and to layered soil, as boundary and interface conditions are enforced by the boundary integral equations.

Internal damping of each soil layer may be introduced by a complex modulus of elasticity.

The boundary element formulation leads first to a displacement/traction relation for limb soil-structure interfaces. Through simple matrix manipulation the dynamic stiffness matrix of a limb foundation may be derived. It could be used in this form to handle mass and stiffness coupling with the structure. Structure-structure interaction does not bring in any more complexity other than higher demands on computer time and storage für the numerical calculation. If the foundation can be considered as rigid, kinematic restrictions on the soil-structure interface are introduced to condense the dynamic stiffness matrix to the rigid body degrees of freedom before coupling with the structure is performed.

In the paper parametric studies are presented for rigid foundations. These include comparison of compliance coefficients of slender 3-dimensional and strip foundations, the effect of topographical disturbances of the soil surface and the influence of embedment on structure-structure interaction, calculated here for two-dimensional problems.

1. Introduction

Soil-structure interaction is commonly performed in the frequency domain, where in a multistep analysis the structure and the soil are treated as separate substructures. The structure including its footing is usually discretized by standard finite element techniques. The soil, with a soil-structure interface of the same shape as the actual foundation, may be discretized by one of the three methods:

- the finite element method with transmitting boundary conditions,
- the half-space method,
- the boundary element method.

The first two methods have been applied successfully and applications can be found in IDRISS et al. /1/. The third method is relatively new in its application to soil-foundation systems and is described in DOMINGUEZ /2/ and OTTENSTREUER and SCHMID /3/. Extensions and additional applications are given in this paper.

2. Boundary integral representation

We assume that the soil is a linear elastic or visco-elastic continuum with finite od infinite extension. In the frequency domain its spatial behaviour is described by the FOURIER transformed wave equation subjected to specified displacement and traction constraints along the boundary, and to the radiation boundary condition if the body extends to infinity. Applying BETTI's reciprocity theorem to the boundary value problem (see Fig. 1) leads to the boundary integral representation, here written in index notation,

$$\int_B \overset{*}{p}_i \; u_i \; dB + \int_\Gamma \overset{*}{t}_i \; u_i \; d\Gamma = \int_B p_i \; \overset{*}{u}_i \; dB + \int_\Gamma t_i \; \overset{*}{u}_i \; d\Gamma , \tag{1}$$

where u_i, t_i, p_i are the components of displacements, tractions and body forces, respectively.

Let $\overset{*}{p}_j$ be a singularity of the body force, equivalent to a unit point load at x_i^α in the direction j and let $\overset{*}{u}_{ij}$ and $\overset{*}{t}_{ij}$ be the corresponding free-field GREEN's functions, i.e. the fundamental solution ef eq. (1) in the full-space. Then, neglecting body forces p_i, eq. (1) leads to the boundary integral equation

$$c \; u_i(x^\alpha) + \fint_\Gamma \overset{*}{t}_{ij}(x,x^\alpha) \; u_i(x) \; d\Gamma = \int_\Gamma \overset{*}{u}_{ij}(x,x^\alpha) \; t_i(x) \; d\Gamma , \tag{2}$$

with $x, x^\alpha \in \Gamma$.

The constant c depends on the singularity of the kernel $\overset{*}{t}_{ij}$ and on the smoothness of the boundary. $\fint$ represents CAUCHY's principal value. Details are given by HARTMANN /4/.

Note, that the inertia forces are retained in the fundamental solution. The fundamental solution is given e.g. in CRUSE and RIZZO /5/.

Since the fundamental solution satisfies the radiation boundary condition, the integrals in eq. (2) extend only along the 'finite part' of the body, where displacements or tractions are specified.

Considerable further simplification may be achieved if GREEN's functions can be found which satisfy the homogeneous boundary conditions. In this case the left hand side integral in equation (2) extends only over that part of the boundary where displacements are given, the right hand side integral extends only over the part where tractions are specified.

In the problem shown we have essentially a boundary value problem of the 2nd kind, with loads prescribed along the surface of the soil, being zero outside the foundation, and with kineamtic constraints along the soil-struture interface.

Eq. (2) now reduces to

$$u_i(x^\alpha) = \int_{\Gamma_I} \overset{*}{u}_{ij}(x,x^\alpha)\; t_j(x)\; d\Gamma \qquad x,x^\alpha \in \Gamma_I \tag{3}$$

In eq. (3) the index I stands for the soil-structure interface Γ_1 and t_j is the unknown contact force, influenced by the kinematic constraints of the foundation.

RÜCKER /6/ used an equivalent formulation to calculate kinematic interaction of surface foundations. In /6/ the GREEN's function for a point load in half-space is given.

3. Boundary element discretization

We subdivide the surface Γ into discrete segments and expand the unknown displacement and traction functions in polynomials

$$\begin{aligned} u_j(x) &= \Omega^\beta(x)\; u_j^\beta \qquad x \in \Gamma_2 \\ t_j(x) &= \Omega^\beta(x)\; t_j^\beta \qquad x \in \Gamma_1 \end{aligned} \tag{4}$$

where Ω^β is the patch function in the patch β.

Eq. (4) into (2) yields the boundary element equation

$$c\; u_i(x^\alpha) + \int_\Gamma \overset{*}{t}_{ij}(x,x^\alpha)\; \Omega^\beta\; d\Gamma\; u_j^\beta = \int_\Gamma \overset{*}{u}_{ij}(x,x^\alpha)\; \Omega^\beta\; d\Gamma\; t_j, \tag{5}$$

or in matrix notation

$$\underline{T}\; \underline{u} = \underline{U}\; \underline{t}. \tag{6}$$

Similarly, eq. (4) into (3) would reduce to

$$\underline{u} = \underline{U}\; \underline{t}. \tag{7}$$

The formulation, given in eqs. (5) and (6) is, with the free-field GREEN's function, valid for any bounded or unbounded homogeneous body with or without interior boundaries. The discretization with boundary elements should extend over the boundary segments Γ_1 and Γ_2 (see Fig. 1). Numerical experiments have shown so far that it is often sufficient to discretize only the soil-structure interfarce or a relatively small area outside the foundation. But more experience has to be gathered. This is especially necessary for stratified soil, where additional discretization has to be introduced along the soil interfaces.

The formulation used e.g. by RÜCKER /6/ where the response of the half-space due to a harmonic point force has been chosen as GREEN's function (eqs. (3) and (7)), is applicable only to surface foundations on the homogeneous half-space. Its advantage, however, is that only the foundation area has to be discretized.

It is interesting to mention here the so called 'decoupled formulation', which can be derived from eqs. (5) and (6). OTTENSTREUER and SCHMID /3/ used it for surface foundations on the half-space. The assumption that vertical and horizontal vibrations are decoupled implies that the $\underline{T}$-matrix in eq. (6) is a scalar matrix and that the discretization outside the foundation has no contribution and may therefore be ommitted. The matrix form corresponds here to eq. (7).

4. Dynamic stiffness of limb and rigid foundations

We assume for the following that in the boundary element the patch function is equal to one. u_j^β and t_j^β may then be interpreted as the displacement and traction component in the center of the element β. Let the area of the element be A^β and the diagonal matrix $\underline{A}$ the areas of all elements.

With the definition of the element (nodal) forces

$$\underline{P} = \underline{A}\ \underline{t} \tag{8}$$

the force displacement relation

$$\underline{u} = \underline{F}\ \underline{P} \quad \text{or} \quad \underline{P} = \underline{K}\ \underline{u} \tag{9}$$

follows.

The compliance $\underline{F}$ and the impedance $\underline{K}$ for a limb soil-structure interface are given by

$$\underline{F} = \underline{T}^{-1}\ \underline{U}\ \underline{A}^{-1} \quad \text{and} \quad \underline{K} = \underline{A}\ \underline{U}^{-1}\ \underline{T}. \tag{10}$$

The dimension of $\underline{F}$ or $\underline{K}$ can always be condensed to the number of the degrees of freedom of the soil-structure interface.

The dynamic stiffness matrix $\underline{K}$ may be used to couple the soil with the mass and stiffness of an elastic structure, using standard coupling techniques.

If the foundation can be considered as rigid the dynamic stiffness can be condensed further to the rigid body degrees of freedom, $\hat{u}$, through the transformation

$$\underline{\tilde{K}} = \underline{a}^T\ \underline{K}\ \underline{a} \tag{11}$$

where the matrix $\underline{a}$ is defined through the kinematic constraint equation

$$\underline{u} = \underline{a}\ \underline{\hat{u}} \tag{12}$$

between the displacements of the soil-structure interface and the rigid body displacement of the foundation.

It may be mentioned here, that for a given load $\underline{P}$, acting on a rigid foundation, the contact forces $\underline{t}$ between foundation and soil follow from

$$\underline{\tilde{P}} = \underline{\tilde{K}}\ \underline{\hat{u}}, \text{ with } \underline{\tilde{P}} = \underline{a}^T\ \underline{P} \tag{13}$$

together with eqs. (12) and (6). Numerical computations show that the results for quadrilateral surface foundations, obtained with the decoupled formulation agree with those given by GAUL /7/.

The following numerical results are calculated in non-dimensional complex form, using the usual dimensionless frequency a_o. The homogeneous soil has a POISSON's constant of $\nu = 0.25$.

5. Numerical results for rigid foundations

5.1 Comparison of rectangular and strip foundation

Rectangular foundations with a large aspect ratio will behave in the same way as strip foundations. The convergence from 3-D towards 2-D vibration behaviour is shown in Fig. 2, where the relation of 3-D to 2-D impedance coefficients is plotted over the aspect ratio a/b. In the 2-D and 3-D calculations only the foundations are discretized by boundary elements, using the 'decoupled formulation'. For an aspect ratio a/b = 10 there is satisfactory agreement of 2- and the 3-dimensional impedance representations.

5.2 Topographical variation

The compliance coefficients of a strip foundation on three different surface configurations are shown in Fig. 3 in dimensionless form, where the non-dimensional coefficients f_{xx}, f_{zz}, f_{xm} are related to the dimensional compliance coefficients through the factors G, G and Gb, respectively. The results of a foundation on a plane surface (case 2) are compared with those of a foundation in a ditch (case 1) and on a dam (case 3). The influence of the topographical variation shows up mainly for low frequency values. The deviation between case 2 and case 3 is more pronounced than that between case 2 and case 1.

5.3 Variation of embedment

For a structure-structure interaction problem the influence of the embedment of structure II (see Fig. 4) on the dynamic stiffness coefficients is shown. The elements of the 6x6 impedance matrix are calculated in dimensionless form, where the non-dimensional impedance matrix is the inverse of the non-dimensional compliance. The figures shown here include only the 3 main diagonal coefficients of foundation II and the cross-coupling coefficients $k_{zx}^{2,1}$, $k_{zz}^{2,1}$, $k_{zm}^{2,1}$ which show the influence of horizontal, vertical and rocking motion of structure I on the force $\underline{P}_z$ of foundation II.

ACKNOWLEDGEMENT

This work has been supported by Deutsche Forschungsgemeinschaft.

REFERENCES

/1/ IDRISS, I.M. et al.: Analysis for structure interaction effects for nuclear power plants. Report by the ad hoc group on soil-structure interaction of the Structural Division of ASCE; ASCE, New York, 1971.

/2/ DOMINGUEZ, J.: Dynamic stiffness of rectangular foundations; Publications No. R 78-20, MIT, Dept. of C.E., 1978

/3/ OTTENSTREUER, M., SCHMID, G.: Boundary elements applied to soil-foundation interaction, in BREBBIA, C.A.(Ed.): 'Boundary Element Methods', Springer-Verlag, Berlin, 1981

/4/ HARTMANN; F.: Elastische Potentiale in Gebieten mit Ecken, Dissertation, Universität Dortmund, 1980.

/5/ CRUSE, T.A. und RIZZO, F.J.: A direct formulation and numerical solution of the general transient elastodynamic problem I; J. Math. Anal. Appl., 22, 244-259, (1968).

/6/ RÜCKER, W.: Dynamic behaviour of rigid foundations of arbitrary shape on a halfspace; Earthquake Eng. struct. dyn., 10, 675-690, (1982).

/7/ GAUL, L.: Zur Dynamik der Wechselwirkung von Strukturen mit dem Baugrund, Habilitationsschrift, Universität Hannover, 1980.

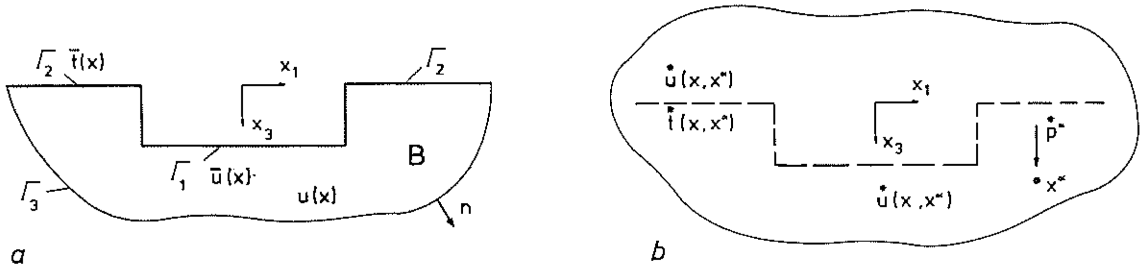

Fig. 1 Soil-foundation interaction

a) Boundary conditions; half-space

b) Point load in full-space

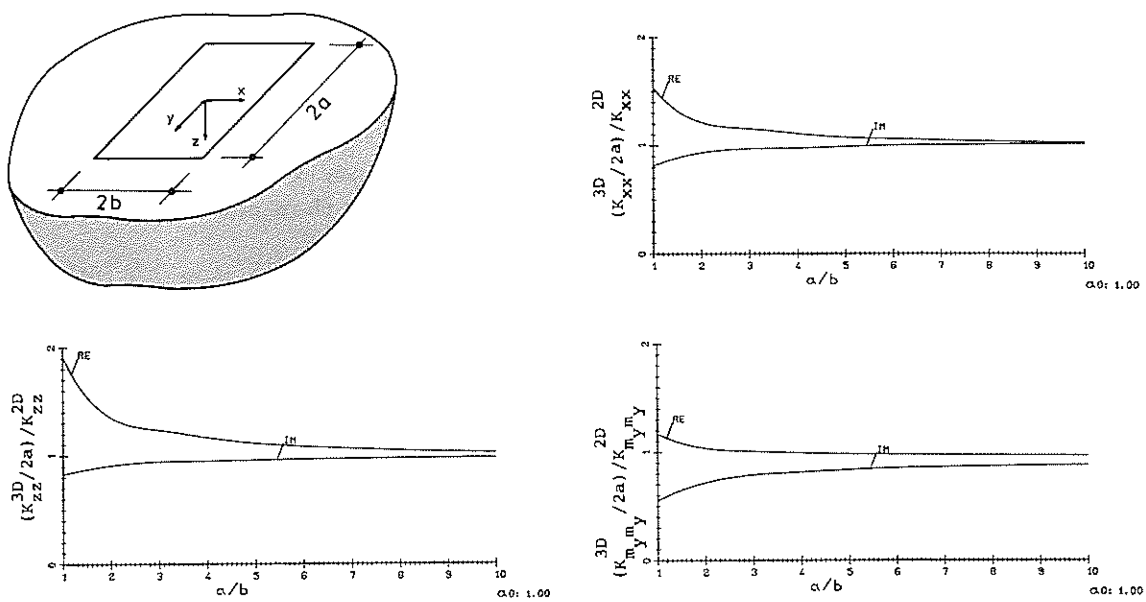

Fig. 2 Comparison of rectangular and strip foundation

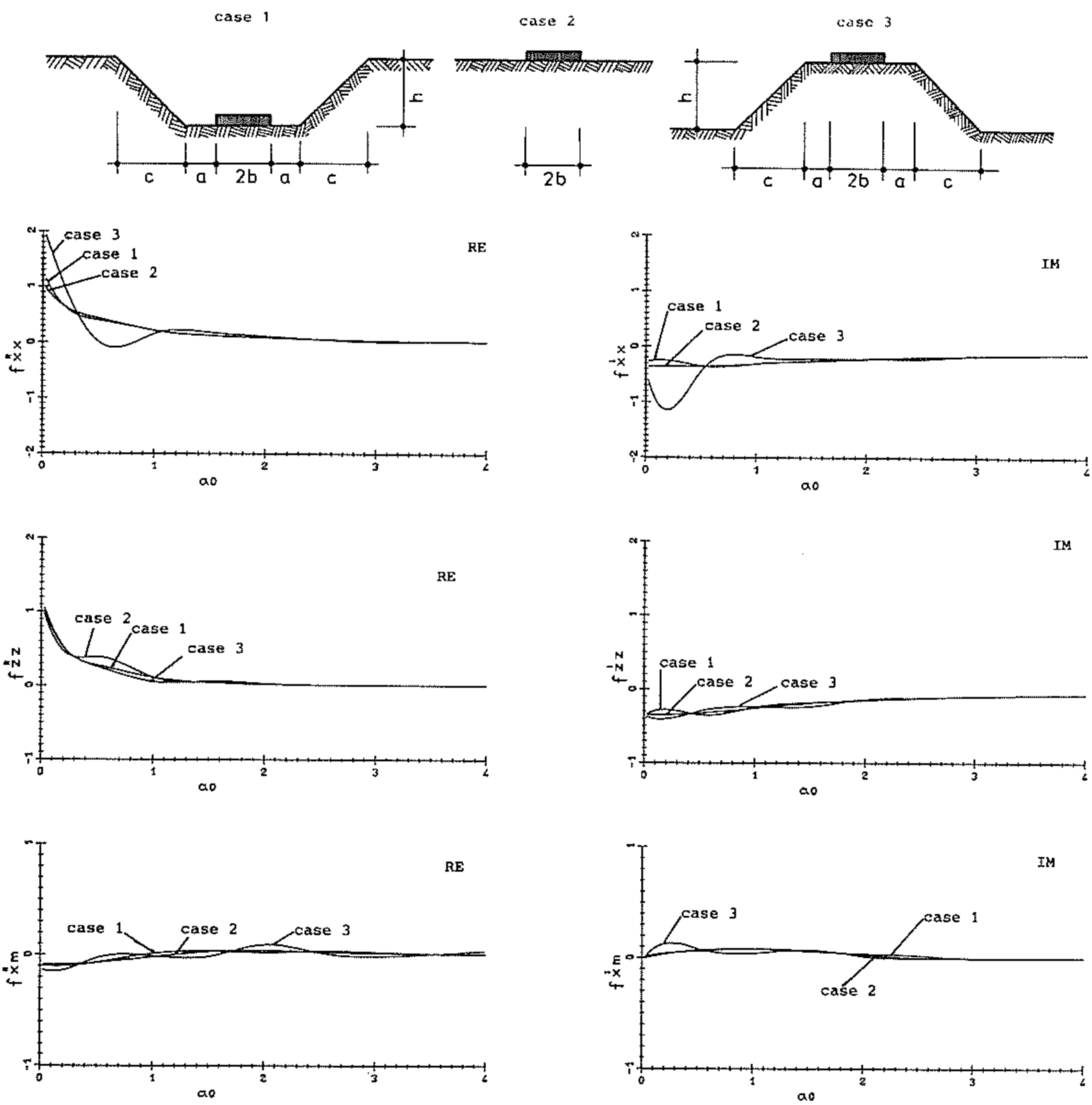

Fig. 3 Influence of topographical variation;
a = 1.$\bar{3}$b, c = 2b, h = 2.$\bar{6}$b

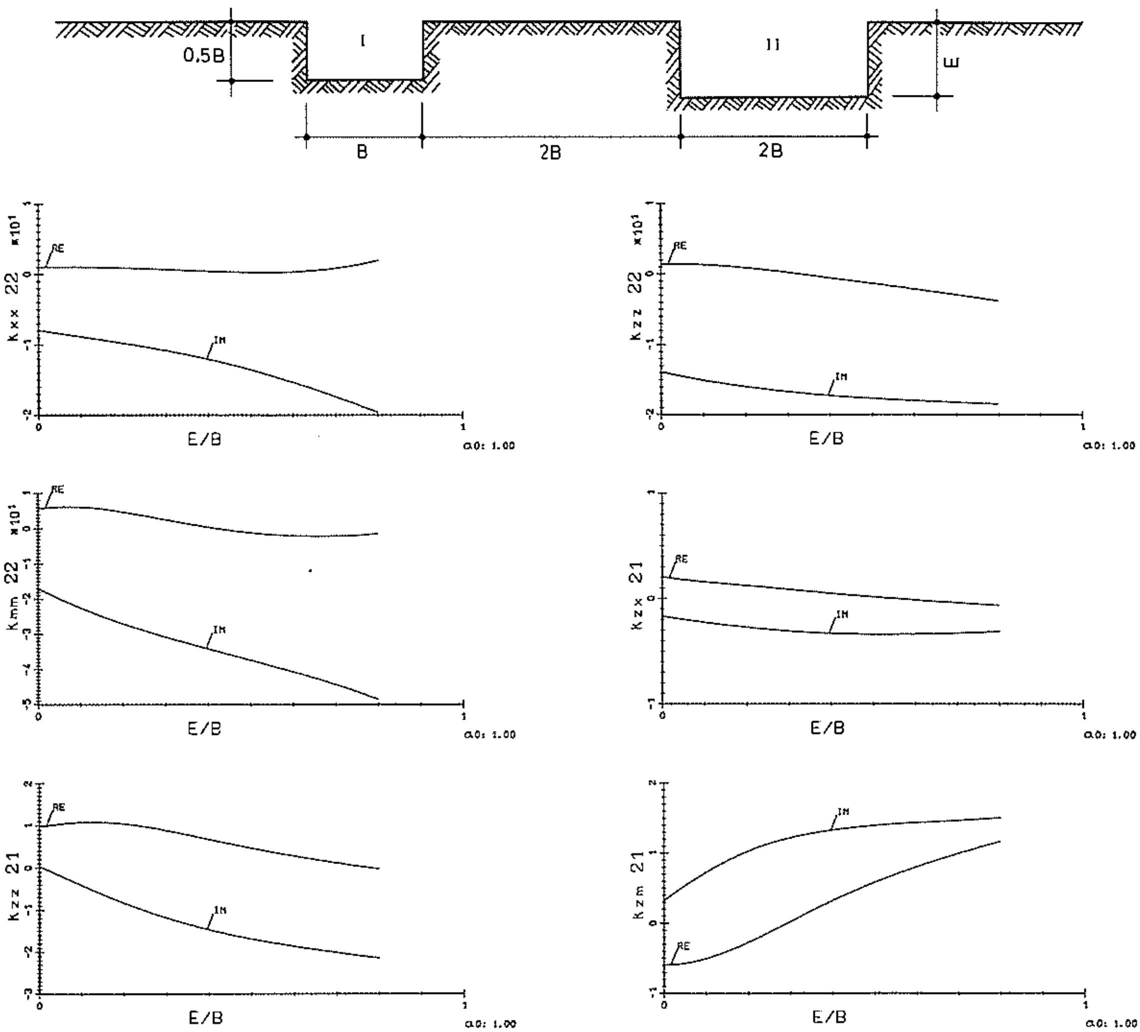

Fig. 4 Variation of embedment