A New Method for Determining Longitudinaly Nonuniform Axisymmetric Residual Stresses in Tubes Using Strain Gages

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ABSTRACT

New equations for calculating residual stress distribution are derived from the theory of elasticity. The initial distribution of the stresses including the shearing stress is computed from longitudinal and radial distributions of strains measured at the inner surface after removal of concentric successive outer layers of material. The equations can also be used for boring out method by simple modification. The end effect is taken into account in this analysis, so that the residual stress distribution in short tubes can be determined. The method demonstrated numerically for a tube with a known stress state. The residual stress distributions obtained by the proposed method agreed well with the known stresses.

KEY WORDS: Residual stress, Strain measurement, Experimental stress analysis, Theory of elasticity, Short tube

INTRODUCTION

This study was carried out to develop a new method for the calculations of the axially symmetric and longitudinally nonuniform residual stresses in tubes. The original residual stresses are computed from the strains measured at an inner surface after removing some outer layers of material in this method. The distributions of strains can be measured by using commercial multiple-element strain gages. The equations consist of a series of trigonometric, Bessel and hyperbolic functions. In the case of a longitudinally uniform stress field, the series can be simplified to the Sachs [1] equations for the turning-off method. These could also be easily extended to the case where material is removed from the inside of a hollow cylinder.

THEORETICAL DEVELOPMENT

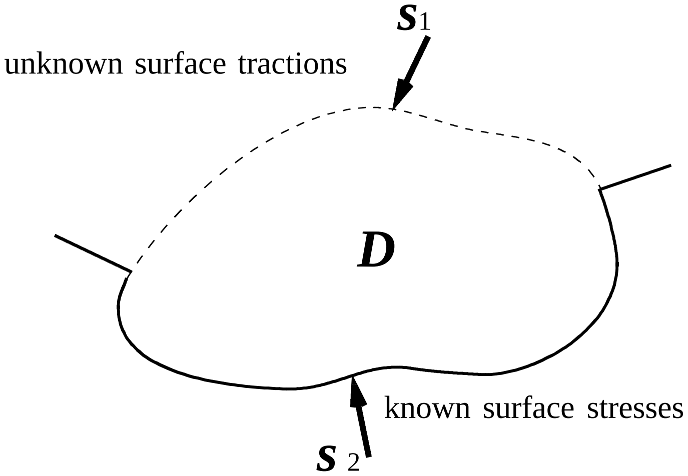

Fig.1 Boundary conditions for mechanical method of residual stress determination.

If the surface displacements or the surface tractions are given, the state of stress and strain of an elastic body is unique, by the theorem of Kirchhoff, when the magnitude of the stress is so small that the strain energy function exists and remains positive definite.

Let S_1 and S_2 be parts of the boundary of an elastic body D (see Fig. 1).

Consider the problem of determining the state of stress and strain in a body of a given shape, which is held strained by unknown surface tractions $\overset{n}{T}_i$ on S_1. The displacements and stresses due to $\overset{n}{T}_i$ are denoted by u_i and σ_{ij}, respectively. The equations for equilibrium are

$$\sigma_{ij,j} = 0, \quad \text{in } D \tag{1}$$

By using Hooke's low $\sigma_{ij} = C_{ijkl}\, u_{k,l}$, the boundary conditions for surface tractions on S_1 and for free surface forces on S_2 are

$$\sigma_{ij} n_j = \overset{n}{T}_i, \qquad \text{on } S_1 \qquad (2)$$

$$\sigma_{ij} n_j = C_{ijkl}\, u_{k,l}\, n_j = 0, \qquad \text{on } S_2, \qquad (3)$$

where n_j is the exterior unit normal vector on S_1 and S_2, the boundary of D. C_{ijkl} in the previous equations is the elastic modulus tensor of the material. The state of stress σ_{ij} and strain $u_{k,l}$ in D is unique, by the theorem of Kirchhoff. There are several integral equations for identification of input forces acting on a general boundary or in a general domain from outputs [2], [3]. These equations have been solved using finite-element-based inverse analysis schemes and other numerical schemes [4], [5]. However, we can evaluate the inputs theoretically form the measured outputs, only when D has a simple shape and simple stress distribution.

The mechanical method of residual stress determination is reduced to identification of inputs $\overset{n}{T}_i$ from outputs $u_{k,l}$ on S_2 under the conditions (1), (2) and (3). This determination may be regarded as an inverse analysis in general. The uniqueness of the solution may be governed by information available for the identification.

The usual procedure in experimental stress analysis is to measure strains and calculate the state of stress from the stress-strain relations. A mechanical method of residual stress determination in which material is removed from the outer of the cylinder and both longitudinal and circumferential strain measurements are made on the inner surface. By substituting these strain values into equations developed from elastic theory of the general axisymmetric solution for short tubes subjected to arbitrary axisymmetric external forces, we can compute accurately radial, circumferential, longitudinal and shearing residual stresses.

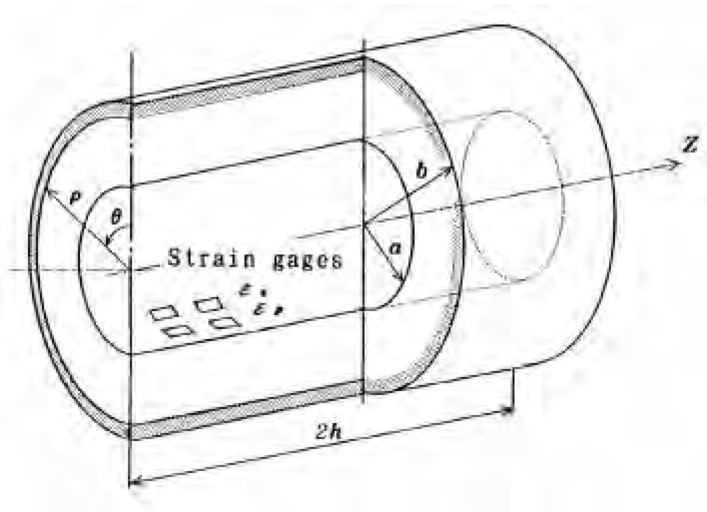

Fig. 2 Geometrical conditions of residual stress analysis in turning off material from b to ρ and locations of stain gages.

Consider an isotropic and homogeneous hollow cylinder with an outer radius b and inner radius a and length $2h$. Let the z axis in $r\theta z$-coordinate system be the axis of the tube. The original radial, circumferential, longitudinal and shearing residual stresses at arbitrary radius r are designated by σ_r, σ_θ, σ_z and τ_{rz}, respectively.

Let us consider the outer radius of the specimen is reduced to ρ from b by removal of some layers of material, as shown Fig.2. On removing the surface layer of thickness $b-\rho$, the forces which have been removed from each square unit of the cylindrical surface of the remaining cylinder are the original residual stresses σ_r, and τ_{rz} at that point before removing started. Removing the force σ_r, which is by convention considered a tensile force, is equivalent to applying an equal compressive force $-\sigma_r$. Similarly, removing the shearing force τ_{rz} is equivalent to applying a opposite shearing force $-\tau_{rz}$.

Now consider a tube with an outer radius ρ and inner radius a and length $2h$ with a traction-free inner surface and free end surfaces but subjected to axisymmetric external forces $-\sigma_r$, and $-\tau_{rz}$ per unit area at the cylindrical outer surface. Stresses $\overline{\sigma}_r$, $\overline{\sigma}_\theta$, $\overline{\sigma}_z$ and $\overline{\tau}_{rz}$ are produced by these external forces in this tube. Symbols $\overline{\sigma}$ and $\overline{\tau}$ represent the stresses induced by external forces, in this analysis. Residual stresses consequently present at a surface of the remaining tube of outer radius ρ can be considered as being the sum of two stresses ;

$$\sigma_r' = \sigma_r + \overline{\sigma}_r,\ \sigma_\theta' = \sigma_\theta + \overline{\sigma}_\theta,\ \sigma_z' = \sigma_z + \overline{\sigma}_z,\ \tau_{rz}' = \tau_{rz} + \overline{\tau}_{rz}, \qquad (4)$$

where $\sigma_r', \sigma_\theta', \sigma_z'$ and τ_{rz}' are the remaining stresses at the surface of the remaining tube. The relaxation of the residual stresses is assumed to be a linearly elastic process. This can be checked by calculation after the experiment is performed, if desired. Considering first the stresses after removing an outer layer, the stresses normal and shearing to a free surface are zero, i.e., $\sigma_r' = \tau_{rz}' = 0$. Therefore, the following equations hold for the original residual stresses from Eqs. (4):

$$\bar{\sigma}_r = -\sigma_r, \quad \bar{\tau}_{rz} = -\tau_{rz} \qquad \text{for } r = \rho \qquad (5)$$

If a particular stress distribution at the inner surface caused by removing an outer layer can be obtained by using the theory of elasticity, we can determine the original residual stresses theoretically from the strains measured at the inner surface. The stresses produced at the inner surface can be expressed as follows:

$$\left.\begin{aligned}
&\bar{\sigma}_r = \bar{\tau}_{rz} = 0 && \text{for } r = a\\
&\sigma_\theta = \frac{E}{1-\nu^2}\left(e_0 + \sum_{n=1}^{\infty} e_n \cos\beta_n z + \sum_{n=1}^{\infty} e'_n \sin\beta_n z\right) && \text{for } r = a\\
&\sigma_z = \frac{E}{1-\nu^2}\left(f_0 + \sum_{n=1}^{\infty} f_n \cos\beta_n z + \sum_{n=1}^{\infty} f'_n \sin\beta_n z\right) && \text{for } r = a\\
&\sigma_z = \tau_{rz} = 0 && \text{for } z = \pm h
\end{aligned}\right\} \quad (6)$$

where: $\beta_n = \frac{n\pi}{h}$ (n=1,2,3, • • •); e_0, e_n, e'_n, f_0, f_n and f'_n are coefficients of Fourier series, and E and ν are Young's modulus and Poisson's ratio, respectively. According to $\sigma_z = 0$ for $z = \pm h$, the terms f_0 and f_n are not independent. As discussed previously, since the radial and shearing stresses are equal to the external forces per unit area applied at the boundary surface, we can obtain the removed residual stresses by computing the radial and shearing stresses at the outer surface of the remaining tube resulting from the boundary condition (6). Using the equations of Hooke's law for plane stress into Eqs. (6), we can obtain strains at the inner surface as follows:

$$\left.\begin{aligned}
&\varepsilon_{\theta a} + \nu\,\varepsilon_{za} = e_0 + \sum_{n=1}^{\infty} e_n \cos\beta_n z + \sum_{n=1}^{\infty} e'_n \sin\beta_n z && \text{for } r = a\\
&\varepsilon_{za} + \nu\,\varepsilon_{\theta a} = f_0 + \sum_{n=1}^{\infty} f_n \cos\beta_n z + \sum_{n=1}^{\infty} f'_n \sin\beta_n z && \text{for } r = a
\end{aligned}\right\} \quad (7)$$

where $\varepsilon_{\theta a}$ and ε_{za} are strains measured at the inner surface of the tube. The strains ε_a and ε_{za} are functions of longitudinal position z and outer radius r. The coefficients e_0, e_n, e_n' and f_0, f_n, f_n' of Fourier series (7) can be expressed by measured stains in the usual manner. We can accordingly express the above boundary conditions (6) in terms of strains in this stress analysis. The left-hand sides of Eqs. (7) have a same expression as the Sachs strain parameters.

The stress distribution of a hollow cylinder of finite length subjected to arbitrary axisymmetric external forces has been derived by Shibahara and Oda [6], [7]. Their stress function satisfying the boundary conditions (6) is

$$\begin{aligned}
\phi = {} & A_0\frac{z^3}{6} + B_0\, z \ln r + C_0\frac{z r^2}{2} + \sum_{n=1}^{\infty}\frac{1}{\beta_n^3}(A_n I_{0r} + B_n \beta_n r I_{1r} + C_n K_{0r} + D_n \beta_n r K_{1r}) \sin\beta_n z\\
& - \sum_{n=1}^{\infty}\frac{1}{\beta_n^3}(A'_n I_{0r} + B'_n \beta_n r I_{1r} + C'_n K_{0r} + D'_n \beta_n r K_{1r}) \cos\beta_n z\\
& + \sum_{s=1}^{\infty}\frac{1}{\gamma_s^3}(E_s \sinh\gamma_s z + F_s \gamma_s z \cosh\gamma_s z + G_s \sinh\gamma_s z + H_s \gamma_s z \cosh\gamma_s z) Z_0(\gamma_s r)
\end{aligned} \qquad (8)$$

where : A_0, B_0, C_0, A_n, B_n, C_n, D_n, A'_n, B'_n, C'_n, D'_n, E_s, F_s, G_s and H_s are arbitrary constants. I_{0r}, I_{1r}, K_{0r} and K_{1r} are $I_0(\beta_n r)$, $I_1(\beta_n r)$, $K_0(\beta_n r)$ and $K_1(\beta_n r)$, respectively. I_0 and I_1 are modified Bessel functions of first kind, zeroth and first orders, respectively. J_0 and J_1 are Bessel functions of first kind, zeroth and first orders, respectively. K_0 and K_1 are modified Bessel functions of second kind, zeroth and first orders, respectively. Y_0 and Y_1 are Bessel functions of second kind, zeroth and first orders, respectively. $Z_i(\gamma_s r)$ is defined by the following equation.

$$Z_i(\gamma_s r) = J_i(\gamma_s r) - J_1(\gamma_s \rho)\frac{Y_i(\gamma_s r)}{Y_1(\gamma_s \rho)} \qquad (i = 0, 1)$$

Symbol $\gamma_s = \lambda_s / a$ and λ_s is the s-th real root satisfying $Z_1(\lambda_s) = 0$. The stress components in terms of the stress function are

$$\left.\begin{aligned} \overline{\sigma}_r &= \frac{\partial}{\partial z}\left[\nu \nabla 2\, \phi - \frac{\partial^2 \phi}{\partial r^2}\right], \quad \overline{\sigma}_\theta = \frac{\partial}{\partial z}\left[\nu \nabla^2 \phi - \frac{1}{r}\frac{\partial \phi}{\partial r}\right], \\ \overline{\sigma}_z &= \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 \phi - \frac{\partial^2 \phi}{\partial r^2}\right], \quad \overline{\tau}_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 \phi - \frac{\partial^2 \phi}{\partial z}\right] \end{aligned}\right\} \quad (9)$$

The symbol ∇^2 is Laplace's operator in cylindrical coordinates. The constants A_0, B_0, C_0, A_n, B_n, C_n, D_n, • • , E_s, etc. of Eq. (8) are determined from the boundary conditions (6).
Substituting the coefficients of the Fourier series (7) into the simultaneous Eqs. (8) and (9), the constants A_0, B_0, C_0, A_n, B_n, C_n, D_n, • • , and H_s can be expressed in terms of the measured strains. Therefore, the true stress distribution that existed before the layers were removed can be derived theoretically by using the strains caused by the removal of the layer of thickness $b - \rho$. The solutions of the simultaneous equations are, however, overdefined simultaneous equations. The constants A_0, B_0, C_0, A_n, B_n, C_n, D_n, • • , and H_s are not independent, they have to be determined as optimum values.

We then can obtain the following expressions for radial and shearing residual stresses by using Eqs. (5) and (9).

$$\begin{aligned} \sigma_r(\rho, z) &= -\overline{\sigma}_r(\rho, z) \\ &= -\frac{\rho^2 - a^2}{2\rho^2}\left[\frac{E e_0}{1-\nu^2} - 2\nu \sum_{s=1}^{\infty} F_s Z_0(\gamma_s a)\frac{\sinh \gamma_s h}{\gamma_s h}\right] \\ &\quad + \sum_{s=1}^{\infty} P_n(\rho)\cos \beta_n z + \sum_{s=1}^{\infty} P_n{}'(\rho)\sin \beta_n z - \sum_{s=1}^{\infty}\{E_s \cosh \gamma_s z + F_s[(1+2\nu)\sinh \gamma_s z \\ &\quad + \gamma_s h \sinh \gamma_s z] + G_s \sinh \gamma_s z + H_s[(1+2\nu)\sinh \gamma_s z + \gamma_s z \cosh \gamma_s z]\} Z_0(\gamma \rho) \end{aligned} \quad (10)$$

For the shearing residual stress,

$$\tau_{rz}(\rho, z) = -\overline{\tau}_{rz}(\rho, z) = -\sum_{n=1}^{\infty}[O_n(\rho)\sin \beta_n z + O_n{}'(\rho)\cos \beta_n z] \quad (11)$$

where, $O_n(r) = A_n I_{1r} + B_n[2(1-\nu)I_{1r} + \beta_n r I_{0r}] - C_n K_{1r} + D_n[2(1-\nu)K_{1r} - \beta_n r K_{0r}]$,

$O_n{}'(r) = A_n{}' I_{1r} + B_n{}'[2(1-\nu)I_{1r} + \beta_n r I_{0r}] - C_n{}' K_{1r} + D_n{}'[2(1-\nu)K_{1r} - \beta_n r K_{0r}]$,

$$P_n(r) = A_n\left(I_{0r} - \frac{I_{1r}}{\beta_n r}\right) + B_n[(1-2\nu)I_{0r} + \beta_n r I_{1r}] + C_n\left(K_{0r} + \frac{K_{1r}}{\beta_n r}\right) - D_n[(1-2\nu)K_{0r} - \beta_n r K_{1r}],$$

$$P_n{}'(r) = A_n{}'\left(I_{0r} - \frac{I_{1r}}{\beta_n r}\right) + B_n{}'[(1-2\nu)I_{0r} + \beta_n r I_{1r}] + C_n{}'\left(K_{0r} + \frac{K_{1r}}{\beta_n r}\right) - D_n{}'[(1-2\nu)K_{0r} - \beta_n r K_{1r}]$$

Under condition of axial symmetry and in the absence of body forces, the following equations of equilibrium in cylindrical coordinates hold for residual stresses as well as other stresses.

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (12), \qquad \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{\partial \sigma_z}{\partial z} = 0 \quad (13)$$

Note that the constants in Eqs. (8) and Fourier coefficient are the functions of ρ. Supposing the removed thickness

$\Delta\rho$ tends to zero, the circumferential residual stress can then be derived from substituting Eqs. (10) and (11) into equilibrium Eq. (12) as follows:

$$\sigma_\theta(\rho, z) = \sigma_r + \frac{\partial \sigma_r}{\partial \rho} + \rho \frac{\partial \tau_{rz}}{\partial z} \qquad (14)$$

The longitudinal residual stress distribution can also be derived from substituting Eq. (11) into equilibrium Eq. (13) as follows:

$$\sigma_z(\rho, z) = -\int_{-h}^{z} \left(\frac{\partial \tau_{rz}}{\partial \rho} + \frac{\tau_{rz}}{\rho} \right) dz \qquad (15)$$

In the case of long hollow cylinders with longitudinally uniform stress distributions, the above equations can be rearranged as follows. For these stress fields, it may be assumed that the residual stresses σ_r, σ_θ and σ_z are functions of $\boldsymbol{r}$ (or ρ) alone in most part of the tube except in the vicinity of the ends. The stress components are independent of $\boldsymbol{z}$ in most part of the tube. Therefore, the constants of Eqs. (8) may be negligible except $\boldsymbol{A}_0$, $\boldsymbol{B}_0$ and $\boldsymbol{C}_0$. The above formulas (10), (14) and (15) can be simplified to the Sachs equations for turning-off method.

AN EXAMPLE BY FINITE ELEMENT MODEL

An axisymmetric residual stress can be estimated from the measured strains, by using the procedure described in the previous section. An axisymmetric finite element analysis of the layer removal process was then carried out with a tube which had longitudinally nonuniform residual stress distribution reported by Nishimura [8]. The finite element model was selected to represent an outer diameter 120 mm, inner diameter 75.5 mm and length 150 mm steel tube with the same residual stress distributions and other characteristics. Strain data were computed using a Young's modulus of 206 GPa and a Poisson's ratio ν=0.3. In the present study, the EPIC-4 general-purpose finite element program developed by Yamada and Yokouchi [9] was employed to simulate the hypothetical measured strain data. On removing up the layers from the outer radius $\boldsymbol{b}$ to a radius of interest, the forces which have been removed from each square unit of surface of the remaining tube are the original radial and shear residual stresses that existed in the tube before removing any layer. The remaining tube with an outer diameter 2$\boldsymbol{r}$, inner diameter 75.5 mm and length 150 mm is used to calculate the strains caused by the removing step. Unloading the radial and shear force distributions from the surface nodes of the model represents the removing step. Because of symmetry of the residual stress distribution, it is sufficient to consider only one half of the axial length. The finite element model of the surface layer removed tube consists of 18 three-noded elements in the radial direction and 30 columns of elements in the longitudinal direction. The total number of elements and longitudinal element density were constant in the procedure of all layer removal. The longitudinal and circumferential strains at the inner surface were monitored at 31 positions, by equal intervals of 5 mm in the longitudinal direction while metal was removed from the outer surface of the tube. Layers were assumed to be removed 21 times from the outer surface until inner surface.

In using the above equations it is convenient to plot the Sachs strain parameters as functions of radius after removal of layers.

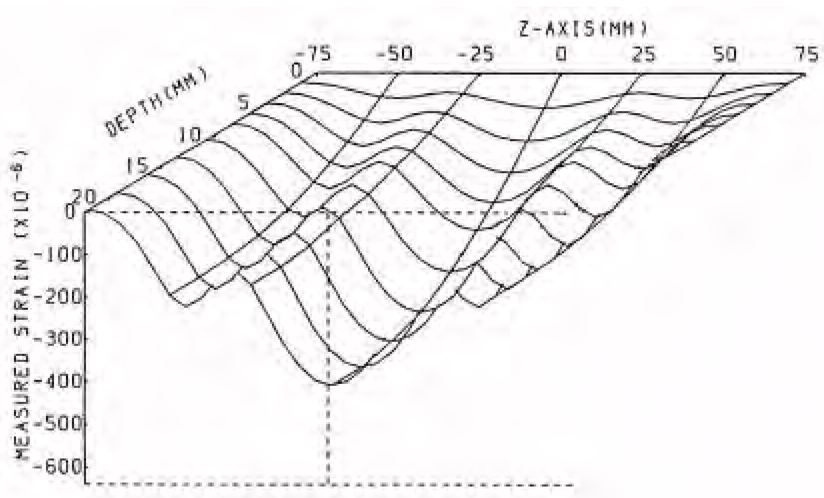

Fig.3 Distribution of $\varepsilon_{za} + \nu\,\varepsilon_{\theta a}$ measured at inner surface of a tube.

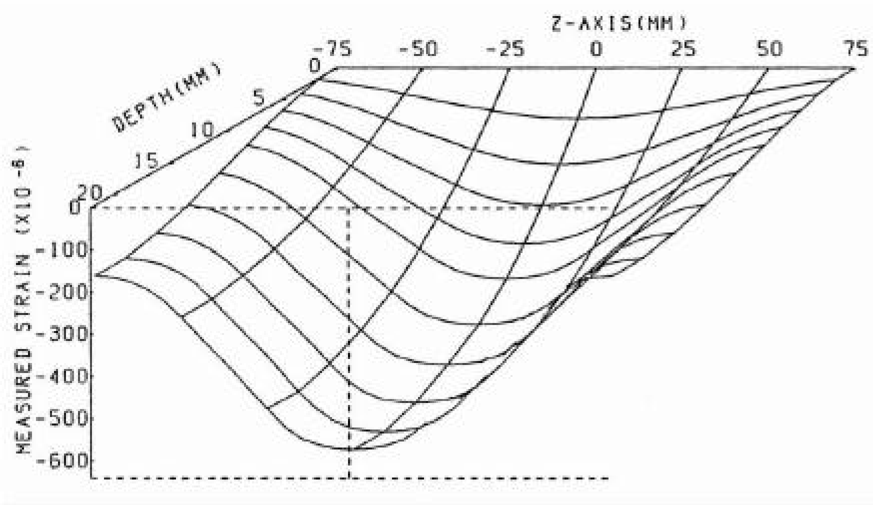

Fig. 4 Distribution of $\varepsilon_{\theta a} + \nu \varepsilon_{za}$ measured at inner surface of a tube.

The distributions of strains computed from the finite element models at the inner surface of the remaining tube are rearranged in the strain parameters of the longitudinal and circumferential directions. Figure 3 shows plots of the computed longitudinal strain parameters at 31 positions in the longitudinal direction and at 11 positions in the radial direction. Figure 4 shows the computed circumferential strain parameters in the same manner as Fig. 3. By using the proposed equations, the calculations of residual stresses from the hypothetical data were made by a computer. The arbitrary constants A_0, B_0, C_0 .., and H_s were determined by using a successive approximation method. The number of terms used in the calculation affects the results. The result gives longitudinally uniform stress distribution in a tube, when n and s = 1. The solution did not converge to a certain condition. When n and s $\geqq$ 4. The number of terms of the series n=3, s=3 were used in the calculations. The calculated longitudinal distributions of the circumferential and longitudinal residual stresses obtained from the new method are shown in Figs. 5 and 6.

Good agreement, in general, has been achieved between the given residual stress distribution provided by the X-ray method, and the "measured" stress field calculated by the strain variations computed by the finite element simulation of turning-off. Although the agreement of σ_θ at the both ends is not quite as good, the residual stress estimate procedure would still be sufficiently accurate for many engineering decisions.

DISCUSSION

The change of distribution of the load is for linear problems equivalent to the superposition of a system of forces statically equivalent to zero force and zero couple. The expectation that such a system, applied to a small part of the surface of the body, would give rise to localized stress and strain only, came to be known as Saint-Venant's principle. The stress or strain due to loading on a small part of the body may be expected to diminish with distance on account of "geometrical divergence," whether or not the resultant is zero [10]. Therefore, it is considered that the measured strains might not be exactly related to a detail of distribution of removed forces on a small part of a tube. Since the measurements of strains are not free from small errors, the calculated original residual stress distributions σ_r and τ_{rz} are not fully exact. Small deviation of the Fourier coefficients may lead to an error that can not be neglected. Markovsky [11] presented how to interpret and solve these types of problems. Further sources of inaccuracies are the calculations of the components σ_θ and σ_z. The derivation of the residual stresses σ_θ and σ_z is somewhat critical, because it requires the differentiation of a pointwise given function.

Stress gradients in radial and longitudinal directions govern the depth of any one removed layer and the longitudinal interval of measuring point. Obviously, a minimum number of layers removed and measuring points is desired, but the number should be great enough to ascertain changes in stress distribution involved. From the conditions that the outer, inner and end boundaries of the tube are free from external forces, the measured stresses must satisfy equilibrium conditions in the radial and longitudinal directions. If the stresses are rotatively symmetric, the equilibrium condition in the radial direction is evidently satisfied. The following equation is required for the boundary conditions in the longitudinal direction, i.e.

$$\sigma_{zm} = \frac{E}{1-\nu^2} \left[f_0 + \sum_{n=1}^{\infty} f_n (-1)^n \right] \qquad (16)$$

The longitudinal strain parameters measured for any radius, must satisfy the condition (16). Both strain measurement methods and X-ray diffraction techniques can be used to determine axially symmetric and longitudinally nonuniform residual stresses in tubes. Comparing the two methods as a technique for measuring residual stresses, it should be noted, first, that material has to be removed in both cases. Chemical etching turned out to be a reliable method for this purpose. The strain measurement methods have no metallurgical limitations but has limitations caused by Saint-Venant's principle. It should also be mentioned that the strain measurement methods can produce usefully accurate estimates of residual stress distributions when the tube has not steep gradients in the stress distribution. For applying the method to a similar type of geometry and residual stress distribution as in the case examined, good results can be expected from the new method presented in this paper. For other conditions the results may not be as good. When a pipe has steep gradients in residual stress distributions and has no metallurgical limitations, the X-ray method is more desirable.

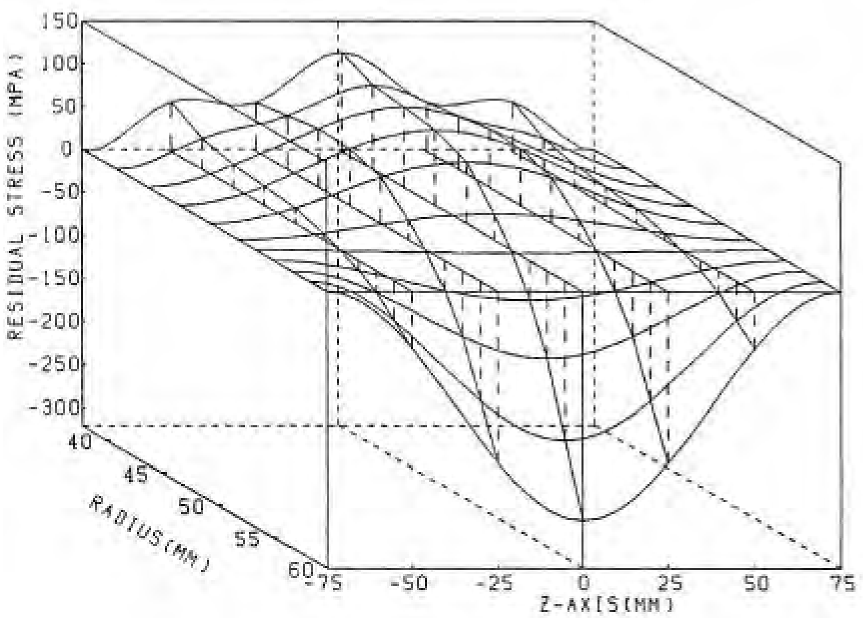

Fig.5, Original distribution of longitudinal residual stress in a tube, calculated from the " measured " strain.

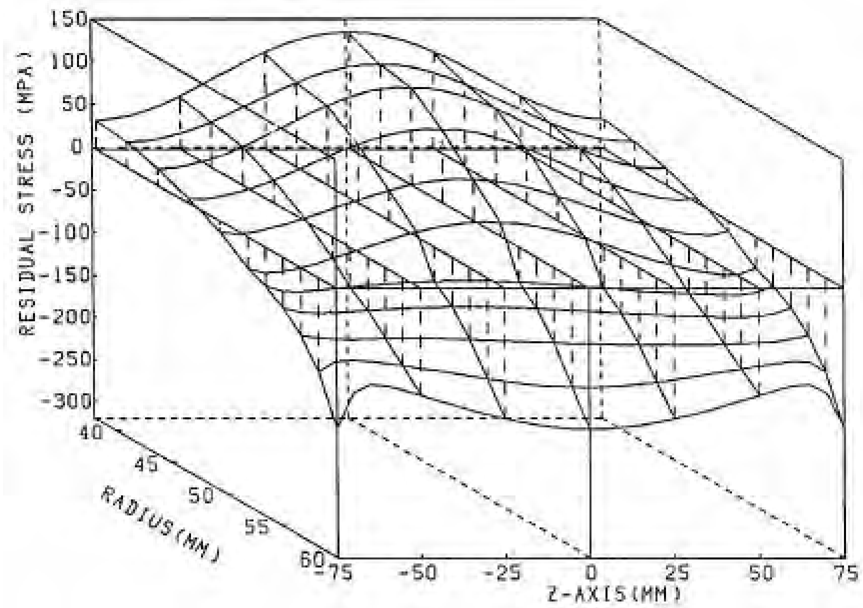

Fig.6, Original distribution of circumferential residual stress in a tube, calculated from the " measured " strain.

Cheng and Finnie [12] studied analytically the accuracy of the layer-removal method for measuring localized residual stresses. In their approach, strips, which may have been cut from a pipe or a plate, have strain-gage rosettes placed on one face and layers removed from the other face. Their analysis shows that the actual stress distribution may be quite different from that predicted by the computational model normally used in the layer-removal method. They recommend that the layer-removal method can be used for measuring residual stresses for cases in which the ratio of the height of the strip to half dimension of the localized residual-stress zone.

CONCLUSIONS

Longitudinally nonuniform residual stress distributions would frequently occur in tubes particularly in ones, which have been received partial heat treatments or metal cold working processes. New equations relating the distribution of longitudinally nonuniform residual stresses to the strains measured during successive removal of layers of material are derived for tubes, including short ones. In the case of a longitudinally uniform stress field, the equations can be simplified to the Sachs equations for turning-off method.

The method is demonstrated for a short tube that had a known residual stress distribution. A finite element analysis of the layer removal process was carried out with the tube to obtain strains to be measured. The original residual stress distributions were then calculated by a computer on the basis of the "measured" strains. The determined residual stress distribution showed good agreement with the known stress state at almost every position.

The great advantage of determining the residual stresses by strain measurements is that not only this can be done relatively easily and with inexpensive equipment, but also this has not metallurgical limitation. The limitation are that diminish with distance on account of "geometrical divergence," by Saint-Venant's principle.

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