Optimal Design of Flange Components Using Parametric Cubic Splines

J. Middleton, H.J. Anderson
Department of Civil Engineering, University College of Swansea, Singleton Park, Swansea SA2 8PP, U.K.

SUMMARY

This paper presents a numerical procedure for nonlinear design optimisation of flanged pipe connections. The geometrical transition region of the flange is defined as a parametric cubic spline and the stress sensitivities are calculated numerically using the finite element method. From these sensitivities a sequential model is developed and this is optimised using a penalty function approach combined with a variable metric unconstrained technique.

Examples of the procedure are shown for different loading conditions and plots of stress reduction are produced for each design stage. The procedure presented is based upon well tried numerical techniques and can easily be extended to other similar problems.

1. Introduction

The design and behaviour of flanged pipe connections is a most important subject matter due to the severe requirements placed upon such components, particularly in the pressure vessel industries. Complex loading conditions together with the difficulties encountered in analysing non-trivial geometries has led to many empirical formulations being proposed which pertain to produce adequate designs (1)(2). Such formulations by their nature are indeed restrictive and in general only at best give an estimate of the overall behaviour of such components.

In this presentation the design problem is posed as one of shape optimisation in which the objective is to bring the maximum stress concentration factor to a minimum value. The shape of the component is defined by a set of design variables which control the flange thickness and the transition region between the flange and tube connection. Optimisation then consists of finding the optimal vector of design variables which minimises the stress concentration factor.

The problem formulation is based upon the well proven finite element technique (3) which is used essentially to produce accurate stress gradients. Using these stress gradients a mathematical model is developed which describes the behaviour of the response to be minimised. The optimisation problem is solved by using the penalty function procedure of SUMT (4) and examples are presented showing its use as a design method.

2. Problem Statement

The flange configuration that will be discussed here consists of an axisymmetric solid of revolution as shown in figure 1. As indicated in the figure the component consists of four integral sections, which are, the raised flange face A, the flange B, the transition region C and tube region D. Such configurations frequently occur in pressure pipework systems and often present the stress analyst with the problem of not only predicting accurate stress fields but poses the related problem of sizing the component to reduce or minimise stress concentrations which are inevitably present.

The majority of flange designs are governed by standard formulations such as presented in ASME VIII (1) and BS5500 (2). Such expressions are based on semi-empirical relationships and by no means reproduce the full stress response but only forward estimates upon which the designer may found a solution. Furthermore these semi-empirical techniques do not take into consideration irregular geometries, flange bolt up behaviour or mechanical and thermal loadings. In order to overcome such deficiencies modern pressure vessel codes allow the use of numerical methods such as the finite element technique (3) which provides the engineer with a most reliable method of predicting displacements and corresponding stress fields throughout the component. With this powerful technique available attention may now be focused on generating techniques which in some way will produce optimum designs which satisfy predetermined specifications with regard to component response.

One such important component response associated with flange components is the reduction of stress concentration effects (5). Using this criterion the optimum design problem can be posed as find the component shape such that the maximum stress concentration occurring within the elastic solid is brought to a minimum value. Mathematically this can be stated in the usual form as

$$\text{Min } Z \equiv \underset{B}{\text{Minimise}} \; |\text{Maximum } S| \tag{1}$$

where 'S' is some function of stress which is to be brought to a minimum and B is the boundary of the solid for which an optimal geometry is being sought. In choosing the function S reference is made to the work of Morrison et al (6) who have shown that maximum shear stress theory or Tresca theory can be accurately related to fatigue failure and this is further endorsed by ASME who adopt this theory when numerical techniques are employed for design purposes. Thus adopting the notation of (1) if $\sigma_1 > \sigma_2 > \sigma_3$ algebraically then the stress intensity S is defined as the largest of the three principal stress differences

$$S = \sigma_1 - \sigma_3 \tag{2}$$

Using equation 2 the optimisation problem may be stated as "given an initial vector of design variables x_n which describes the component boundary find the optimal vector x^*_n such that the maximum value of stress intensity 'S' occurring within the solid is minimised".

3. Discrete formulation

Since the finite element method is based upon a discrete sampling technique where the stresses 'S_i' are sampled only at a finite number of points i then equation 1 must be formulated in terms of the discretised state. The discretised form of equation 1 can be expressed as

$$Z(x_n) \equiv \underset{x_n}{\text{Minimise}} \left| \underset{i}{\text{Maximum}}\ S \right| \tag{3}$$

where x_n characterises the set of design variables which describe the boundary and i represents the points at which the stresses S are sampled. It should be noted that no behavioural constraints will be imposed on equation 3, in fact the only additional constraints added will be those directly applied to the design variables and hence on the geometry.

In order to express equation 3 in explicit form the following approximation will be considered

$$S(x_n) = \sum_{n=1}^{N} a_n x_n + a_{N+1} \tag{4}$$

where the unknowns a_n are found from

$$\frac{\partial S(x_n)}{\partial x_n} = a_n, \ n=1, N \tag{5}$$

The stress gradients $\frac{\partial S(x_n)}{\partial x_n}$ are found as follows. Consider the following stiffness equation in the context of the finite element approximation

$$K\delta = F \tag{6}$$

differentiating equation 6 with respect to x_n yields

$$\frac{\partial K\delta}{\partial x_n} + \frac{K\partial\delta}{\partial x_n} = \frac{\partial F}{\partial x_n} \tag{7}$$

Rearranging gives

$$\frac{\partial\delta}{\partial x_n} = K^{-1} \left| \frac{\partial F}{\partial x_n} - \frac{\partial K\delta}{\partial x_n} \right| \tag{8}$$

from which $\frac{\partial\delta}{\partial x_n}$ can be found at a relatively small computational cost by using the previously decomposed stiffness matrix K (7).

A more accurate approximation to equation 4 would obviously be advantageous hut initially the use of higher order terms if precluded due to the excessive computational effort required to produce second order derivatives. In fact higher order terms can he included after the primary linear approximation has been developed and furthermore this can be accomplished without recourse to additional finite element analysis. Consider the following expression

$$S(x_n) = \sum_{n=1}^{N} a_n x_n^{\,2} + \sum_{n=1}^{N} a_{n+N}\, x_n + a_{2N+1} \qquad (9)$$

on differentiating equation 9 at the jth design

$$\frac{\partial S(x_n)^j}{\partial x_n} = 2a_n x_n^{\,j} + a_{N+n} \qquad (10)$$

is obtained and similary by moving to the j+1th design point an identical set of equations can be found

$$\frac{\partial S(x_n)^{j+1}}{\partial x_n} = 2a_n x_n^{\,j+1} + a_{N+n} \qquad (11)$$

Equations 10 and 11 form a linear set of equations and these can be solved simultaneously to find the coefficients a_n. Such refinements are advantageous in that the design space is no longer restricted to the linear region and furthermore convergence is generally faster due to the fact that the nonlinear model is more accurate.

In order to control the accuracy of the approximation model the design space is restricted to ensure that the updated design vector $x_n^{\,j+1}$ remains within the neighbourhood of $x_n^{\,j}$ where the objective function $S(x_n)^{j+1}$ is still considered to be accurate. This restricted area is defined by adding a set of side constraints, usually termed move limits, to the vector x_n as follows

$$g_L = x_n^{\,j+1} + |\Delta x_n^{\,L}| - x_n^{\,j} \geq 0 \qquad n=1,N \qquad (12)$$

$$g_u = x_n^{\,j} + |\Delta x_n^{\,u}| - x_n^{\,j+1} \geq 0 \qquad n=1,N \qquad (13)$$

These equations effectively set up a barrier which defines the current feasible design space in which optimisation of the approximation function is considered to be accurate. This process is repeated sequentially by updating $S(x_n)$ after each local optimisation solution and is terminated when $S(x_n)^j - S(x_n)^{j+1} \leq \varepsilon$ where ε is some predetermined acceptable tolerance.

4. Component geometry

The component boundary is defined in terms of the master nodes 1 to 9 and the design vector x_1, x_2 and x_3, as shown in figure 2. The transition section between nodes 6 and 7 is defined by fitting a parametric cubic spline through the variables x_2 and x_3 with end locations at the master nodes, positions 6 and 7. Cubic splines have been chosen in this application since continuity and end slope vectors are easily satisfied and the shapes generated also lend themselves to product manufacture (8).

The loading conditions which can be applied to the component are listed as follows and are shown schematically in figure 2.

1. Internal Pressure
2. End thrust
3. Bolt loading
4. Thermal loading

Loadings types 1, 2 and 4 are easily dealt with as prescribed nodal input data. Bolt loading however is of a more complex nature due to the behaviour at the contact region between the flange faces. In an attempt to model this behaviour a triangular distribution of load is assumed to act over a predetermined length of the flange T, as shown in fig. 2. The load distribution over this region is then generated such that equilibrium of the axial forces is automatically satisfied. Guidance to typical flange contact lengths can be found in reference (2).

To avoid the possibility of producing undesirable shapes and as a means of allowing predetermined control of the transition region a set of side constraints will be added as defined in fig. 3. These constraints also prevent highly distorted shapes from evolving such as could occur if the spline variables x_2 and x_3 were moved to some extreme position. The set of side constraints controlling the transition region are described as follows

$$\begin{aligned} \ell_1 &= L_1 \leq x_1 \leq L_2 \\ \ell_2 &= L_3 \leq x_2 \leq L_5 \\ \ell_3 &= L_3 \leq x_3 \leq L_4 \\ \ell_4 &= x_3 \leq x_2 \end{aligned} \tag{14}$$

These constraints together with the move limits of equations 12 and 13 are the only control on minimising S(x) there being no stress constraints since stress is the quantity to be brought to a minimum value.

5. Optimisation

The optimisation problem can now be expressed as

$$\begin{aligned} &\text{Minimise } \left(\underset{i}{\text{Maximum}}\ S\right) \\ &\text{subject to } \ell_i,\ i=1,4 \\ &\qquad\qquad g_L,\ L=1,N \\ &\qquad\qquad g_U,\ U=1,N \end{aligned} \tag{15}$$

The well proven SUMT algorithm will be used to find the optimum value of equations (15). This technique has found wide applications in solving such problems due to its versatility in dealing with general nonlinear equations (4). In this technique the constrained minimisation problem is transformed into one of unconstrained minimisation by adding the constraints to the objective function through a prescribed penalty term G. A full explanation of this technique is given by Fiacco and McCormick in reference (9).

6. Design Examples

Typical design examples are shown in figures 4 and 5. A plot of reducing stress is shown agains the number of design steps taken and the initial and optimal shape evolved is shown. The final design shape is shown by the thickened flange and transition profile. Example 1 (fig. 4) treats the conditional of bolt load only which is often critical if high initial loads are required to reduce long term losses. In example 2 the case of bolt loading, internal pressure and a thermal loading conditions are applied. In both cases considered the optimal designs produced show appreciable reduction in shear stress and typically converged after six design steps. It is also interesting to note that example 2 shows a reduction of material content as well as stress which is due to the effect of thermal loading on the flange thickness. In both examples shown a two by two Gauss point rule was applied for numerical integration with material properties being the Elastic Modulus $E = 30 \times 10^6$, Poisson's ratio $\nu = 0.3$ and the coefficient of expansion $\alpha = 12 \times 10^{-6}/°C$.

7. Conclusion

The optimal design of a flange component using parametric cubic splines to describe the transition region between the flange and tube connection has been investigated. Accurate modelling of the optimisation problem is formed numerically by the application of the finite element technique using the well tried 8 noded isoparametric element.

From the examples presented it is shown that optimal design can be evolved typically in six design steps. Input and output has been simplified such that the user requires minimum effort in data preparation. Output is presented in the form shown in figure 4 and 5 with a plot of stress reduction and fully meshed shapes of the initial design and the optimal solution for the given prescribed loading condition.

8. References

1. ASME Boiler and Pressure Vessel Code. Section III and VIII.
2. British Standard 5500. Specification for Unfired Fusion welded pressure vessels.
3. O. C. ZIENKIEWICZ, The Finite Element Method, 3rd Edition, McGraw-Hill, 1977.
4. J. L. KUESTER, J. H. MIZE, "Optimization Techniques with Fortran" McGraw-Hill, 1977.
5. J. F. HARVEY, Pressure Component Construction, Van Nostrand, 1980.
6. J. L. M. MORRISON, B. CROSSLAND, J. S. C. PARRY, "The strength of Thick Cylinders Subjected to Repeated Pressure" ASME Transactions, Paper 59-A-167, 1959.
7. A. FRANCAVILLA, C. V. RAMAKRISHNAN, O. C. ZIENKIEWICZ, Optimisation of shape to minimise stress concentration. J. Strain Analysis 10(2), p.63, 1975.
8. P. M. PRENTER, Splines and Variational Methods, John Wiley & Sons, 1975.
9. A. V. FIACCO, G. P. McCORMICK, Nonlinear Programming: Sequential Unconstrained Minimization Techniques, John Wiley & Sons, New York, 1960.

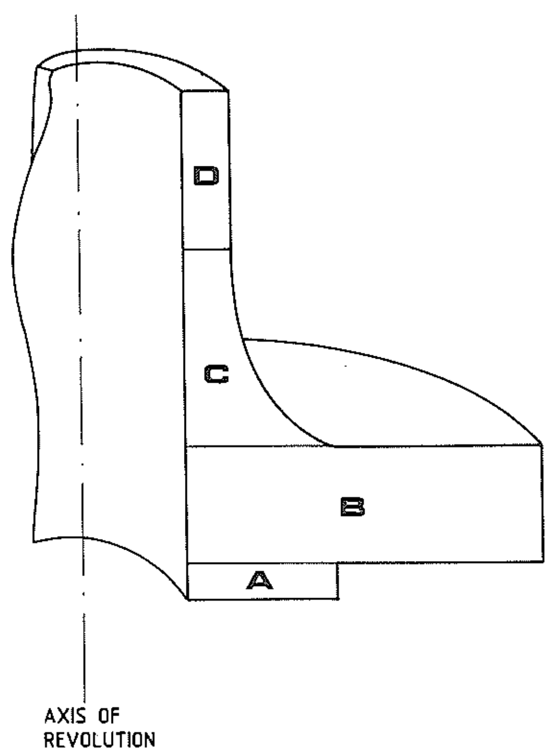

FIGURE 1

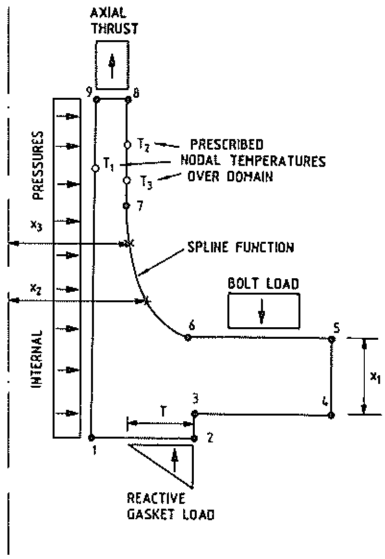

FIGURE 2

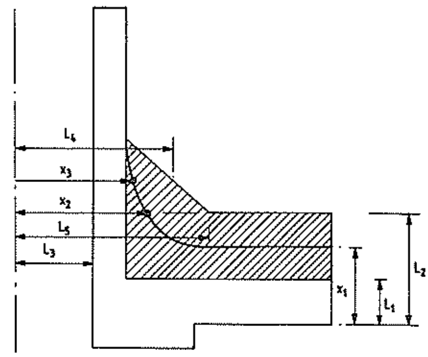

FIGURE 3

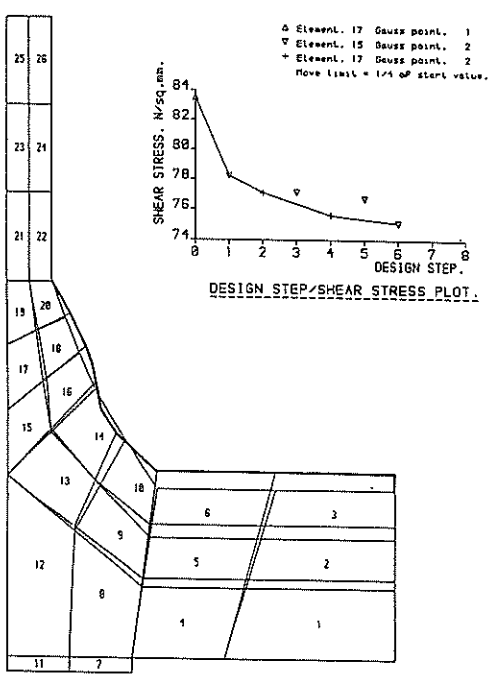

FIGURE 4. INITIAL AND FINAL MESH PLOT.
MINIMISE SHEAR STRESS BOLT LOAD ONLY.

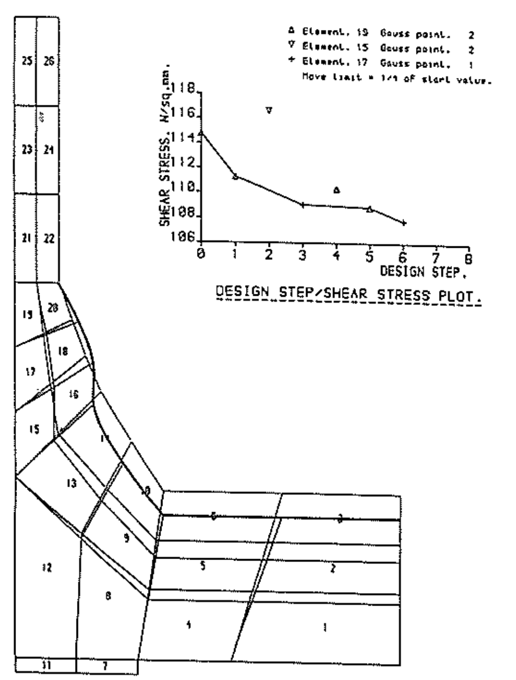

FIGURE 5. INITIAL AND FINAL MESH PLOT.
MINIMISE SHEAR STRESS BOLT+PRESS+TEMP. LOADIN(