The Influence of Failure Mechanism Mode on the Damage Field in Creeping Plates

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1 INTRODUCTION

One of the two design criteria of structural metal components which operate in creep conditions is estimation of the time to failure due to creep rupture. Microscopic and metallographic observations indicate that the creep rupture of polycrystalline metals proceeds through the nucleation and growth of microcracks and voids in the crystalline structure. The effect of growth the microdefects causes progressive weakening of material which results stress redistribution, strain rates increase and rupture by coalescence and creation of macroscopic cracks.

Essentially there are two microscopic mechanism mode of microdefects growth [1,2]. The first one in which voids growth on the grain boundary is caused by the diffusion of vacancies along the boundary perpendicular to directions of the maximum principal stress. In this case the rate of growth of microdefects is governed by the magnitude of the maximum principal stress. The second mechanism in which damage is related to grain slide which enables the growth of microdefects at triple grain boundary junctions. In this case the rate of of growth of microcracks is effective stress dependent. But there are materials, particularly creep resistant alloys, in which failure mechanism is of mixed mode i.e. it is a combination of the two.

In the paper a numerical investigation of plates in bending for steady-state creep theory coupled with damage is done by FEM and an explicit scheme for time integration. The main interest is focused on influence of different failure mechanism modes on the damage field, localization and time to appear the first macrocracks of analysed structures.

2 CONSTITUTIVE EQUATIONS

The total strains ε_{ij} are assumed to be decomposed into its elastic ε^e_{ij} and creep ε^c_{ij} parts as follows

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^c_{ij}. \tag{2.1}$$

The elastic strains are related to stresses σ_{ij} by the Hooke law

$$\varepsilon^e_{ij} = C_{ijkl}\sigma_{kl}. \tag{2.2}$$

Creep-damage material behaviour under multiaxial stress state are described by steady-state creep theory coupled with the Kachanov-Rabotnov damage growth law generalized for three-dimensional state of stress [1,3]

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$$\partial\varepsilon^c_{ij}/\partial t = \gamma\{\sigma_e/(1-\omega)\}^n \partial\sigma_e/\partial\sigma_{ij}, \tag{2.3}$$

$$\partial\omega/\partial t = A\{\sigma_{eq}/(1-\omega)\}^m, \tag{2.4}$$

where $\sigma_e = \sqrt{3s_{ij}s_{ij}/2}$ - effective stress ($s_{ij} = \sigma_{ij} - \sigma_{kk}/3$),

$$\sigma_{eq} = \alpha\sigma_1 + (1-\alpha)\sigma_e, \tag{2.5}$$

ω - scalar damage parameter ($0\le\omega\le1$), σ_{eq} - equivalent stress, σ_1 - maximum principal tensile stress, γ, A, n, m, α ($0\le\alpha\le1$) - material constants, t- time.

This form of equivalent stress enables the numerical modelling of different failure mechanism modes. For α=1 we have the maximum principal stress law, whereas α=0 corresponds to the effective stress law, and $0<\alpha<1$ corresponds to the mixed law. For prescribed stress state the equation (2.4) yields isochronous surfaces connecting stress states with equal rupture time t_R. In the case of plane stress state these surfaces reduce to isochronous curves, general form of which is shown in Fig. 1.

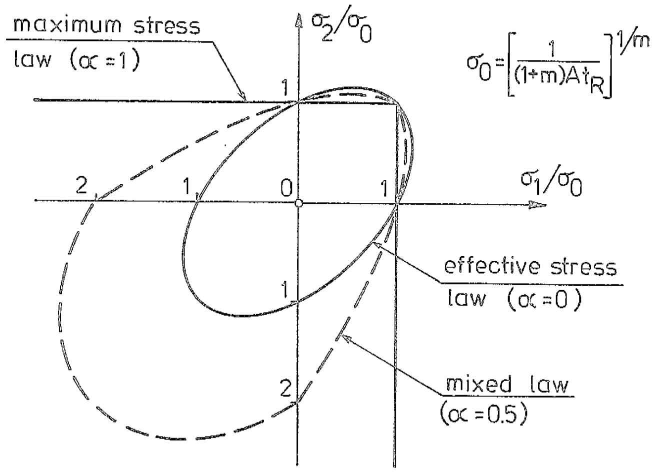

Figure 1. Plane stress isochronous rupture curves

3 BASIC SET OF EQUATIONS AND SOLUTION STRATEGY

Employing classical theory of thin plates, the basic set of equations of creeping plate with damage field is as follows

$$\left.\begin{aligned} &\operatorname{div} \underline{\sigma} = \underline{0}, \\ &\underline{\varepsilon} = \underline{\varepsilon}^e + \underline{\varepsilon}^c, \\ &\underline{\varepsilon}^e = \underline{D}^{-1}\underline{\sigma}, \quad \dot{\underline{\varepsilon}}^c = \underline{\Gamma}(\underline{\sigma},\omega)\underline{\sigma}, \quad \dot{\omega} = A\{\sigma_{eq}/(1-\omega)\}^m, \end{aligned}\right\} \tag{3.1}$$

where $\underline{\varepsilon}^T = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}] = [-z\partial^2w/\partial x^2, -z\partial^2w/\partial y^2, -2z\partial^2w/\partial x\partial y]$,

$\underline{\sigma}^T = [\sigma_x, \sigma_y, \sigma_{xy}]$,

$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\sigma_{xy}^2}$, $\sigma_1 = (\sigma_x + \sigma_y)/2 + \sqrt{(\sigma_x-\sigma_y)^2 + 4\sigma_{xy}^2}/2$,

$$\underline{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu/2 \end{bmatrix}, \quad \underline{\Gamma}(\underline{\sigma},\omega) = \gamma\,\frac{\sigma_e^{\,n-1}}{(1-\omega)^n}\begin{bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

w(x,y) - deflection of a middle surface of a plate, E - Young's modulus, ν - Poisson's ratio, dots stand for time derivative of respective variables.

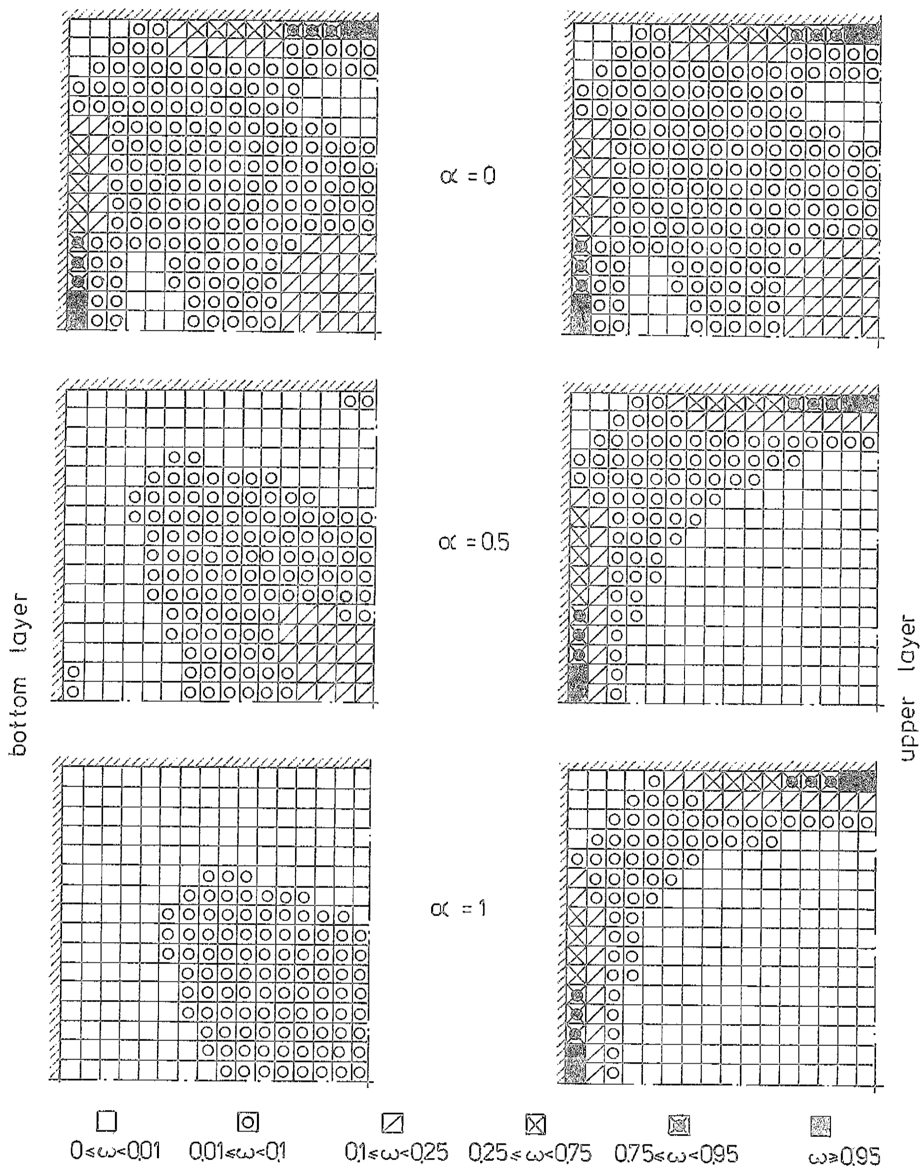

Figure 2. Distributions of damage parameter at the rupture time on outer layers for plates with clamped edges.

The above set of equations was solved by FEM for structure discretization and Euler's time integration procedure. Details of the algorithm and conditions for numerical stability and accuracy of solution are given in [4,5]. The calculations proceeds until damage parameter at a certain point of a plate reaches its critical value set to one. The time at which it takes place is considered as rupture time t_R for the structure.

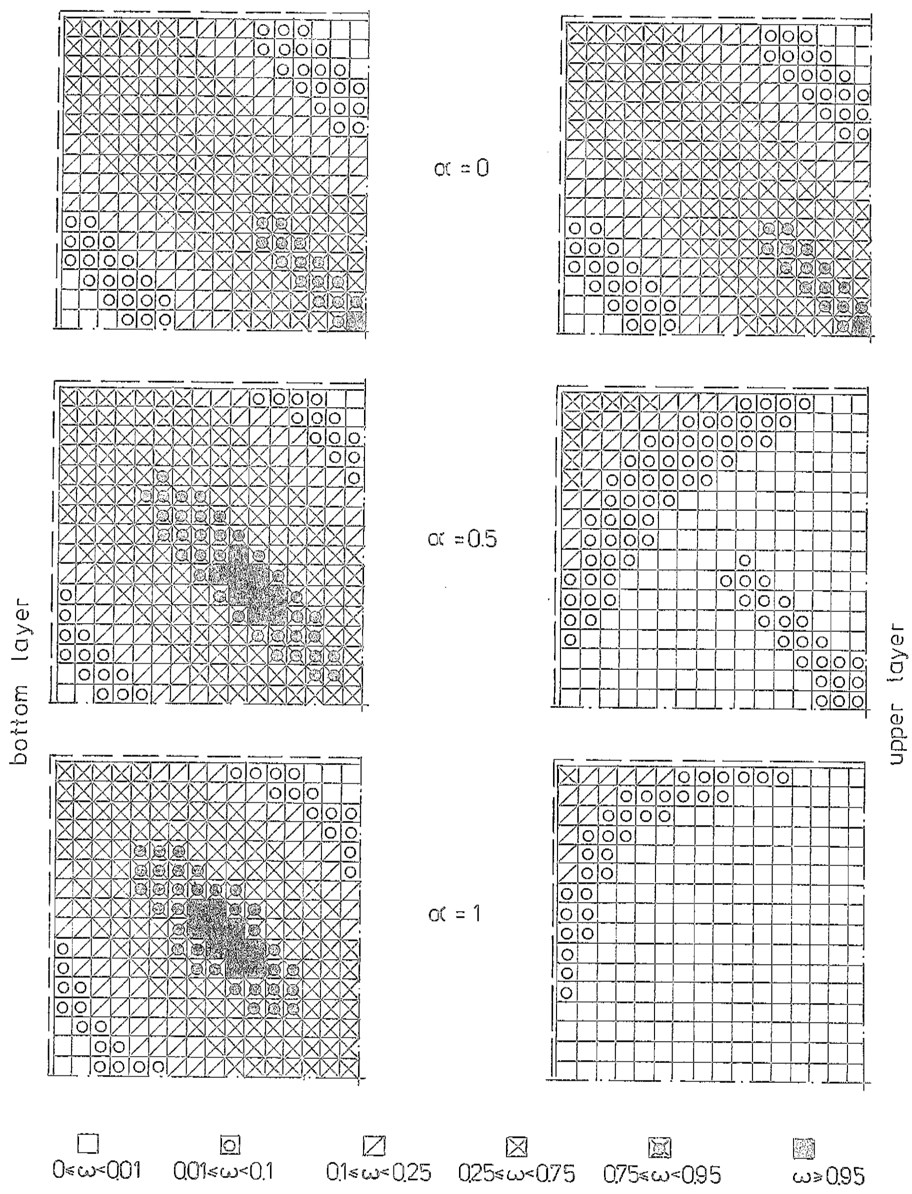

Figure 3. Distributions of damage parameter at the rupture time on outer layers for plates with simply supported edges.

4 NUMERICAL RESULTS

The computer code used in the solution of considered problem employed rectangular finite elements with twelve degrees of freedom in which the Hermitian interpolation formulae for shape functions were used. For numerical volume integration two points Gaussian quadrature was applied with plate thickness subdivided into ten layers.

Figures 2 - 5 show some results of computations for square plates with simply supported and clamped edges. The plates have the side length 1.0 m, the thickness is 0.01 m, and are uniformly loaded with p=0.2 MPa at temperature of 675 K. The material is Ti-6Al-2Cr-2Mo titanium alloy which material constants are: $E=0.102\cdot10^{6}$MPa, $\gamma=1.38\cdot10^{-24}(\mathrm{MPa})^{-n}\ h^{-1}$, $A=1.08\cdot10^{-20}(\mathrm{MPa})^{-m}h^{-1}$, $\nu=0.33$, n=6.8, m=5.79. Coefficient α which determines failure mechanism mode was assumed 0, 0.5 and 1. Due to symmetry of the structure only quadrant of a plate divided into 256 elements was considered.

Results of calculations show considerable influence of failure mechanism mode on the damage field, location of the first macrocracks and rupture time of the structures. Rupture time of analysed plates appeared to be the shortest one for $\alpha=1$ as well for clamped as simply supported plates. The longest rupture time is when $\alpha=0$ for plate with clamped edges and when $\alpha=0.5$ for plate with simply supported edges. The place where the first macrocracks occur is mainly determined by boundary conditions. However, in the plate with simply supported edges it changes its location with different values of α (as shown in Fig.5).

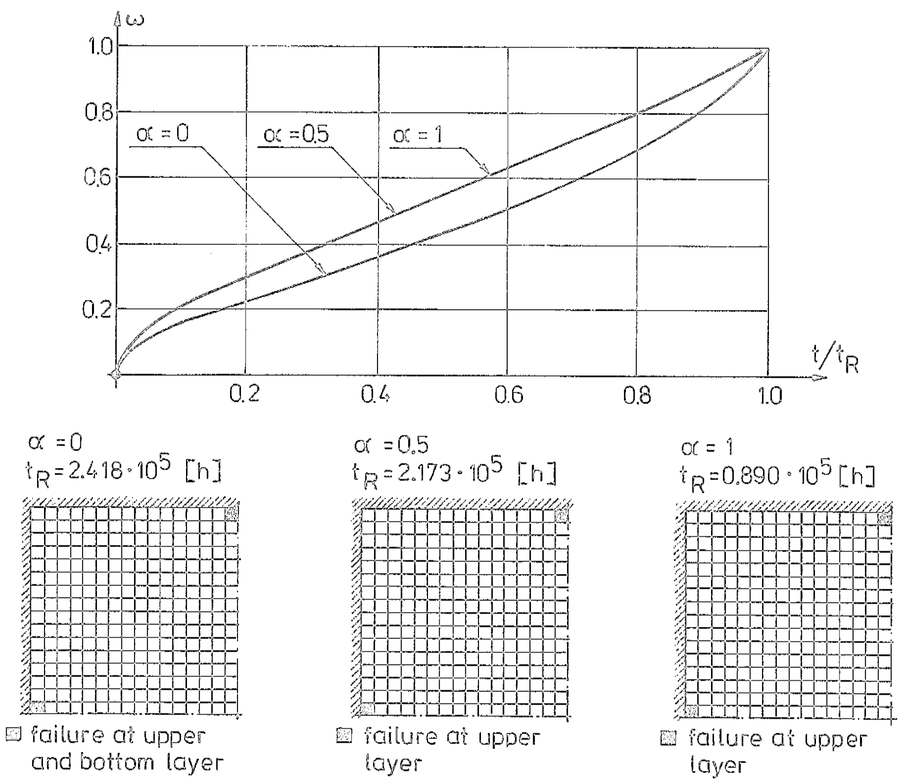

Figure 4. Damage parameter at failure points as time function and locations of the first macrocracks for plates with clamped edges.

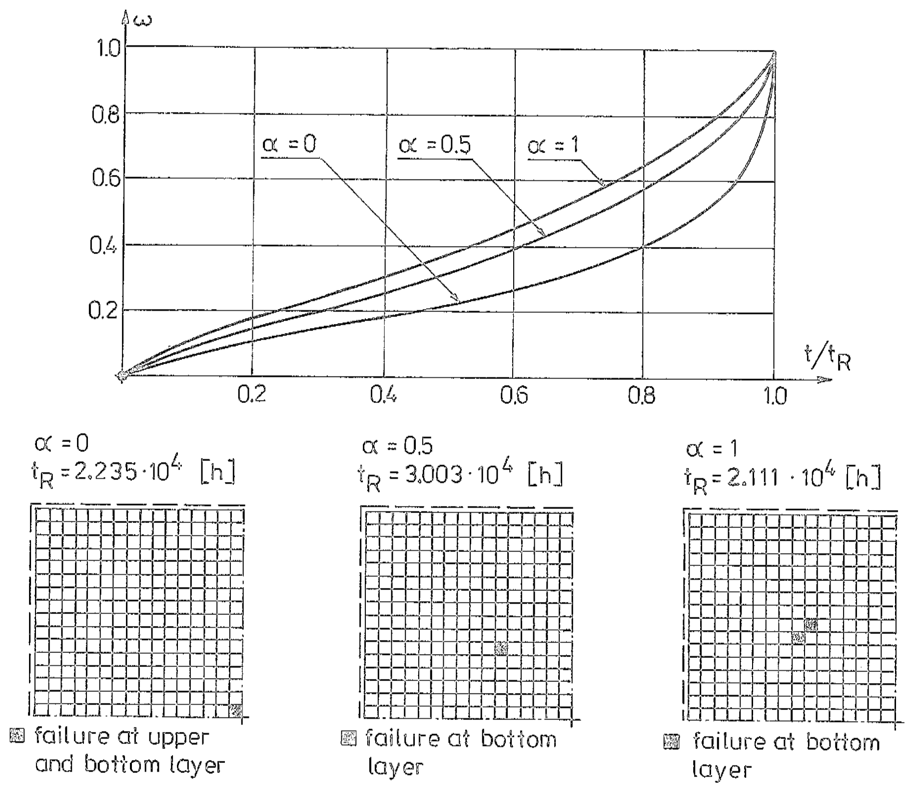

Figure 5. Damage parameter at failure points as time function and locations of the first macrocracks for plates with simply supported edges.

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