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Partial Safety Factors for Combined Loading

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ABSTRACT

This paper is an extended summary of Ref. /1/, to which the reader is asked to refer to for further details.

The Load and Resistance Factor Design (LRFD) format (Eq. (02)), which is a generic form of most current mechanical and structural design criteria, is assumed to be appropriate for load combination purposes. Attention is focussed on the fact that whilst load factors are calibrated by suitable procedures these procedures have raised certain problems which are examined.

A new calibration methodology is formulated and the relevant operative aspects are discussed. In this approach, the structure is designed to have a high reliability under permanent and quasi-permanent loads. Consistent degrees of safety for different design situations are achieved by calibrating the combination factors to be applied to the short-duration loads. Some numerical applications are finally performed.

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1. Introduction

During the last decade the problem of the combination of time-varying stochastic loads acting upon a structure has been treated in great depth.

State-of-the-art papers on this topic have been presented at some recent international conferences (/2/ and /3/ among others) where the developments of Refs. /4/ to /18/ are summarized.

The objective of these theoretical developments is to consistently account for load and structure uncertainties in the design of structural components. "For practical reasons, this goal of probabilistic design has to be achieved within the deterministic framework of current design criteria"/19//20/. All theoretical results must therefore be translated into suggestions for a Load and Resistance Factor Design (LRFD) format /21/, i.e. for the generic form of most current design criteria /22//23//24/. The problem of translating probabilistic load combination concepts into partial safety factors has been dealt with in Ref. /25//21/ /23//24//27//28/.

The current policy of calibrating safety factors for insuring consistent reliability for all the design situations is criticized and some undesired features of the results of present calibration procedures are emphasized. This paper puts forward for consideration the concept of differential safety levels. This methodological approach provides a set of calibrated load safety factors which are in agreement with the factors adopted by present criteria of structural design. Some numerical examples illustrate this result.

2. Load combination

It is suitable, if one wishes to consider the problems of linear and non-linear structural mechanics in the same theoretical context, that the load combination is investigated in the space $\mathbb{F}^n$ of the n loads, $\mathbb{F}^n$ being a sub-space of the Euclidean space $\mathbb{R}^m$ of all the m design variables.

For example: let two be the number of the loads on a structure and let Q_1 and Q_2 denote these loads. The failure of the structure is defined by the achievement of a limit state. Suppose that the points $W(q_1,q_2)$, $A_1(q_1,0)$ and $A_2(0,q_2)$ of Fig.1 are checked and found to be points of the safety domain $\mathbb{D}$ of the structure (i.e. the loads $Q_1 = q_1$ and $Q_2 = q_2$ do not activate any mode of failure when they act separately or together). As the safety domain $\mathbb{D}$ is generally convex (except for some limit-states including member stability), the belonging to $\mathbb{D}$ of W, A_1 and A_2 means that the actual safety domain certainly contains the set $\mathbb{D}_1$ of boundary Γ_1, $\mathbb{D}_1$ being defined in Fig. 1a). By contrast, if the points A_1, A_2, B_1 and B_2 are checked and found to be points of $\mathbb{D}$, the actual safety domain certainly contains the set $\mathbb{D}_2$ of boundary Γ_2. The set $\mathbb{D}_2$ is defined in Fig. 1b) and is contained into $\mathbb{D}_1$.

Present semiprobabilistic design criteria /21//22/ conceive the safety verifications as the comparison of a factored resistance with a factored load combination or load effect (Load and Resistance Factor Design (LRFD) format) /23//24/. Here the loads are not defined by their nominal values (Q_{ni}) but by their design values (Q_{di}). According to Ref. /21/, these design values are the product of a suitable partial safety factor γ_{Qi} and the 95% fractile Q_{Ki} of the distribution of the random variable Q_i. The design value U_D of the generalized resistance with which the load effect $U(\gamma_{Qi} Q_i)$ must be compared is its lower characteristic value U_K (i.e. the 5% fractile) multiplied by a suitable resistance factor $\gamma_R \leq 1$:

$$U(\gamma_{Q_1} Q_1, \gamma_{Q_2} Q_2, \ldots) \leq U_D = \gamma_R U_K \tag{1}$$

where U(.) denotes the limit state function. For structure with deterministic properties subject to two loads, eq. (2) requires that the safety verifications are performed in the points A_1, W and A_2 with q_1 and q_2 = the design values Q_{d1} and Q_{d2} of the loads. The safety of the other points of $\mathbb{D}_1$ is ensured by the convexity of the boundary Γ_D of $\mathbb{D}$.

The same LRFD format allows for the fact that the simultaneous occurrence of more actions with their design value is an unlikely event by substituting eq. (2) with

$$U(\gamma_D D_K, \gamma_{Q_0} Q_{K_0}, \gamma_{Q_1} \psi_{o1} Q_{K_1}, \ldots, \gamma_{Q_{r-1}} \psi_{o(r-1)} Q_{K_{r-1}}) \leq U_D \tag{2}$$

where γ_D, γ_{Q_i} = load factors; D_K = characteristic dead load; Q_{K_i} = characteristic time-varying load over a given reference period (f.i. 50 years); ψ_{oi} = load combination factor ($\psi_{oi} \leq 1$); r = number of time-varying loads. The symbol Q_{K_0} denotes the principal variable load of the combination; any individual time-varying load must be rotated in eq. (2), each load taking the position of the principal variable load. Therefore, eq. (2) means a system of inequalities as Q_{K_0} must be rotated into the r values Q_{K_i} and the number of time-varying loads in combination must be increased from 1 to r. For a structure with deterministic properties subject to two time-varying loads, eq. (2) requires therefore that the safety verifications are performed in the points A_1, A_2, B_1 and B_2 with q'_1 and q'_2 = the factored design values $\psi_{o1}\, Q_{d1}$ and $\psi_{o2}\, Q_{d2}$ of the loads. The safety of the other points of $\mathbb{D}_2$ is ensured by the convexity of Γ_D.

The uncertainty of both the loads and structural properties is allowed for in the LRDF formats. Extension of the above considerations to this case is possible by bearing in mind that eqs. (1) and (2) check that the load effect on the l.h.s. is lower than the factored characteristic value of the resistance on the r.h.s.. Therefore, for $\gamma_R = 1$, the points A_1, W and A_2 if Fig. 1a) and the points A_1, A_2, B_1 and B_2 of Fig. 1b) are checked to be point of the region $\mathbb{P}_K$ having as the boundary the contour surface Γ_K of the CPF for which $P_\Gamma(Q_i) = 5\%$. Thus $\mathbb{D}_1$ and $\mathbb{D}_2$ can be regarded as a set of points certainly contained in $\mathbb{P}_K$ instead of in the deterministic safety domain $\mathbb{D}$; and moreover also in the case of a structure with random properties eq. (1) expresses verifications in the points A_1, W and A_2 and eq. (2) in the point A_1, A_2, B_1 and B_2.

3. Values of the load factors

The load factors should be chosen on the basis of consistency with general desired reliability levels, statistical data and the selected safety checking format /23/. Consistency is attained in Refs./19//25/ and /28/ by designing the structures to achieve the target limit-state probability p_{f_o}. The associated reliabilities will deviate from the target reliability $(1 - p_{f_o})$ given that the set of load factors are constant for several design situations. The optimal set will be therefore selected which minimizes the extent of this deviation /25//21/.

Some paradoxical situations which arise from the policy of insuring consistent levels of safety for all different design situations will be considered in some detail; i.e.: 1) the value of the load factors depends on the number of loads in combination; 2) the load factor on a time-varying load in combination increases with its uncertainty and consequently the load factors of the other loads in combination decrease independently of the importance of that load in the load combination.

The first statement is a direct consequence of the considerations that $\mathbb{D}$ or $\mathbb{P}_K$ must be increased in size as the number of loads acting upon the structure increases. For example, let a deterministic structure be subject to two loads Q_1 and Q_2 and let the verifications (2) be conducted in the point $A_1(\gamma_{Q_1}Q_{K_1},0)$, $W_{1,2}(\gamma_{Q_1}Q_{K_1}, \gamma_{Q_2}Q_{K_2})$ and $A_2(0, \gamma_{Q_2}Q_{K_2})$ of Fig. 1. Furthermore one can assume that these verifications involve the following equation:

$$\mathrm{Prob}\left[(Q_1,Q_2)\in\mathbb{D}_1(Q_1,Q_2)\right] = 1 - p_{f_o} = \mathbb{H}_o \tag{3}$$

where $\mathbb{H}_o$ is the target reliability. The introduction of a third load Q_3 involves that safety has to be checked in the points $A_1'(\gamma_{Q_1}'Q_{K_1},0,0)$, $A_2'(0,\gamma_{Q_2}'Q_{K_2},0)$, $W_{1,2}(\gamma_{Q_1}'Q_{K_1},\gamma_{Q_2}'Q_{K_2},0)$, $A_3(0,0,\gamma_{Q_3}Q_{K_3})$, $W_{1,3}(\gamma_{Q_1}'Q_{K_1},0,\gamma_{Q_3}Q_{K_3})$, $W_{2,3}(0,\gamma_{Q_2}'Q_{K_2},\gamma_{Q_3}Q_{K_3})$ and $X(\gamma_{Q_1}'Q_{K_1},\gamma_{Q_2}'Q_{K_2},\gamma_{Q_3}Q_{K_3})$ (see Fig. 2) with /28/

$$\gamma_{Q_1}' > \gamma_{Q_1} \quad \text{and} \quad \gamma_{Q_2}' > \gamma_{Q_2} \tag{4}$$

so that

$$\mathrm{Prob}\left[(Q_1,Q_2,Q_3)\in\mathbb{D}_1(Q_1,Q_2,Q_3)\right] = 1 - p_{f_o} - \mathbb{H}_o \tag{5}$$

Note that also γ_{Q_1} is greater than the load factor which applied to Q_{K_1} insures p_{f_o} for the structure subject only to Q_1. A consequence of eq. (4) is that present policy of calibrating the load factors leads to the selection of one set of load factors for all likely combinations of loads /21//25/. That is more, by using present calibration processes,the results $\gamma_{Q_1}^o$ and $\gamma_{Q_2}^o$ could be much lower than γ_{Q_1} and γ_{Q_2} when Q_3 is the time-varying load characterized by the higher uncertainty. Necessarily then $\gamma_{Q_3}^o$ is much greater than $\gamma_{Q_1}^o$ and $\gamma_{Q_2}^o$ i.e. the safety must be mainly checked against the load apparently more dangerous. But this result does not take into account that also low values of Q_3 may cause the activation of the modes of failure relevant to Q_1 and Q_2 when added to high values of Q_1 and Q_2. The deviation from the target value of the reliability associated with the resulting design becomes therefore more and more significant. A calibration policy capable of avoiding the inconsistencies referred to above is given in the next section. This approach is founded on the proposal of differential safety levels for different design situations. The use of differential target reliability in past practice has already been recognized /21/. The levels of reliability implied by the use of current design standards and specifications have been estimated in Ref. /20/: $\beta_o = 3.0(p_{f_o} = 1.35\ 10^{-3})$ is the target value of the reliability index β /26/ for load combinations involving only gravity loads; $\beta_o = 2.5(p_{f_o} = 6.21\ 10^{-3})$ for those involving additive wind and $\beta_o = 2.0(p_{f_o} = 4.01\ 10^{-2})$ for those combinations involving earthquake loads. These different levels of target reliability, however have only be used in the context of the calibration process proposed by Ref. /25/. In the next section, attention is focussed on the margins of target reliability between different design situations.

4. An approach to the calibration of the load combination factors

The conclusion reached by the considerations summarized in Fig. 2 lead to the fact that the adding of one load upon a structure causes the relevant reliability to decrease. Therefore, if one whishes to maintain the same design against a first set of load when an additional action is introduced, one must start with a design whose reliability is higher than the target value $(1 - p_{f_o})$. That can be done according to past practice, in which all loads are enhanced by the same factor independently of their uncertainty.

Starting with a very reliable design under permanent loads (say $p_f = \frac{p_{f_o}}{10\ N}$, with N = 10 to 20 ≃ the maximum number of load on the structure), γ_{Q_i} must not be increased as the number of loads is increased: the only effect is that the safety margin is reduced. Moreover, to perform the verification (1) in the point A_1, $w_{i,j}$ defined in this way ensures that the actual safety domain $\mathbb{D}$ or the region $\mathbb{P}_K$ surely contains the prism $\mathbb{D}_1(Q_i)$ of Fig. 2b) (for i=1,2,3). This prism can therefore be employed as approximate definition of $\mathbb{D}$ or of the CPF in order to obtain a conservative estimation of the probability of failure of the structure.

The same policy can also be followed in order to state the load factors associated with the long-duration (or quasi-permanent) actions (i.e. the load which can be described with square-wave Poisson processes with very small arrival rate and very long duration of each occurrence): in this case one assumes $\psi_{oi} = 1$ in eq. (2). Thus, the global probability of failure p_f increases to the value p_{f_u} and the margin to p_{f_o} is further reduced.

One additional short-duration action on the structure can then use the margin $(p_{f_o} - p_{f_u})$. This margin should be divided in several parts if several short-duration loads act on the structure. Once γ_{Q_1}, γ_{Q_2} and γ_{Q_3} have been selected in such a way that $(p_{f_{Q_1}} + p_{f_{Q_2}} + p_{f_{Q_3}}) < p_{f_0}$ the load combination factors ψ_{oi} are the only factors to be calibrated. In this case the scheme in Fig. 1b) is adopted. On the basis of this consideration, the calibration of the load combination factors ψ_{oi} may be suitably performed according to the following procedure. Let us approximate the boundary Γ_K of the region $\mathbb{P}_K$ of the structure by the boundary Γ_2 of a region of the type of $\mathbb{D}_2$ in Fig. 1b). The surface Γ_2 is assumed to be formed by the three orthonormal planes Π_i

$$Q_i = \gamma_{Q_i} Q_{K_i} = \pi_K \tag{6}$$

and by an oblique plane Ξ of suitable direction cosines /1/. Then the CPF of the structure is modelled by a set of idealized contour lines, each line being formed by planes paralleles to the ones of Γ_2. For any idealized contour line $P_\Gamma(Q_i) = p$, the distances π_i and ξ from the origin of the planes Π_i and Ξ respectively are given by

$$\begin{aligned} P_{\Pi_i}(\pi_i) &= p \\ P_\Xi(\xi) &= p \end{aligned} \tag{7}$$

The functions $P_{\Pi_i}(\pi_i)$ and $P_\Xi(\xi)$ are predefined cumulative distribution functions with a given equal value of the coefficient of variation δ_R and 5% fractile π_K and $\chi_K \xi'$ where χ_K is presently unknown and ξ' is given.

The probabilities of failure of a structure subject to loads with characteristic values Q_{K_i} can now be calculated by the convolution integrals of the probability density function of the relevant loads with the CPF expressed by eq. (7). Successive iterative analyses permit one to determine the value $\chi_K^o \xi'$ for which the probability of failure becomes equal to its relevant target value.

It can be shown /1/ that χ_K^o is independent of the values Q_{K_i} which are used in the calculations. Therefore, one estimation of χ_K^o is sufficient, there are not deviations from the target reliability and hence no optimization over different design situations is required by the approach here formulated. Once χ_K^o has been determined, the intersection of the relevant plane Ξ with the planes Π_i gives the six points $B_1(Q_1,Q_2)$, $B_2(Q_1,Q_2)$, $B_1(Q_1,Q_3)$, $B_2(Q_1,Q_3)$, $B_1(Q_2,Q_3)$ of Fig. 3. One of the coordinates of these points is $\gamma_{Q_i}Q_{K_i}$, while the other two must be read $Q_{K1} = \psi_{oj}\gamma_{Q_j}Q_{K_j}$ and $Q_{K_2} = \psi_{o1}\gamma_{Q_1}Q_{K_1}$.

5. Numerical examples and conclusions

The numerical examples have been developed for a structure acted by a live load (L) by an earthquake excitation (E) and by the actions of snow (S) and wind (W). The probability of failure of the structure when these loads do not act on it is assumed to be much lower than p_{f_o}. The probabilistic definition of the actions is given in Table I, where δ = coefficient of variation of the maximum over 50 years; ν = mean rate of arrival per year; d = average duration, in days, of each occurrence. The load safety factors $\gamma_L,\gamma_E,\gamma_S$ and γ_W have been taken equal to 1.5 and the characteristic values of the loads are the 95% fractiles of the ditributions of the maximum over 50 years. The conditional probability of failure has its plane with distance from the origin with normal distribution and coefficient of variation δ_R = 5% or 10%.

The results of Table II show a large dependence of the load combination factors on the properties of the stochastic processes which model the loads. The simplicity of the calibration procedure proposed in Section 3 and the modest amount of calculations required, however, make it possible to propose the use of this approach not only for code purposes but also in current practice. It can be employed, in fact, as a preliminary study having as the objective the determination of the load combinations which one must consider in structural analyses.

Table I - Probabilistic definition of the actions

Type of load	L	E	W	S
Type of CDF	Ext.I (max)	Ext.I (max)	Ext.I (max)	Ext.I (max)
cov of the max over 50 years δ_{Q_i}	25%	70%	18.6%	20%
mean arrival rate ν_i |per year|	12	2	24	6
mean duration of each occurrence d_i |days|	2	7E-4	1	1-20

Table II - Some results obtained by the calibration procedure proposed in this paper

d_S	δ_R		p_{fQ_i}	ψ_{oi} L,E	W	S
1	5%	L	.52 E-03	–	.049	.049
		W	.19 E-03	.049	–	.050
		S	.25 E-03	.049	.050	–
		p_{f_o}			0.0015	
20	5%	L	.52 E-03	–	.199	.199
		W	.19 E-03	.199	–	.200
		S	.25 E-03	.199	.200	–
		p_{f_o}			0.0025	
20	10%	L	.32 E-03	–	.040	.040
		W	.13 E-03	.040	–	.040
		S	.16 E-03	.040	.040	–
		p_{f_o}			0.0025	
20	5%	E	.36 E-02	–	.050	.049
		W	.19 E-03	.050	–	.050
		S	.25 E-03	.049	.049	–
		p_{f_o}			0.0075	

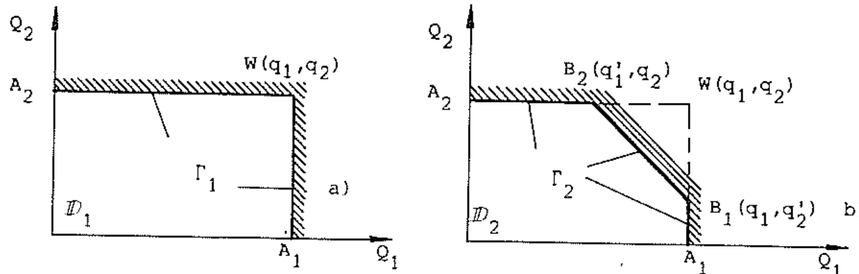

Figure 1 - Points where the safety is checked according to current design criteria

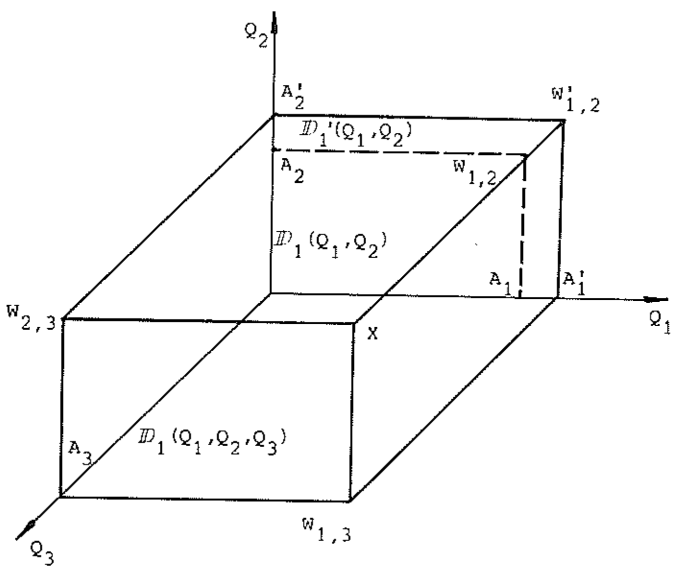

Figure 2 - Safety verifications as the number of loads acting on a structure increase by one.

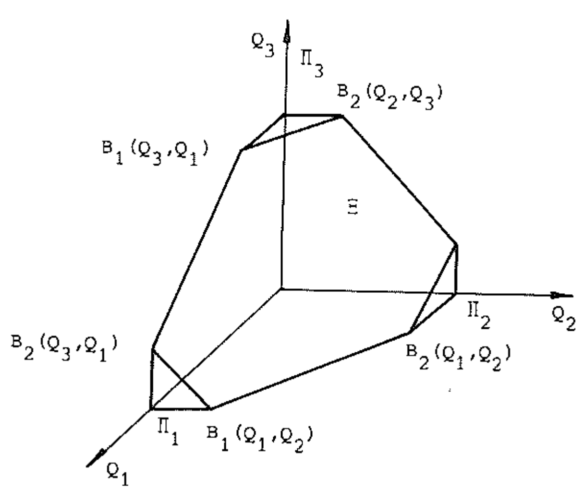

Figure 3 - Points $B_i(Q_j, Q_K)$ where the verification (2) has to be performed according to the calibration process proposed in Section 4.

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