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Effects of Yield Variations on Strain Concentrations in Inelastic Response

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Abstract

Simplified and detailed inelastic analyses have been used to assess strain concentrations. Evaluations of beams, cylinders, notched sections, perforated plates, and shells revealed that predictions of inelastic peak strains are sensitive to the choice of yield strength. The extent of sensitivity, however, depends upon the type of structure, the material strain hardening properties, the type of loading, and the level of loading. An assessment of these effects and some guidance on the conditions where they are significant were accomplished.

1. Introduction

The peak strain induced in a component operating in the creep regime is an important parameter for counting creep-fatigue damage. Design analyses accordingly seek to accurately quantify the strain levels and ranges with special attention to concentrations. Concentrations can occur due to geometrical and metallurgical notches, local yielding and plasticity, creep, and elastic follow-up mechanisms. Accordingly, linear elastic models do not provide good tools for strain concentration evaluation. However, inelastic models also have limitiations. The uncertainities and ranges of variation of the material behavior, structural idealization, loading and thermal history predictions, etc., lead to the desire for understanding the sensitivities of analysis results to these variabilities. This compact will provide some results regarding strain concentration relative to the yield strength variations of interest to designers of elevated temperature structures.

2. Load-Controlled Bending Strain Concentrations in Smooth Section Beams

Radomski and White [1] demonstracted that the plastic strain concentration factor is dependent on level of load (relative to that to cause nonlinear and yielding response), the extent of strain hardening, the type of load, and the type of structure. Using their results one can investigate the effects of yield strength on the strain concentration factor under monitonic load. For example, consider the stainless steel results (Figure 1). If the yield strength is 20% lower it can be shown (Table 1) that the increase in strain concentration is about 5 to 10 percent.

3. Deformation-Controlled Strain Concentrations in Smooth Cylinders

Houtman [2] and Severud [3] demonstrated that the plastic strain concentration factors for thermal shocked and biaxially stressed cylinders are different from that of beams. However, considering an elastic, perfectly plastic material (no strain hardening) under an equi-biaxial stress field, a 20 percent lower yield strength will give rise (Table 2) to about the same amount, 5 to 10 percent, of increase in strain concentration [3,4].

4. Strain Concentration Factors for Notched Sections

Neuber's [5], equation, eq, (1), can be used to conservatively estimate the local inelastic stress and strain concentrations.

$$K_\sigma \cdot K_\epsilon = K_T^2 \tag{1}$$

where, $K_\sigma = \frac{\sigma}{S}$ = Stress Concentration Factor

$K_\epsilon = \frac{\epsilon}{e}$ = Strain Concentration Factor

K_T = Elastic Stress Concentration Factor

σ = Local Stress, S = Gross Stress

ϵ = Local Strain, e = Gross Strain

Rearranging e.g., (1) gives

$$K_\epsilon = \left(\frac{S}{\sigma}\right) K_T^2 \tag{2}$$

This eq, (2) implies that the strain concentration factor for a notched section is inversily proportional to the local stress. Thus, for a given gross stress S, and notch factor K_T, a reduction in inelastic strength will result in a proportional increase in strain concentration factor K_ϵ.

For strainless steels, the minimum yield strength is typically about 80 percent of the average material value. Stresses in the inelastic range for a minimum yield strength material are also about 80 percent of those of the average material. Thus, for a 20 percent weaker material, the strain concentration factor increases about 25 percent.

5. Strain Concentrations in Perforated Plates

O'Donnell and Porowski [6] evaluated local plastic strain concentrations in the ligaments of perforated plates in both triangular and square penetration patterns. The concentration factors vary significantly depending on loading (e.g., equi-biaxial, shear, or uniaxial), ligament efficiency factor, and load level relative to initial local yield level. The findings [6] were based on elastic-plastic solution with no strain hardening included.

The effect of a 20 percent reduction in yield strength on the strain concentration factor of a perforated plate under pressure loads with ligament efficiency of 10 percent, ranges up to a 50 percent increase (Table 3) if strain hardening is neglected. For a material like stainless steel with good strain hardening properties, strain concentration increases of much less than the 50 percent would be expected.

6. Conditions That Increase Strain Concentration Sensativity to Yield Variations

It is easy to see that a material with little strain hardening can develop high plastic strain concentrations and thus a small change in yield strength can lead to large charges in strain concentration factor.

For structures operating in the creep conditions, a low yield strength may result in significantly more or less creep strain than that of a high yield strength material. If a low yield strength limits the deformation-induced stress to a lower level, creep strains will be less. However, if stresses are higher with a higher yield strength material, more creep strain can result.

An example of the first case could be a pipeline loaded by restraint of thermal expansion. An example of the second case could be a pressurized cylindrical tube under intermittant thermal shocks and sustain operation in creep conditions.

The sensitivity of creep strain accumulation to yield strength can be much more than that expected from plastic strain concentrations. For example, consider stainless steels with a power law representation of secondary creep rate, eq, (3).

$$\dot{\epsilon} = B_{\sigma}^{n} \tag{3}$$

where, ϵ = creep rate
σ = effective stress
B = material constant
n = material constant, about equal to 7 for stainless steel

Structures of such materials can have creep strain rates of 20 percent or 500 percent of nominal for stresses that are 80 percent or 125 percent of nominal respectively. The significance of these different creep strain rates is of course dependent on the amount of time, stress relaxation, and accumulated creep strain.

7. Conclusions

Choice of yield strength employed in evaluating strain concentrations in inelastic plastic and creep response can significatnly affect the analysis findings. The type of structure and loading influence the sensativity of yield strength on strain concentrations. For monotonically loaded beams, cylinders, and notched structures, of materials with strain hardening comparable to stainless steels, a lower yield strength results in proportionally higher plastic strain concentration factors. For elastic, perfectly plastic materials, the plastic strain concentration factor will be very sensative to yield strength. Yield strength variations potentially are the most influential for conditions of creep strain accumulation.

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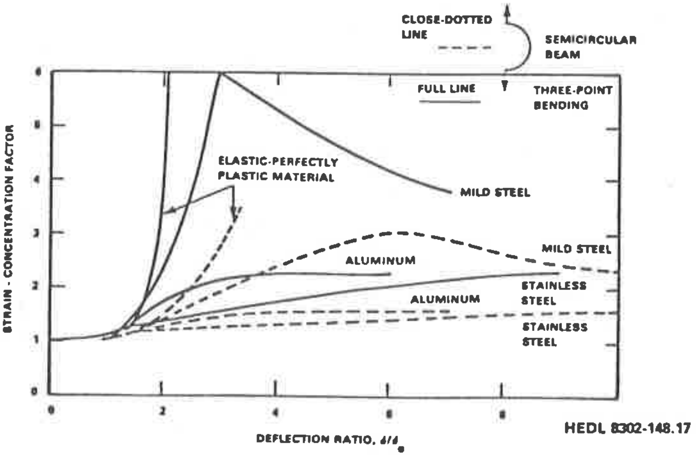

FIGURE 1. Strain Concentation During Inelastic Action (From M. Radomski and D. J. White, Reference 1).

TABLE 1. LOAD-CONTROLLED BENDING STRAIN CONCENTRATIONS ON SMOOTH SECTION BEAMS

Defl Ratio, δ/δ_0	Yield Strength	Strain Concentration Factor, K_ε Straight Beam	Curved Beam
2	Nominal	1.35	1.15
	80% Nom.	1.42	1.22
3	Nominal	1.50	1.25
	80 % Nom.	1.65	1.35
4	Nominal	1.70	1.37
	80% Nom.	1.85	1.43

Notes: Strain ratio $\varepsilon/\varepsilon_0 \approx \delta/\delta_0$ for material with $\sigma = A\varepsilon^m$

δ = Deflection for applied load

δ_0 = Deflection for 0.2% offset yield stress load

K_ε = Multiplier of elastically calculated pseudo strain to compute inelastic strain based on work of Radomski and White [1].

TABLE 2: DEFORMATION-CONTROLLED STRAIN CONCENTRATIONS IN SMOOTH CYLINDERS

Strain Ratio, $\varepsilon_{eff}/\varepsilon_y$	Yield Strength	Strain Concentration Factor, K_ε
2	Nominal	1.26
	80% Nom.	1.31
3	Nominal	1.37
	80% Nom.	1.43
4	Nominal	1.43
	80% Nom.	1.46

Notes: K_ε values based on equiaxial thermal shock stress field. ε_{eff} is effective multiaxial strain; ε_y is yield strain of elasto-plastic material model. Based on work by Severud [3].

TABLE 3: STRAIN CONCENTRATIONS IN PERFORATED PLATES

Deformation Ratio, $\varepsilon/\varepsilon_0$	Yield Strength	Plastic Strain Concentration Facator, K_p For Ligament Efficiency of 10% Equibiaxial	Shear Loading
1.0	Nominal	1.0	1.0
	80% nom.	1.4	1.1
1.2	Nominal	1.3	1.1
	80% nom.	1.9	1.3
1.5	Nominal	1.9	1.3
	80% nom.	2.3	1.6

NOTE: Based on work by O'Donnell and Porowski [6].