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Some Cases of Acoustical Problems in Fluid-Structure Interaction Based on a 2D Potential Symmetric Formulation

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ABSTRACT

This paper presents a symmetric variational formulation for the calculation of dynamic problems regarding a fluid-structure interaction. The discretization of the problem is obtained through the method of finite elements as applied to the functionals of a solid, a fluid and an interface. This paper also makes a comparison between the coupled and uncoupled dynamic behaviours for a given test and its verification through analytical solutions.

INTRODUCTION

The interaction between the fluid and the structure greatly affects the structural response and can be found in many engineering applications, such as nuclear reactors, offshore platforms, ships, turbines, barrages, etc. It is crucial that its effects be taken into account.

The validation of the formulation in this paper has been developed by various authors [1], and it includes a Lagrangian description for the solid and an Eulerian description for the fluid. Other sorts of formulation are to be found in the specialised literature [2,3,4,5,6], and they consider only the hydrostatic pressures or the velocity potential as variables for the fluid, and the nodal displacements for the solid. Other authors [7,8,9], however, have used Lagrangian formulations for the nodal displacements of the solid and the fluid.

This paper will only analyse inviscid and irrotational fluids, which are limited by pre-determined domains and have slight displacements. The variables involved are a displacement X_s, a pressure p and a potential Π (similar to a displacement potential for fluid), which is also useful for symmetrizing the problem.

The dynamic behaviour of systems in frequency domain of free vibrations for simple problems is studied, always making comparison with analytical results and results provided the literature. Problems such as ill conditioning of matrixes have been pinpointed and solved, and the results obtained have proved to be in accordance with the values predicted.

THEORICAL FORMULATION

Fluid-Structure Coupled Treatement

Considering a structure with linear and homogeneous elastic behaviour within a domain V_s and an inviscid and irrotational fluid within a domain V_f. And, Σ is the surface comprising

V_s and V_f, where the fluid-structure interaction takes place. The slight displacements made by the structure are determined by the degree of freedom of displacement $X_s(x,y,t)$. The fluid description is obtained through the following variables of pressure: P and Π ($\Pi = -p/\omega^2$).

1. Functionals associated to the Fluid-Structure System

The terms of the functional associated to X_s-P-Π formulation are essential for finite elements discretization, which is easily employed in classical calculation programs. From the physical point of view, however, it clearly shows the various energy magnitudes of the problem, particularly the terms involved in the fluid-structure coupling [10,11,12].

a) Functional of the Structure Equation.

$$\Im_s = \frac{1}{2}\int_{V_s} \bar{\bar{\sigma}} \otimes \bar{\bar{\varepsilon}} dv - \frac{1}{2}\omega^2 \int_{V_s} \rho_s (x_s)^2 dv - \int_\Sigma p x_s \cdot n dv \tag{1}$$

The boundary conditions regarding the displacement shall be neglected.

b) Functional of the Fluid Equation.

$$-\frac{1}{\omega^2}\Im_f = -\frac{1}{2\omega^2}\int_{V_f} \rho_s (\mathrm{grad} p)^2 dv + \frac{1}{2}\int_{V_f} \frac{1}{\rho_f c^2} p^2 dv - \int_\Sigma p x_s \cdot n dv \tag{2}$$

2. Functional of the Coupling

The functional $\Im$ of the coupled system must be determined in such a way that the first variation of $\Im$ regarding an arbitrary increase of X_s and p lead to equilibrium equations. In order to achieve that, the functional $\Im$ is obtained through the association of $\Im_s$ and $\Im_f$ so that there is a coincidence with the coupling term. And, with a view to obtaining the classical expression $\Im' = \Im_1 - \omega^2 \Im_2$, a supplementary (or dual) variable $\pi = -p/\omega^2$ is associated to the pressure variable, thus leading to final functional $\Im'$ related to fluid-structure system:

$$\begin{aligned} \Im' &= \frac{1}{2}\int_{V_s} \bar{\bar{\sigma}} \otimes \bar{\bar{\varepsilon}}\, dv + \frac{1}{2}\int_{V_f} \frac{1}{\rho_f c^2} p^2\, dv \\ &-\omega^2\left[\frac{1}{2}\int_{V_s} \rho_s (x_s)^2\, dv - \frac{1}{2}\int_{V_f} \frac{1}{\rho_f} (\mathrm{grad}\, \pi)^2 dv - \int_{V_f} \frac{1}{\rho_f c^2} p\pi\, dv - \int_\Sigma \pi\, x_s \cdot n\, d\Sigma\right] \end{aligned} \tag{3}$$

The first two terms represent the strain energy of structure and of fluid (compressibility). The third term represents the kinetic energy contained in the structure. The three last terms represent the kinetic energy of the fluid (the complexity of the inertial term associated to the fluid is due to the fact that the dual variables p and π are employed to determine it).

Finite Element Discretization

We discretize the variational indicated expressions through a standard isoparametric interpolation function for X_s-P-Π variables,

$$x_s \cong x_{s,h} = \sum_{i=1}^{n} h_i x_{s,i}\,,\ p \cong p_h = \sum_{i=1}^{n} h'_i p_i \ \text{ and } \ \pi \cong \pi_h = \sum_{i=1}^{n} h''_i \pi_i \tag{4}$$

where, h_i, h'_i, h''_i are the various shape functions involving solid and fluid variables.

3. Discretization of the Solid Domain

By using the linear elastic element 2D of plane stress state (PSS) in order to discretize the solid domain, we obtain the most general configuration of the problem,

$$x_{s,h} = \underline{H}\vec{X}_{s,h} \tag{5}$$

where n is number of element nodes; u, v, w are displacement function of solid element for directions x, y and z, respectively; u_i, v_i, w_i represent the nodal displacements of solid element in node i for corresponding directions; h_i is the interpolation function for node i; [H_u H_v H_w] are the shape functions associated to all element nodes regarding the subscript displacement, i.e., the functions H_u of displacement function u are:

$$H_u = \begin{bmatrix} h_1 & h_2 & \dots & h_n \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{bmatrix}; \text{ and } \underline{H} = \begin{bmatrix} h_1 & 0 & 0 & h_2 & 0 & 0 & \dots & h_n & 0 & 0 \\ 0 & h_1 & 0 & 0 & h_2 & 0 & \dots & 0 & h_n & 0 \\ 0 & 0 & h_1 & 0 & 0 & h_2 & \dots & 0 & 0 & h_n \end{bmatrix}$$

is the shape function matrix for an element containing n nodes; $\vec{X}_{s,h}^T = [u_1\ v_1\ w_1\ u_2\ v_2\ w_2 \dots u_n\ u_n\ u_n]$ is the nodal displacement vector for an element containing n nodes.

4. Discretization of the Fluid Domain

Similarly, for the elements of the fluid, we have,

$$p_h = \underline{h}'\vec{P}_h \text{ e } \pi_h = \underline{h}''\vec{\Pi}_h \tag{6}$$

where, n is the number of elements nodes; p_h and π_h correspond to approximated functions of pressure and the variable π of fluid domain; p_i and π_i are pressure and π variables of the fluid in node i; h'_i and h''_i represent interpolation function of pressure and of variable π in node i;

5. First Variation of the Functional

By replacing the polynomial approximations in each term of functional (3), it is possible to compose the discretized functional of the fluid-structure problem:

$$\begin{aligned} \Im'_h &= \frac{1}{2}X_{s,h}^T \cdot \int_{V_s} \underline{B}^T \underline{DB} dv \cdot X_{s,h} + \frac{1}{2}\frac{1}{\rho_f c^2} P_h^T \cdot \int_{V_f} h^T h dv \cdot P_h \\ &- \omega^2 \left[\frac{\rho_s}{2} X_{s,h}^T \cdot \int_{V_s} H^T H dv \cdot X_{s,h} - \frac{1}{2}\frac{1}{\rho_f} \Pi_h^T \cdot \int_{V_f} \nabla h^T \cdot \nabla h dv \cdot \Pi_h \right. \\ &\left. - \frac{1}{\rho_f c^2} P_h^T \cdot \int_{V_f} h^T h dv \cdot \Pi_h - X_{s,h}^T \cdot \int_{\Sigma} H^T (\vec{n} \cdot h) d\Sigma \cdot \Pi_h \right] \end{aligned} \tag{7}$$

By minimising the approximated functional (7) regarding $X_{s,h}$, P_h and Π_h, we arrive at a matrix expression for the fluid-structure interaction (FSI),

$$\left(\begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{pp} & 0 \\ 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M_{xx} & 0 & -M_{x\pi} \\ 0 & 0 & -K_{p\pi} \\ -M_{x\pi}^T & -K_{p\pi}^T & -M_{\pi\pi} \end{bmatrix} \right) \cdot \begin{Bmatrix} X_{s,h} \\ P_h \\ \Pi_h \end{Bmatrix} = 0 \tag{8}$$

where, K_{xx} and M_{xx} are the stiffness and structure mass matrixes, K_{pp} is the compressibility matrix of the fluid ("stiffness matrix"); $M_{\pi\pi}$ describes the inertial effects of the fluid, and $M_{x\pi}$. and $K_{p\pi}$ correspond to the coupling matrixes X_s-Π and P-Π, respectively.

Matrix system (8) shall provide the natural modes for the fluid-structure coupling. The solution in the time domain will be easily obtained.

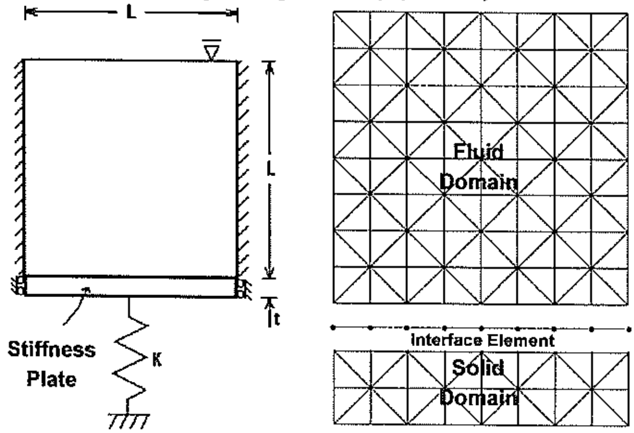
NUMERICAL RESULTS

We will show some typical examples of 2D problems in which it is possible to apply the proposed formulation and obtain theoretical solutions to validate them.

Some Cases of Stiffness Cavity with Movable Boundaries

This set of problems include a stiffness cavity with a movable boundary (a stiffness plate) which is discretized through linear triangular finite elements (LTFE) in a plane stress state (PSS) on an elastic support (springs), and the fluid is also discretized by LTFE.

1. Case 1A - Square Open Cavity (CO/CC) with a Piston



Problem's Data:

Geometric Characteristics:
$L_x = L_y = L = 1m$; $t = 0.05m$

Fluid Characteristics:
$\rho_f = 1000\ kg/m^3$; $c = 1500\ m/s$

Solid Characteristics:
$E = 2.10\cdot10^{11}\ N/m^2$; $\nu = 0.3$
$\rho_s = 7800\ kg/m^3$

Figure 1 - Scheme of Case 1A and Finite Element Mesh of Problem.

The coupled and uncoupled analytical solutions of the problem are [10,13], respectively,

$$\omega = \pi c\left({n_x^2}/{L_x^2} + {n_y^2}/{4L_y^2} \right)^{1/2},\ n_x = 0,1,2,\cdots;\ n_y = 1,3,5,\cdots\ \&\ \lambda^2\left(\mu + \frac{1}{\lambda}\mathrm{tg}\lambda\right) = \alpha \qquad (9)$$

where $\mu = m_s / m_f$ = ratio between the piston mass and the fluid mass; $\alpha = (KL)/(\rho_f c^2 \cdot S)$ = ratio between the spring stiffness and the fluid volume stiffness; $\lambda = (\omega \cdot L)/c$ = ratio between the cavity length and the wave length for a given natural frequency.

Table 1 - Frequencies Results of Case 1A.

Uncoupled Modes				
	ω - rad/s			
	Mode	Num.	Ref.[14]	Theoric.
Piston	-	14.32	15.19	14.32
Fluid	1 - 10	2359.36	2359.35	2356.19
	2 - 11	5360.29	5317.50	5268.58
	3 - 20	7154.72	7154.55	7068.58
	4 - 21	8748.01	8719.01	8495.38
	5 - 12	10060.19	9968.31	9714.84

Coupled Modes				
	ω - rad/s			
	Mode	Num.	Ref.[14]	Theoric.
Fluid Piston	1	7.62	7,59	8,04
	2	3597.83	3597,82	3586,65
	3	5288.32	5316,83	5268,58
	4	7873.30	7873,08	7758,90
	5	8614.61	8717,87	8495,38
	6	100052.22	--	--

The mesh shown in Figure 1 helps discretize the problem, where the fluid domain has 128 elements containing 81 nodes and the solid domain has 32 elements containing 27 nodes. Eight interface elements containing 9 nodes are employed.

By analysing the modes and frequencies in the system*, we can realise that the first coupled frequency derives from the additional mass effect, this frequency being given approximately by $\omega^2 = K/(m_s + m_f)$. This is an indication that for low frequencies the compressibility effect can be neglected. It is also possible to notice that there is an uncoupling

* Due to the impossibility of to use colour graphics in this congress, we decide to choose a simplified representation of mode shapes.

of the transversal modes, which is due to the unidirectional translation displacements of the piston as a stiff body.

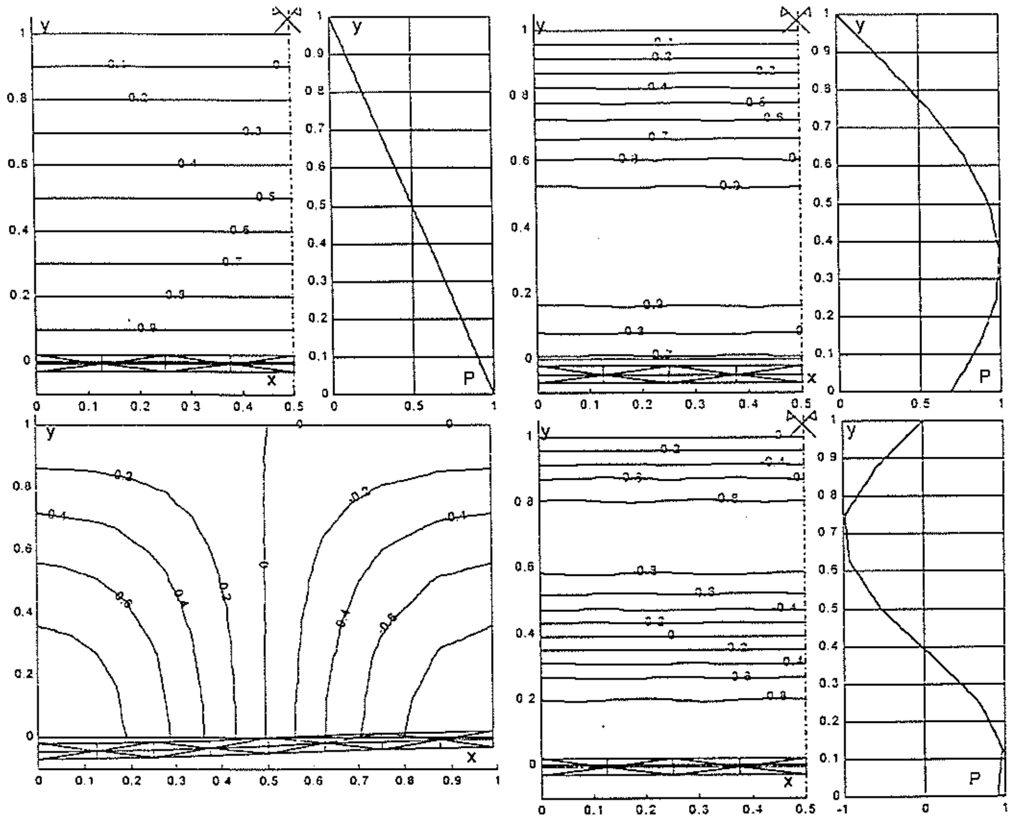


Figure 2 - Coupled Vibration Modes of Case 1A: (a) 1st, (b) 2nd , (c) 3rd and (d) 4rd modes.

2. Case 1B - Square Cavity with a Double Piston

The cavity mentioned above is employed again, with two plates in opposite ends. The coupled and uncoupled solution [10] (phase and phase opposition) are, respectively,

Uncoupled $$\omega = \pi c \left({n_x^2}/{L_x^2} + {n_y^2}/{L_y^2} \right)^{1/2}, \quad n_x = 0,1,2,\cdots; n_y = 0,1,2,\cdots \tag{10}$$

Coupled $$\lambda^2 \left[\mu - \frac{1}{\lambda} \left(\frac{1}{\tan\lambda} - \frac{1}{\operatorname{sen}\lambda} \right) \right] = \alpha, \ \lambda^2 \left[\mu - \frac{1}{\lambda} \left(\frac{1}{\tan\lambda} + \frac{1}{\operatorname{sen}\lambda} \right) \right] = \alpha \tag{11}$$

The coupled analytical solution is applied to the longitudinal modes, while the transversal modes are identical to the uncoupled solution, Eq.(10). These results are displayed in Figure 4 and Table 2.

The remarks above about the coupling effect can also be fully applied in this case. A coupling can only be found in those cases where the transversal mode component corresponds to zero. The existence of two mobile plates enables the fluid column to have a certain mobility, thus generating a mode (phase) of additional mass, whose frequency is $\omega_1^2 = 2\pi\, K/(2m_s + m_f/2)$.

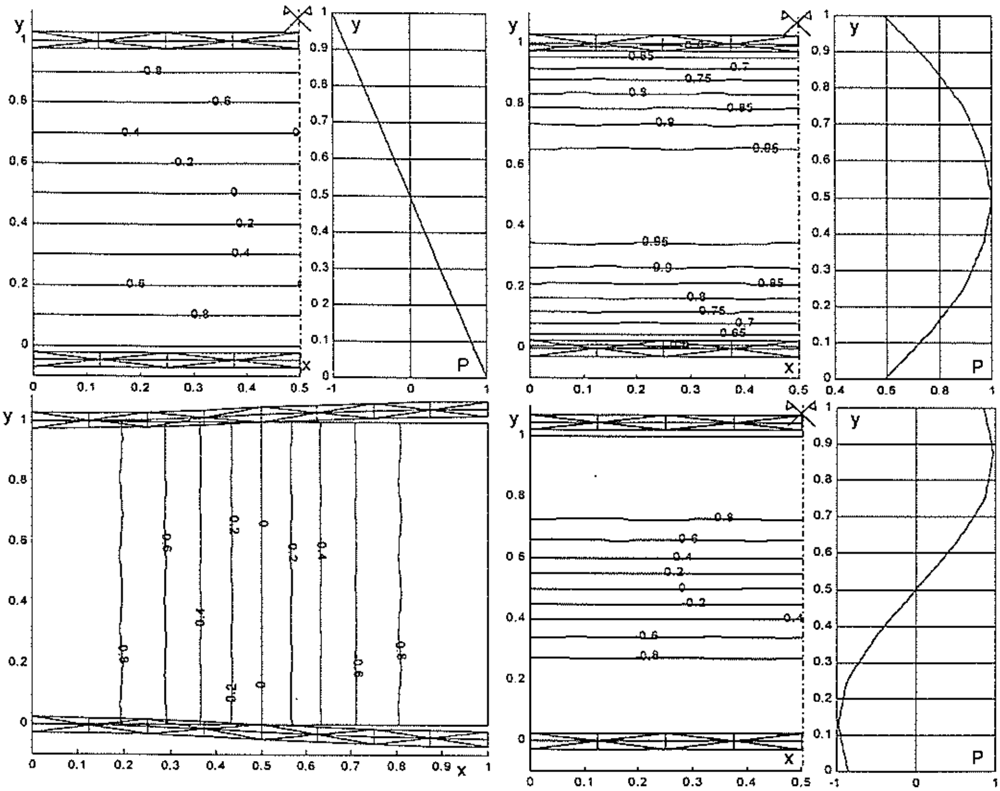


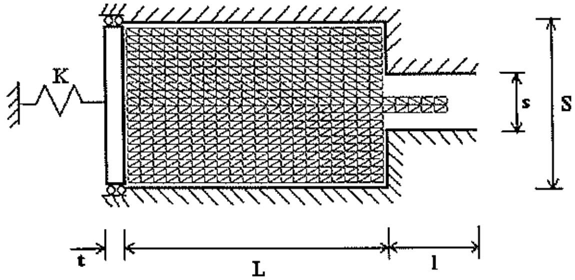
Figure 3 - Coupled Vibration Modes of Case 1B: (a) 1st, (b) 2nd , (c) 3rd and (d) 4rd modes.

Table 2 - Frequencies Results of Case 1B.

Coupled Modes				
	ω - rad/s			
	Mode	Numerical	Ref.[14]	Theoric.
Fluido-Placa	1	9.48	10,06	9,48
	2	2821.61	2821,72	2816,31
	3	4735.56	4737,14	4712,39
	4	6409.07	6409,20	6346,84
	5	6771.16	6776,79	6664,32

3. Case 2 - Tube Connections (Helmholtz resonator)

Case 2 is useful for studying the section variation at the connection of two pipes. A large pipe with length L and section S is connected to another smaller pipe, with length l, variable section s and walls with the same degree of stiffness. A stiffness plate supported by a spring with constant K is coupled to the end of larger pipe. The plate is able to move freely on the longitudinal direction of the pipe. The end of the smaller pipe is open. The frequencies and mode shapes of this acoustic cavity are calculated, and the section of smaller pipe is s = 0,1m.



Problem's Date:

Geometric Characteristic:

$L = 1m$; $l = 0.50m$; $t = 0.05m$

$S = 1m^2$; $s = 0.10m^2$

Fluid Characteristics:

$\rho_f = 1000\ kg/m^3$; $c = 1500\ m/s$

Solid Characteristics:

$E = 2.10 \cdot 10^{11}\ N/m^2$; $\nu = 0.3$

$\rho_s = 7800\ kg/m^3$

Figure 4 - Scheme of Case 2 (Helmholtz resonator) and Finite Element Mesh.

In this case, the frequencies to be calculated are those on the longitudinal direction, on the axis where the two pipes are positioned. The coupled and uncoupled solutions are, respectively,

$$\cos(2\lambda)\cos(\lambda/2) - \tfrac{1}{100}\cdot \mathrm{sen}(2\lambda)\mathrm{sen}(\lambda/2) = 0, \qquad \lambda = \omega_n L/c \tag{12}$$

$$\alpha - \theta_n\left[\theta_n \mu + \left(\mathrm{sen}\,\theta_n \cos\tau\theta_n + \sigma\cos\theta_n\,\mathrm{sen}\,\tau\theta_n / \cos\theta_n \cos\tau\theta_n - \sigma\,\mathrm{sen}\,\tau\theta_n\,\mathrm{sen}\,\tau\theta_n\right)\right] = 0 \tag{13}$$

where, $\sigma = S/s$ is the area ratio, and $\tau = L/l$ is the length ratio between the pipes.

Table 3 brings together the numerical and analytical calculations for this case.

Table 3 – Numerical an Theoretical Frequencies Results of Case 2

ω	s(m²) = 0,1					
(rad/s)	Desacoplado			Coupled		
Mode	Numerical	Ref.[15]	Theoric.	Numerical	Ref.[15]	Theoric.
1	11,32	11,04	11,32	3,64	3,39	3,59
2	426,45	426,45	443,50	1278,58	1008,61	1019,35
3	2425,48	2425,59	2427,50	3441,78	2855,85	2854,25
4	4725,34	4725,35	-	4721,24	4725,24	-
5	4731,71	4731,71	4712,39	5683,25	5004,61	4981,93

Again, the numerical results prove to be very close to the analytical values. Both the cases involving couplings and uncouplings have come very close to the expected results.

CONCLUSIONS AND PERSPECTIVES

The solution of the problem as a whole is extremely simplified due to the use of symmetrical matrixes. A classical algorithm (QZ method) [14] has been employed to solve eigenvalue problems. One of the problems was a difficulty in reaching an accordance with analytical solution due to numerical errors caused by the ill-conditioning of matrixes.

The results obtained have been excellent as compared to analytical calculations. The minor discrepancies were partly due to discretization and ill-conditioning of eigenvalue problem.

The results obtained through the comparison between the coupling and uncoupling cases show how important the influence of the solid-fluid interaction was on the main frequencies in the system and modal shapes.

Some of the prospect uses of this formulation are, mainly, the solution of more complex problems, associating other types of finite elements for fluid and solid, the analysis of the modal sensibility in the problem and inclusion of other types of boundary conditions (free surface, radiation, etc...).

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