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Non-linear dynamic analysis with non-proportional damping

Claret, A.M.[1], Rodrigues, A.G.[1], Venâncio Filho, F.[2]
1) School of Mines, Ouro Preto, Minas Gerais, Brazil
2) COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, Brazil

ABSTRACT: In this paper an extension of a method for the treatment of non-proportional damping is presented which incorporates non-linear behavior of the material. Nonlinearity and non-proportional damping are considered through a pseudo-force term in the RHS of the modal equations. These equations are then solved by a partial iterative process.

1 INTRODUCTION

Dynamic analysis of structural systems by the modal superposition method in its classical formulation presents two main advantages over time domain methods: a good interpretation of the system behavior through the analysis of the natural frequencies and normal modes and economy of computational effort due to the possibility of mode truncation.

In general it is assumed in the dynamic analysis of structural systems that the distribution of damping forces is like the inertia and elastic ones. When this is the case, the system damping is called proportional. On the other hand, in composite systems, when the damping properties of the component elements are quite different, in nature or in magnitude, the damping is non proportional. Proportional damping occurs when the damping matrix [c] is diagonalized by the modal matrix [Φ], i.e., when the triple product $[\Phi]^t[c][\Phi]=[C]$, [C] being the diagonal generalized damping matrix. In many cases of non-proportional damping the foregoing assumption has been used and the results have been shown greatly in errors (Claret & Venâncio-Filho 1991, Warburton & Soni 1977).

Non-proportional damping is typical of structural systems composed of materials with different damping properties as, for example, soil-structure interaction systems and systems with discrete dampers. On the other hand, nonlinearity is due to high levels of strain in the soil.

A previous work by Landau (1983) has shown that the pseudo-force method is an efficient alternative for the dynamic analysis of structural systems with nonlinearity. In this method, nonlinearity is considered in the RHS of equations of motion and a purely iterative procedure is carried out to ensure equilibrium within a time step. Therefore there is no need to update the structural properties, once the LHS of the equations remains unchanged.

Recently Claret & Venâncio-Filho (1991) and Ibrahimbegovic & Wilson (1989) introduced a pseudo-force method for the treatment of non-proportional damping. In this method, the pseudo-force concept is applied in order to solve the coupled (in face of non-proportional damping) modal equations.

The present paper gathers the pseudo-force concept for the simultaneous consideration of non-proportional damping and nonlinearity which have been in the past individually considered. This gathering is, in passing, suggested by Léger (1993). The coupled modal equations due to non-proportional damping and taking into account nonlinearity are solved by the pseudo-force method through a partial iterative process. Examples of the analysis of soil-structure interaction systems are presented.

2 THE PSEUDO-FORCE METHOD WITH NONLINEARITIES AND NON-PROPORTIONAL DAMPING

Consider the dynamic equilibrium equation of a linear MDOF system with N degrees of freedom

(1) $$[m]\{\ddot{v}\}+[c]\{\dot{v}\}+[k]\{v\}=\{p(t)\}.$$

In this equation, [m], [c] and [k] are, respectively, the (NxN) mass, stiffness and damping matrices; $\{\ddot{v}\}$, $\{\dot{v}\}$ and $\{v\}$ (nodal coordinates), and $\{p(t)\}$ are the (Nx1) vectors of accelerations, velocities, displacements and external loads. If nonlinearity is present, equation 1 is modified as follows:

(2) $$[m]\{\ddot{v}\}+[c]\{\dot{v}\}+([k_E]+[k_N])\{v\}=\{p(t)\}.$$

$[k_E]$ and $[k_N]$, in equation 2, are, respectively, the elastic stiffness matrix and the non-linear stiffness matrix which depends on the present deformation state of the system. When the non-linear term $[k_N]\{v\}$ on the LHS of equation 2 is transferred to the RHS that equation transforms in

(3) $$[m]\{\ddot{v}\}+[c]\{\dot{v}\}+[k_E]\{v\}=\{p(t)\}+\{q(t)\}$$

where

(4) $$\{q(t)\}=-[k_N]\{v\}$$

is the pseudo-force term which takes into account the nonlinearity. The LHS of equation 3 is then purely linear.

Equation 3 can be solved by a total or a partial iterative process. In the total process, the iterations are performed along the total time interval in which the system response is desired. On the other hand, the partial process considers the total time interval divided in sub-intervals and the iterations are successively performed along that sub-intervals.

In order to consider also non-proportional damping, the pseudo-force method introduced by Claret & Venâncio-Filho (1991) is applied. For this, equation 3 must be transformed to modal coordinates by the modal transformation

(5) $\{v\}=[\Phi]\{Y\}$

where $[\Phi]$ is the (NxK) modal matrix (K<N), $\{Y\}$ is the (Kx1) vector of modal coordinates. Introducing now $\{v\}$ and its derivatives from equation 5 into equation 3, pre-multiplying both sides of the resulting equation by $[\Phi]^t$, and taking into account that the normal modes are normalized, the following equation in modal coordinates is obtained:

(6) $\{\ddot{Y}\}+[C]\{\dot{Y}\}+[\Lambda]\{Y\}=\{P(t)\}+\{Q(t)\}.$

In equation 6, $[\Lambda]$ is the (KxK) diagonal matrix formed by the natural frequencies squared, $\{P(t)\}$ and $\{Q(t)\}$ are the (Kx1) vectors of generalized forces and pseudo-forces given respectively by

(7a) $\{P(t)\}=[\Phi]^t\{p(t)\}$

and

(7b) $\{Q(t)\}=[\Phi]^t\{q(t)\}.$

Moreover, in equation 6, $[C]$ is the (KxK) generalized damping matrix given by

(8) $[C]=[\Phi]^t[c][\Phi].$

As non-proportional damping is considered, $[C]$ is non-diagonal. This matrix is now splitted as

(9) $[C]=[C_d]+[C_f]$

where $[C_d]$ has the diagonal elements of $[C]$ and $[C_f]$ has zero diagonal elements and the corresponding off-diagonal elements of $[C]$. Substituting for $[C]$ in equation 6 from equation 9 one obtains

(10) $\{\ddot{Y}\}+[C_d]\{\dot{Y}\}+[\Lambda]\{Y\}=\{P(t)\}+\{Q(t)\}-[C_f]\{\dot{Y}\}.$

This equation, in modal coordinates, incorporates nonlinearity through the term $\{Q(t)\}$ and non-proportional damping through the term $-[C_f]\{\dot{Y}\}$. The individual decoupled equations (on the LHS) from equation 10, relative to each normal mode, are solved by a partial iterative process.

3 THE PARTIAL ITERATIVE PROCESS

The modal equation relative to the rth generic mode is firstly considered. This equation follows, from equation 10, as

$$(11)\quad \ddot{Y}_r + C_{rr}\dot{Y}_r + \Lambda_r Y_r = P_r(t) + Q_r(t) - \sum_{s=1,s\neq r}^{K} C_{rs}\dot{Y}_s .$$

The total response time interval is discretized in the time t_0, t_1,...,t_j,...t_J, the generic time interval being $\Delta t_j = t_{j+1} - t_j$. Equation 11 is then solved by a partial iterative process with the assumption of a linear variation of the RHS along the generic time interval. τ is the current time in the interval. The first subscript indicates the mode and the second, discretized time and the superscripts, the order of the iteration. $\dot{Y}_{rj}$ and Q_{rj} without superscripts are final values. Equation 11 can be integrated by the piecewise exact method according to Clough & Penzien (1993). The pseudo-forces are calculated with the present state of deformation. In the kth iteration, it is assumed that

$$(12)\quad \dot{Y}_r^{(k)}(\tau) = \frac{\dot{Y}_{r(j+1)}^{(k-1)} - \dot{Y}_{rj}}{\Delta t_j}\tau$$

and

$$(13)\quad Q_r^{(k)}(\tau) = \frac{Q_{r(j+1)}^{(k-1)} - Q_{rj}}{\Delta t_j}\tau .$$

For the first iteration the initial values are considered through the following linear extrapolations of the velocity and pseudo-force from the values in two subsequent previous time:

$$(14)\quad \dot{Y}_{r(j+1)}^{(0)} = 2\dot{Y}_{rj} - \dot{Y}_{j-1}$$

and

$$(15)\quad Q_{r(j+1)}^{(0)} = 2Q_{rj} - Q_{j-1} .$$

4 EXAMPLES

As a first example, the response of a ten degree of freedom shear building analysed by Kawamoto (1983) is examined. This structure has an elastic-perfectly plastic material model. The 1st period was 3.65 seconds and the 10th period, 0.28 seconds. A seismic excitation represented by a single cycle sine wave of duration 2 seconds and amplitude of 10 in/sec^2 is used as loading. To induce a localized nonlinear behavior and participation of higher modes in the response, a yield stress of 100 ksi is specified for each member, except at level four where it is 1 ksi. Rayleigh damping is employed with damping ratios $\xi_1 = \xi_4 = 5\%$ resulting in a proportional distribution of damping forces. Ten iterations are used in the analysis. Figure 1 shows the relative displacements of 10th degree of freedom which is the same obtained by Kawamoto (1983).

As a second example, the response of a seven degree of freedom shear beam type structure (Figure 2a) is analysed by the present method. The soil springs are described by an elastic-perfectly plastic material model having a yield stress of 3.0E+4 N. The fundamental period is 0.068 seconds and the 7th period, 0.0067 seconds. An external load represented by the function 8.0E+5 sin (60t) is applied at level 7. Rayleigh damping is employed by specifying a damping ratio os 5% for the 1st and 7th modes. Discret viscous dampers of constantes 8.5E+5 Ns/m are attached at levels 1 and 2, resulting in a non-proportional damping with a coupling index (Claret & Venâncio-Filho 1991) of 83.12%. Ten iterations are used and all modes are considered in the response. Figures 2b, 2c and 2d show, respectively, the relative displacements of the 1st, 2nd and 7th degrees of freedom.

5 CONCLUSIONS

The proposed method gathers the concept of pseudo-force for simultaneous consideration of non-proportional damping and nonlinearity. The numeric examples show two systems with localized nonlinearity and high non-proportional damping and the results indicates that the proposed method is well suited for nonlinear dynamic analysis with non-proportional damping.

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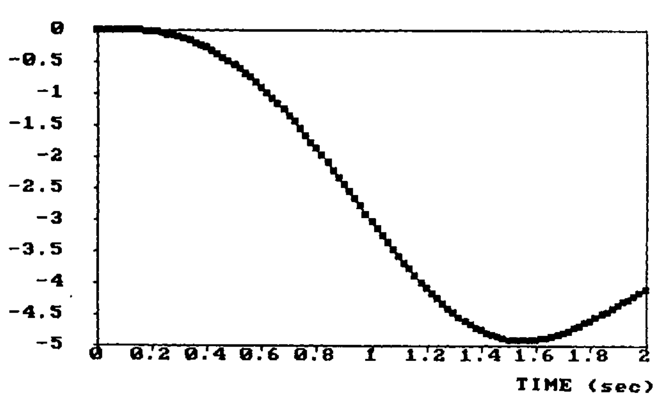

Figure 1. Relative displacements of 10th degree of freedom.

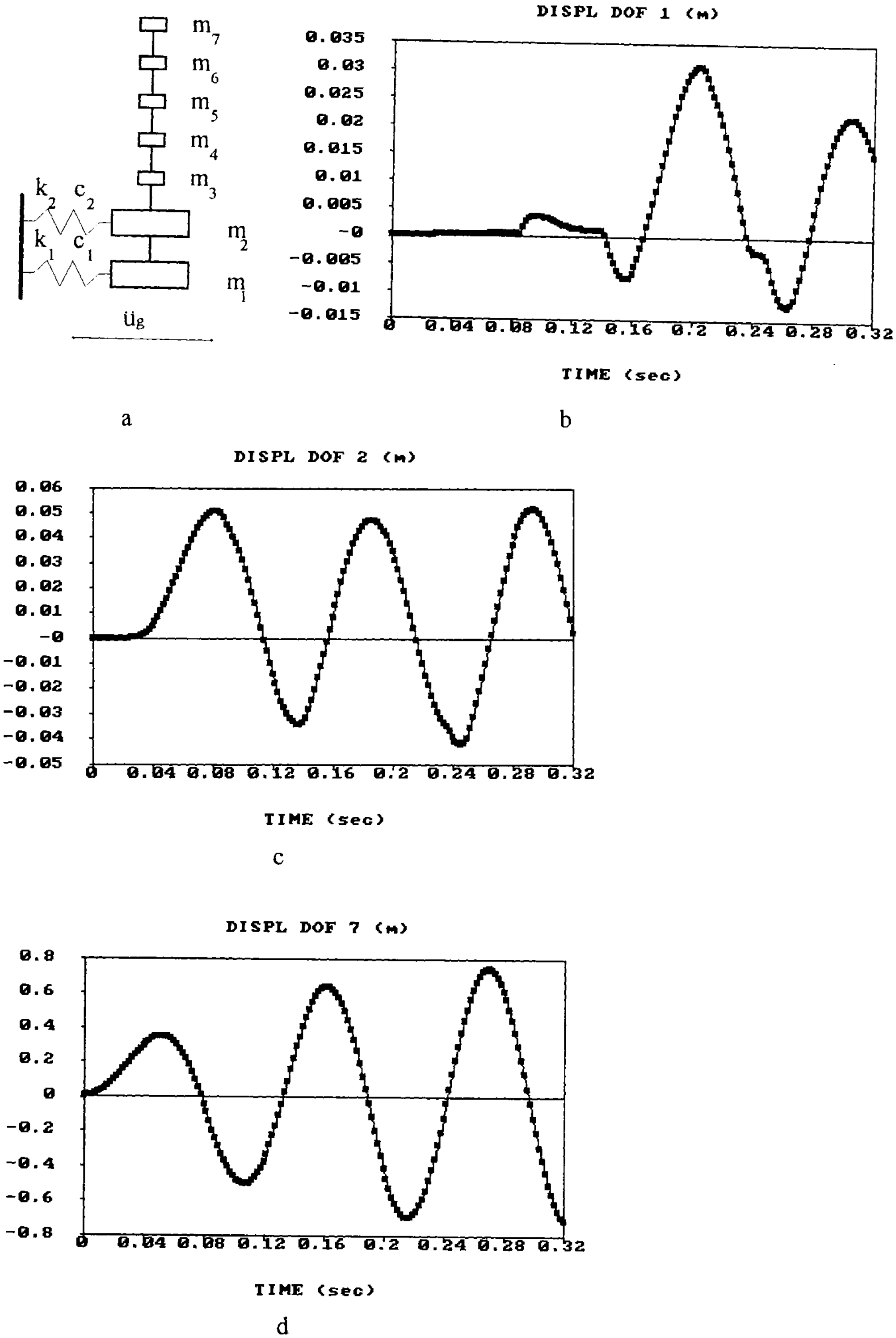

Figure 2. Shear beam type building of seven degrees of freedom (a); relative displacements of 1st foundation mass (b); relative displacements of 2nd foundation mass (c); relative displacements of 7th degree of freedom.