ASEISMIC DESIGN OF STRUCTURES WITH NUCLEAR REACTORS - METHOD OF EARTHQUAKE RESPONSE ANALYSIS FOR COMPOSITE STRUCTURES EVALUATED FOR DAMPING EFFICIENCIES BY MATERIAL AND STRUCTURE TYPE

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ABSTRACT

The purpose of this paper is to summarize analytical procedures which have been employed in the evaluation of dynamic properties and dynamic response values for earthquake motions putting emphasis on estimating the damping capacities of a special structure like a nuclear power plant which is made of various materials such as concrete, steel, special alloys, etc., and various structural types such as walls, shells, frames, trusses, etc., considering the interaction between foundation of structure and ground at time of earthquake motion.

The formulation of equations is discussed including the procedures for defining the complex stiffness matrix by which the damping capacities of structures can be estimated as hysteretic damping.

This paper further elaborates on techniques for solving the eigenvalue equation and the proof of orthogonality properties of the complex stiffness matrix.

Finally, the numerical results of dynamic properties and dynamic response analyses are obtained by using computer programs based on the abovementioned theory using some different earthquake motions for a nuclear power plant to be constructed in Japan and further considerations and discussions of usage are presented.

1. INTRODUCTION

A structure with a nuclear reactor consists of Nuclear Reactor at the center, steel Reactor Pressure Vessel, steel concrete Shield Wall, flask shaped Dry Well and Reactor Building of reinforced concrete walls. The dynamic properties of this structure are characterized as follows:

(a) These are composed of different structures and having a common foundation, they are interconnected which makes analysis very complicated. Therefore, these structures of which dynamical models are different, interact on each other at time of motion.

(b) Each has a different weight, stiffness and damping capacity.

Especially, Reactor Building has heavy weight in comparison with others. Response values of Reactor itself, which is the most important part of this structure, are affected by the dynamical properties of Reactor Building.

(c) Reactor Building being spatially constructed of many walls which react with the external force acting in various directions makes calculation of stiffness of the building difficult.

(d) Reactor Building being massive and having a short period, the dynamic properties (natural period, damping capacity) are affected by the interaction between foundation and ground. There is theoretically no difference in dynamic properties at time of earthquake motion between a structure like this and normal structures but it must be analyzed by the best method which can be considered for its dynamic properties.

(e) Reactor Building has heavy weight and rigid stiffness of which response values are quite large. On the other hand, Nuclear Reactor has light weight and less rigid stiffness of which response values are very small. Thus the difference in these two structures causes several problems at the time of analysis together. For the reasons mentioned above, the dynamic analysis must be done in the following two steps:

    1st step : dynamic analysis based on structures of heavy weight
    2nd step : dynamic analysis based on structures of light weight using results of 1st step

Considering the abovementioned dynamic properties of structures, a study has been made which will be explained hereinbelow.

(a) The dynamic analysis is performed using the model analysis method based on the lumped mass-spring system.

(b) The stiffness of Reactor Building and Dry Well are calculated by the finite element method (F.E.M.).

(c) Damping capacity is evaluated as hysteretic damping, which can be evaluated by expressing with complex numbers. Dynamic analysis and calculation of eigenvalues are performed using the complex stiffness.

(d) The stiffness and damping capacity of ground are analyzed according to Tajimi's theory.

Here a series of programs for dynamic analysis of earthquake motions are developed using an IBM computer S/360 J-75.

2. STIFFNESS OF STRUCTURES

<u>Reactor Building</u>

Reactor Building is spatially constructed with plain and shell shaped reinforced concrete walls. Therefore, stiffness of these walls are calculated by F.E.M., which is the best and most popular method for evaluating stiffness of structures.

Dry Well

Dry Well is a typical thin shell structure of which dynamic properties must be calculated considering movements in bending and shearing, which is similar to movement of free end of cantilever with change in shape of shell. Stiffness of this is also calculated using F.E.M..

Other Structures

Stiffnesses of other structures like Reactor Pressure Vessel and Shield Wall are calculated using the bending shear deflection theory.

Details about the stiffness calculation of structures are explained in the paper, "Aseismic Design of Nuclear Reactor Building — Stress Analysis and Stiffness Evaluation of the Entire Building by the Finite Element Method," [1].

3. DAMPING PROPERTIES OF STRUCTURES

Generally, the damping properties of structures are usually assumed as follows:

(a) viscous damping due to viscous properties of materials molecules
(b) hysteretic damping due to imperfect elasticity of members of structures
(c) structural damping due to slip motion at joints of members of structures
(d) radiation damping due to interaction of structure and ground at time of motion

In conventional response analyses of structures in general, the damping mechanism of a structure has been evaluated as viscous damping proportional to the motion velocity of the structure. There are various reasons for this, but one would be that this can be readily expressed in mathematical terms. In recent years, vibration test data of a large number of actual buildings have been obtained, and upon analyses of these data, it is found that in some structures the ratios (h/ω) between natural circular frequencies (ω) and damping ratios (h) are not necessarily constant.

From recent experimental study on dynamic properties of reinforced concrete frames the damping properties of reinforced concrete structures have been reported as hysteretic damping.

From analyses of the above data, it is theorized that the damping mechanism of a structure does not consist only of viscous damping, but also of hysteretic damping due to imperfect elasticity of the frame itself and radiation damping caused by the interaction between foundation of structure and ground. A special structure such as one containing a nuclear reactor, from the very fact that materials and structural types differ, would possess complicated damping ratios; analysis of earthquake response of such a structure must be performed by methods allowing adequate evaluation of the above damping ratios. For the reason that the requirements of such response analysis are satisfied, and the reason that, according to the dynamic theory of elastic semi-infinite medium, the radiation damping can be evaluated by expressing the stiffness of ground by complex numbers, an experiment was made expressing the stiffnesses of structure and ground by complex numbers ($K_R + iK_I$) and evaluating the damping ratios of the structure as hysteretic damping.

If the stiffness matrices are expressed by complex numbers, the equation of single freedom motion for the dynamic analysis of the structures can be expressed as follows:

$$M\ddot{x} + (K_R + iK_I)x = -M\ddot{x}_o \qquad (1)$$

where M : mass, $\ddot{x}$: acceleration of earthquake motion.
Details of the equation of motion will be explained elsewhere.

Imaginary Parts of Stiffness Matrices[2]

Therefore, in Eq. (1), the solution of equation may also be assumed as follows:

$$x = Ae^{-h\omega t}\, e^{i\omega t} \qquad (2)$$

Substituting Eq. (2) into Eq. (1), the following equation is obtained.

$$(-h\omega + i\omega)^2 + \frac{K_R + iK_I}{M} = 0 \qquad (3)$$

Separating the above equation into real part and imaginary part:

$$\omega^2(1 - h^2) = \frac{K_R}{M} \qquad 2\omega^2 h = \frac{K_I}{M} \qquad (4)$$

and assuming the damping ratio (h) to be negligibly small, the approximate formula can be introduced as follows:

$$\omega^2 = \frac{K_R}{M} \qquad h = \frac{1}{2} \cdot \frac{K_I}{K_R} \qquad (5)$$

Therefore, damping ratio (h) is apparently connected with only K_R and K_I and the imaginary parts of complex stiffnesses are obtained by the following equation

$$K_I = 2hK_R \qquad (6)$$

Properties of Hysteretic Damping Ratios

On the assumption that imaginary parts of complex stiffness may be proportional to the real part

$$K_R + iK_I = (1 + i2h)K_R \qquad (7)$$

in which the sign h expresses the damping ratio to be established for the particular system. Substituting the equation $x = ue^{i\lambda t}$ into Eq. (7), the original equation can be expressed as follows:

$$(1 + i2H)K_R U = \lambda^2 MU \qquad (8)$$

therefore, $K_R U = \omega_o{}^2 MU$ where ω_o : undamped natural circular frequency.

Expressing the undamped eigenvalue of j-th mode by the sign ω_{oj}, the damped eigenvalue of j-th mode (λj) is obtained by Eq. (8) as follows:

$$\lambda_j^2 = (1 + i2h)\omega_{oj}^2 \qquad (9)$$

Substituting $\lambda_j = \omega_j + i\omega_j h_j$ into Eq. (9) and separating the resultant equation into real part and imaginary part, the following two equations are obtained:

$$\omega_j^2 = \omega_{oj}{}^2/(1 - h_j^2) \qquad (10a)$$

$$h_j = h\omega_{oj}^2/\omega_j^2 \tag{10b}$$

Eliminating the sign ω_{oj} from the above two equations, the damping ratio of j-th mode can be expressed by the following equation:

$$h_j = (-1 + \sqrt{1+4h^2})/2h \tag{11}$$

The damping ratio (h_j) is constant by hypothesis and assuming the damping ratio (h) to be small enough compared with 1, it can be concluded that the damping ratio of each mode is equal to the damping ratio established by material and structural type.

Complex Stiffness Matrices of Multi-Degrees of Freedom

In calculating the complex stiffness matrices of a composite structure such as one containing a nuclear reactor, at first, the entire structure is divided into a number of groups (G) by damping ratios expressed by the sign $_GH$ and the complex stiffness matrices $(K_R + iK_I)_G$ in the local coordinate system for each group are prepared. The individual complex stiffness matrix contributing to each group is calculated by Eq. (12) from the individual real stiffness matrix ($_GK_R$) which is calculated by F.E.M. and other methods.

$$[K_R + iK_I]_G = (1 + i2_GH)[_GK_R] \tag{12}$$

In the second stage, the complete complex stiffness matrix can be obtained by superposing of the individual complex stiffness matrix in the global coordinate system of the entire structural assemblage by means of the following equation.

$$[K_R + iK_I]_S = \sum_G [K_R + iK_I]_G \tag{13}$$

As a result, the equation of motion of the entire structure can be obtained as follows:

$$[M]_S\{\ddot{x}\} + [K_R + iK_I]_S\{x\} = -[M]\{\ddot{x}_o\} \tag{14}$$

4. INTERACTION BETWEEN FOUNDATION AND GROUND

Foundation - Ground System

The ground may act in the two directions of propagating earthquake motions and supporting structures at time of motion. Therefore, it assumes radiation energy caused by interaction between foundation and ground. The effect of the interaction mentioned above must be considered during seismic analysis of important structures such as a nuclear reactor building. In dynamic analysis of the foundation-ground system, two effects, one on the underside and the other on the lateral side of foundation must be considered. Consequently, the foundation-ground system must be idealized by a method which can estimate the stiffness of ground analytically with accuracy at the time of the abovementioned effects. To accomplish the analysis required, the foundation assumed to be attached to the ground by means of some springs and dashpots which consist of lateral spring-dashpots and a rotational spring-dashpot as shown in Fig. 1 and Fig. 4. The analysis in this paper is carried out with the application.

Calculation of Stiffness of Ground

In preceding studies, there have been two well known techniques for calculation of stiffnesses: one of them deals with the ground statically and the other deals with the ground dynamically being based on the application of the wave propagation theory. Aiming for practical uses in dynamic analyses of structures, the coefficient of subgrade reaction, from which the stiffnesses can be calculated, is defined as a linear relation between stress and strain of soil and expressed analytically as a function of stress of ground, the shape of a foundation and its area. In addition to effect of spring, the interaction also contains the effect which is known as dissipation of energy and by which the effect of spring is decreased and effect of damping resultingly grows. For the purpose of estimating the effect of damping capacity, the stiffness must be calculated considering dynamic properties of the interaction.

This problem of dynamic properties of interaction has been solved using the experimental results of forced vibration tests by means of exciter. This has been also analytically and numerically expressed based on the theoretical displacement of foundation on an elastic semi-infinite caused by a harmonic force as expressed by H. Tajimi and T. Kobori-R. Minai.

From the abovementioned investigations, the stiffness of ground can be expressed in the form of a complex number as shown below:

$$ {}_{F}K\,(\omega) = {}_{F}K(\omega) + i\,{}_{F}C(\omega) \qquad (15) $$

The complex number ${}_{F}K(\omega)$ can be analytically obtained as a function of the exciting natural circular frequency (ω) of which ${}_{F}K(\omega)$ and ${}_{F}C(\omega)$ are respectively described in the form of two curved lines as shown in Fig. 5. As it is very difficult to calculate the eigenvalues of structures in the form of Eq. (15) considering the interaction at time of earthquake motion, Eq. (15) must be changed into the same complex number form as the stiffnesses of upper structures which are independent of the excitation by natural circular frequency. Therefore, firstly the complex stiffness is calculated for the fundamental natural circular frequency (ω_1) by the equation below.

$$ {}_{F}K(\omega_1) = {}_{F}K(\omega_1) + i\,{}_{F}C(\omega_1) \qquad (16) $$

By using this result, eigenvalues can be approximately calculated. But estimating the stiffness being independent of natural circular frequencies, the radiation damping of ground for higher order modes is usually evaluated to be less than the actual value.

Actually, as ω_1 has not yet been determined at time of calculation of the stiffness, at first, the stiffness must be calculated for the approximate frequency and further approximate eigenvalues can be calculated using this result.

After these successive procedures are iterated several times, elaborate eigenvalues can be obtained. As the abovementioned procedures require a great deal of labor and time, for practical purposes of dynamic analysis, it is better to use ω_1 which was calculated neglecting damping capacities.

As the other method for calculation of damping capacity without theoretical calculations, some values of damping ratio are determined by engineering judgment and using this result the stiffness and damping capacity are commonly calculated.

Because of lack of sufficient space to describe in detail the theory of complex stiffness expressed as a function of exciting frequencies, only the numerical results calculated by Tajimi's theory[3] will be introduced. The abovementioned investigations made by Tajimi and Kobori - Minai, described the interaction between foundation and ground on the underside of a foundation, but theories regarding effect of surrounding ground on the lateral sides have not yet been established.

Therefore, for practical purposes the resistance effect of surrounding ground must be commonly estimated by means of coefficient of subgrade reaction and some values of damping ratio are determined by engineering judgment and stiffness and damping capacity must be calculated using these damping ratios.

5. EQUATION OF MOTION

The structures are idealized by the lumped mass-spring system shown in Fig. 1. The equation of motion of the multi-degrees of freedoms system, which is allowed to sway and rock under earthquake motions ($\ddot{x}_o$), can be mathematically expressed using the coordinate system and parameters indicated in Fig. 1 as follows:

$$[M]\{\ddot{x}\} + [K]\{x\} = -[M]\{C\}\ddot{x}_o \tag{17}$$

where Eq. (17)

$[M]$: mass matrix

$[K]$: complex stiffness matrix

$\{x\}$: complex displacement vector

$\{\ddot{x}\}$: complex acceleration vector

$\{C\}$: vector of external excitation

where the matrix [M]

$$[M] = \begin{bmatrix} m_{rr} & & & & & & 0 \\ & m_B & & & & & \\ & & m_2 & & & & \\ & & & \cdot & & & \\ & & & & m_i & & \\ & & & & & \cdot & \\ 0 & & & & & & m_n \end{bmatrix} \tag{18}$$

m_i : mass of point "i"

m_B : total mass of foundation

I_B : inertia moment of foundation around axis at point "BE"

I_i : inertia moment of i-th mass around axis at its gravity center

$$m_{rr} = I_B + m_B(H_B - H_{BE})^2 + \sum_{i=2}^{n} I_i \tag{19}$$

where the matrix [K]

$$[K] = \begin{bmatrix} k_{rr} + ik'_{rr} & k_{rs} + ik'_{rs} & k_{r2} + ik'_{r2} & --- & k_{ri} + ik'_{ri} & --- & k_{rn} + ik'_{rn} \\ k_{sr} + ik'_{sr} & k_{ss} + ik'_{ss} & k_{s2} + ik_{s2} & --- & k_{si} + ik'_{si} & --- & k_{rn} + ik'_{sn} \\ k_{2s} + ik'_{2r} & k_{2s} + ik'_{2s} & k_{22} + ik'_{22} & --- & k_{2i} + ik'_{2i} & --- & k_{2n} + ik'_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ k_{ir} + ik'_{ir} & k_{is} + ik'_{is} & k_{i2} + ik'_{i2} & --- & k_{ii} + ik'_{ii} & --- & k_{in} + ik'_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ k_{nr} + ik'_{nr} & k_{ns} + ik'_{ns} & k_{n2} + ik'_{n2} & --- & k_{ni} + ik'_{ni} & --- & k_{nn} + ik'_{nn} \end{bmatrix} \tag{20}$$

$$k_{rr}+ik'_{rr} = k_R+ik'_R + \int_0^{H_{Bn}} (k_s(y)+ik_s(y))(y-H_{BE})^2)\ dy + \sum_{r_2=2}^{n} \sum_{r_1=2}^{n} (k_{r_2r_1}+k'_{r_2r_1}) \times (H_{r2}-H_{BE})(H_{r1}-H_{BE}) \tag{21}$$

$$k_{rs}+ik'_{rs} = \int_0^{H_{Bn}} (k_s(y)+ik'_s(y)(y-H_{BE}))\ dy + \sum_{r_2=2}^{n} \sum_{r_1=2}^{n} (k_{r_2r_1}+ik'_{r_2r_1}) \times (H_{r2}-H_{BE}) \tag{22}$$

$$k_{ss}+ik'_{ss} = \int_0^{H_{BE}} (k_s(y)+ik_s(y))\ dy + \sum_{r_2=2}^{n} \sum_{r_1=2}^{n} (k_{r_2r_1}+ik'_{r_2r_1}) \tag{23}$$

$$k_{si}+ik'_{si} = k_{is}+ik'_{is} = -\sum_{r_1=2}^{n} k_{r_1 i} \tag{24}$$

$$k_{ri}+ik'_{ri} = k_{ir}+ik'_{ir} = -\sum_{r_1=2}^{n} k_{r_1 i}\ (H_i-H_{BE}) \tag{25}$$

where vectors $\{\ddot{x}\}$, $\{x\}$ and $\{C\}$

$$\{\ddot{x}\} = \begin{Bmatrix} \ddot{\theta} + i\ddot{\theta}' \\ \ddot{x}_{BE} + i\ddot{x}'_{BE} \\ \ddot{x}_2 + i\ddot{x}'_2 \\ \vdots \\ \ddot{x}_i + i\ddot{x}'_i \\ \vdots \\ \ddot{x}_n + i\ddot{x}'_n \end{Bmatrix} \quad (26) \qquad \{x\} = \begin{Bmatrix} \theta + i\theta' \\ x_{BE} + ix'_{BE} \\ x_2 + ix'_2 \\ \vdots \\ x_i + ix'_i \\ \vdots \\ x_n + ix'_n \end{Bmatrix} \quad (27) \qquad \{C\} = \begin{Bmatrix} 0 \\ 1.0 \\ 1.0 \\ \vdots \\ 1.0 \\ \vdots \\ 1.0 \end{Bmatrix} \quad (28)$$

6. EIGENVALUE PROBLEM

Eigenvalue Calculation

For determination of dynamic properties, the damped free vibration equation (29), obtained from Eq. (17), needs to be analized as follows:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{29}$$

In order to obtain the eigenvalues and the eigenvectors, a solution for Eq. (29) may be made in the following form:

$$\{x\} = \{x\}e^{i\lambda t} \tag{30}$$

where the sign λ expresses the complex natural circular frequency and the sign $\{x\}$ expresses the complex eigenvectors. Substituting Eq. (30) and the equation $\{x\} = [M]^{-\frac{1}{2}}\{Y\}$ into Eq. (29), Eq. (29) is changed as follows:

$$[K][M]^{-\frac{1}{2}}\{Y\} = \lambda^2[M][M]^{-\frac{1}{2}}\{Y\} \qquad (31)$$

Premultiplying both sides by $[M]^{-\frac{1}{2}}$:

$$[M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}}\{Y\} = \lambda^2[M]^{-\frac{1}{2}}[M][M]^{-\frac{1}{2}}\{Y\}$$

or in other form $\quad [\widetilde{K}]\{Y\} = \lambda^2\{Y\} \qquad (32)$

here $[K]$ is symmetric and $[\widetilde{K}]$ is also symetric since

$$[\widetilde{K}]^T = ([M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}})^T = [M]^{-\frac{1}{2}}[K]^T[M]^{-\frac{1}{2}} = [\widetilde{K}]$$

now in order to obtain the solution, Eq. (32) must be analyzed as follows:

The method of analyzing Eq. (32) consists of two parts;[5]

(a) 1st step: The given matrix $[\widetilde{K}]$ is reduced to almost triangular (Hessenberg) form $[H]$ by elementary similarity transformations. It follows that

$$[T]^{-1}[\widetilde{K}][T] = [H] \qquad (33)$$

Substituting Eq. (33) into Eq. (32), Eq. (34) is obtained as follows:

$$[H]\{Y\} = \lambda^2[I]\{Y\} \qquad (34)$$

(b) 2nd step: The iterative search for the eigenvalues of $[H]$ is performed. A natural extension of Hyman's method may be used to evaluate $P(\lambda^2) = \det(H - \lambda^2 I)$ and any number of derivatives in an accurate and stable way. By hypothesis, eigenvalues of matrix $[H]$ are correspondent to those of matrix $[M]$ and $[K]$.

The j-th eigenvalue (λ_j) can be solved through the above procedure and can be expressed by a real part (λ_{Rj}) and by an imaginary part (λ_{Ij}) as follows:

$$\lambda_j = \lambda_{Rj} + i\lambda_{Ij} \qquad (35)$$

Therefore, the j-th natural frequency (ω_j) and the j-th damping ratio (h_j) are given as follows:

$$\omega_j = \lambda_{Rj} \qquad (36)$$

$$h_j = \lambda_{Ij}/\lambda_{Rj} \qquad (37)$$

Eigenvector Calculation

By replacing successively the eigenvalues (λ_j) in Eq. (34), the eigenvectors $\{x_j\}$ are obtained by analyzing the simultaneous linear equation through a procedure known as the Gauss reduction. The j-th eigenvectors (x_j) can be expressed by a real part (x_{Rj}) and by an imaginary part (x_{Ij}) as follows:

$$\{x_j\} = \{x_{Rj}\} + i\{x_{Ij}\} \qquad (38)$$

Orthogonality Property of Complex Matrix[4]

Matrix [M] and [K] are respectively real and complex symmetric matrix, let λ_i and λ_j be eigenvectors and let $\{x_i\}$ and $\{x_j\}$ be complex eigenvectors corresponding respectively to λ_i and λ_j. Here $i \neq j$

Then $$[K]\{x_i\} = \lambda_i[M]\{x_i\} \text{ and } [K]\{x_j\} = \lambda_j[M]\{x_j\} \quad (39)$$

If the first equation in premultiplied by $\{x_j\}^T$ and the second by $\{x_j\}^T$, the following are respectively obtained:

$$\{x_j\}^T[K]\{x_i\} = \lambda_i\{x_j\}^T[M]\{x_i\} \quad (40)$$

and $$\{x_i\}^T[K]\{x_j\} = \lambda_j\{x_i\}^T[M]\{x_j\} \quad (41)$$

Now, if transpose of each side of Eq. (40) is taken, remembering that [K] and [M] are $[K]^T = [K]$ and $[M]^T = [M]$, the following equation is obtained:

$$\{x_i\}^T[K]\{x_j\} = \lambda_i\{x_i\}^T[M]\{x_j\} \quad (42)$$

Finally, substracting Eq. (41) from Eq. (42), the following equation is obtained.

$$(\lambda_i-\lambda_j)\{x_i\}^T[M]\{x_j\} = 0 \quad (43)$$

Therefore, since $\lambda_i \neq \lambda_j$, by hypothesis, it follows that

$$\{x_i\}^T[M]\{x_j\} = 0 \qquad i \neq j \quad (44)$$

On the other hand, since $\lambda_i = \lambda_j$, by hypothesis, it follows that

$$\{x_i\}^T[M]\{x_j\} = C \qquad i = j \quad (45)$$

where C is a complex number.

If the matrix [K] is a symmetric real matrix and the matrix [M] is a unit matrix, C is equal to 1.

7. RESPONSE ANALYSIS

The dynamic response analysis of a structure is performed by solving the equation of motion Eq. (17). There are two different techniques for the dynamic response analysis. One of them is step by step direct integration of Eq. (17) and the other is the mode superposition method well known as the modal analysis method. The dynamic response analysis in this study is performed by the modal analysis method, using eigenvalues and eigenvectors.

The basic concept of the modal analysis method is that the displacement vectors $\{x\}$ and the earthquake motion vectors $\{C\}$ can be expressed as a linear combination of vibration modes as follows:

$$\{x\} = q_1\{x_1\} + q_2\{x_2\} + \ldots + q_j\{x_j\} + \ldots + q_{m-1}\{x_{m-1}\} + q_m\{x_m\} \quad (46)$$

$$\{C\} = \beta_1\{x_1\} + \beta_2\{x_2\} + \ldots + \beta_j\{x_j\} + \ldots + \beta_{m-1}\{x_{m-1}\} + \beta_m\{x_m\} \quad (47)$$

In the above equations q_j and β_j represent a modal displacement and a participation coefficient of j-th mode. The modal amplitudes consisting of acceleration, velocity and displacement are functious of time (t) and β_j is expressed by the following equation:

$$\beta_j = \sum_{r=1}^{n} m_r x_{rj} \Big/ \sum_{r=1}^{n} m_r x_j{}^2 \quad (48)$$

Substituting Eqs. (46) and (47) into Eq. (17)

$$\sum_{j=1}^{m}[M]\{x_j\}\ddot{q}_j + \sum_{j=1}^{m}[K]\{x_j\}q_j = -\ddot{x}_o \sum_{j=1}^{m}[M]\{x_j\}\beta_j \tag{49}$$

Then premultiplying by the transpose of an arbitrary modal vector x_i^T which is not the same as the j-th mode and taking advantage of the orthogonality properties, a single uncoupled equation of motion for j-th mode is obtained.

$$\ddot{q}_j + \lambda_j^2 q_j^2 = -\beta_j \ddot{x}_o \tag{50}$$

or

$$\ddot{q}_{jo} + \lambda_j{}^2 = -\ddot{x}_o \tag{51}$$

where $q_{jo} = q_j/\beta_j$

For arbitrary loading earthquake motions, the solution of each modal response Eq. (51) can be performed by the Duhamel integral.

When the modal responses consisting of accelerations, velocities and displacements for all significant modes have been determined at any time "t", the response values of mass points at this time are then obtained by Eqs. (52), (53) and (54).

$$\{x\} = \sum_{j=1}^{m} \beta_j\, q_{jo}\, \{x_j\} \tag{52}$$

$$\{\dot{x}\} = \sum_{j=1}^{m} \beta_j\, \dot{q}_{jo}\, \{x_j\} \tag{53}$$

$$\{\ddot{x}\} = \sum_{j=1}^{m} \beta_j\, \ddot{q}_{jo}\, \{x_j\} + \ddot{x}_o \tag{54}$$

If the stiffness matrix [K] of Eq. (17) is complex, accelerations, velocities and displacements which are obtained through the above procedures are also expressed by complex numbers. In practice, the response values are expressed by the absolute values of complex numbers. It may be necessary to include all mode shapes to obtain adequate accuracy by modal superposition, but the response values can be approximated reasonably well by superposing the first few modes essential for calculation by judging whether the participation functions are large or small for each structure of the entire structure.

8. NUMERICAL RESULTS

For further consideration and the usage of the abovementioned theory, the following analyses were performed using computer programs based on the abovementioned theory for a nuclear power plant to be constructed in Japan.

Model for Dynamic Analysis

Fig. 2 shows a section (x-x section) of a typical nuclear power plant. An equivalent lumped mass-spring system, shown in Fig. 3, is constructed to approximate many sizes of structures and another consisting of two lateral springs-dashpots and a rotational spring-dashpot shown in Fig. 4, is also constructed to approximate the interaction effect. According to the computer programs, the entire structure is divided into a number of blocks (B) and each block is further divided into a number of groups (G) according to damping capacities by the use of damping ratios ($_G H$).

This nuclear power plant was divided into one block and seven groups as shown in Table-IV and Fig. 3.

Calculation of Dynamic Properties

The stiffness matrix of each group and stiffnesses of ground were individually calculated as follows:

(a) The stiffness matrices (real part) of Reactor Building (G-1) and Dry Well (G-2) were calculated by F.E.M. and those of Shield Wall (G-3), Truss (G-4), Skirt (G-5), Reactor Pressure Vessel (G-6) and Stabilizer (G-7) were calculated by the bending shear deflection theory.

(b) The complex stiffness matrix of each group was calculated by Eq. (12) putting the damping ratio ($_GH$) of each group shown in Table IV into this equation. The damping ratio of each group can be estimated from the damping ratios shown in Table III considering the material and structural type of each group.

(c) The complex stiffness of ground was calculated in two ways.

o Case A : The real part of complex stiffness was approximately calculated by the method based on Tajimi's theory and the imaginary part of this was done putting the damping ratios into Eq. (12). The damping ratios used in this calculation are shown in Table III.

o Case B : The complex stiffness was theoretically calculated by the method based on Tajimi's theory putting the fundamental natural circular frequency into Eq. (16). These are shown as case 3 in Table III.

These numerical results are shown in Tables I and II respectively.

Calculation of Natural Periods, Damping Ratios and Participation Functions

The calculation was performed in three ways.

case 1 : neglecting swaying and rocking of foundation (FIX)

case 2 : considering swaying and rocking of foundation (R&S) and using the stiffness of ground shown in Table I.

case 3 : same as case 2 but using the stiffness shown in Table II.

The numerical results are shown in Table V to Table VII and in Fig. 6 to Fig. 9. Table VII shows the natural periods and damping ratios of case 1 and case 3. Judging from the results, the damping ratio of each mode, of which mode shape is largest in each mode, is nearly equal to the damping ratio ($_GH$) of the group which has the most large mode shape. If two groups, of which damping ratio ($_GH$) are not equal, have a large mode shape respectively in the same mode, the damping ratio of this mode is between these two damping ratios ($_GH$) of groups according to their motions. This is the reason why the damping ratios of higher modes do not amount to overdamping.

Table V shows the first five periods and damping ratios of which mode shapes are especially large in all modes. It is shown in Table V that the damping ratios of each mode in case 3 increases gradually as mode number increases from 1st mode to 3rd mode. It may be the reason why the damping ratio of higher mode is affected by radiation damping caused by the interaction. But it is also shown in Table V that the damping ratio of case 3 does not increase as mode number increases higher than number 3. This may be explained by the fact that the stiffness of ground was evaluated using the approximate fundamental natural circular frequency by hypotheses, and that the stiffness of ground is constant as a result.

Table VI shows the comparison of natural periods and damping ratios among three cases of whole structures.

Fig. 6 shows the first three participation functions of Reactor Building.

Fig. 7, Fig. 8 and Fig. 9 show those of the entire structure for case 3.

Calculation of Dynamic Response Values for Earthquake Motions

The dynamic response values were calculated for case 2 and case 3 to make clear the relationship between the damping ratio and the response.

(a) El Centro 1940 N-S component (max. acceleration = 300 gal)

(b) Taft 1952 E-W component (max. acceleration = 300 gal)

Fig. 10 and Fig. 11 show the max. values of displacement and overturning moment for case 3 respectively. Fig. 12 shows the comparison between case 2 and case 3 for the overturning moment to the Taft 1952 E-W component.

Fig. 12 shows that the values of case 3 are smaller by about 10% than those of case 2.

9. CONCLUSIONS

In short, this method can be described in the following 3 steps.

(a) Evaluate the damping capacities of individual structures as hysteretic damping by material and by type of structure and evaluate the radiation damping of interaction between foundation and ground.

(b) Calculate the natural periods and damping ratios of each mode for the entire structure by using the complex stiffness matrix based on step one.

(c) Make the dynamic analysis of entire structure for earthquake motions by using the results obtained in step two.

Up till now, analytically and theoretically, it has been very difficult to evaluate damping capacities which are more accurate and satisfactory; by this method it may be seen this has become possible.

ACKNOWLEDGEMENT

The authors wish to thank Dr. Hiroshi Tajimi, Professor of Building Structure, Nihon University and Mr. I. Funahashi, Research Manager of Takenaka Technical Research Laboratory for their valuable advices, and the cooperation of structural engineers engaged in the design of the nuclear reactor building mentioned in this paper.

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TABLE I CASE A :
STIFFNESS *of* GROUND
$\{K_S\ (t/m)\quad K_R\ (t\cdot m/rad)\}$

	STIFFNESS
K_{S2}	$2.34\times10^{4}+i\,2.34\times10^{3}$
K_{S1}	$3.06\times10^{5}+i\,3.06\times10^{4}$
K_R	$2.44\times10^{12}+i\,2.44\times10^{11}$

TABLE II CASE B :
STIFFNESS *of* GROUND
$\{K_S\ (t/m)\quad K_R\ (t\cdot m/rad)\}$

	STIFFNESS
K_{S2}	$2.34\times10^{4}+i\,2.34\times10^{3}$
K_{S1}	$3.24\times10^{5}+i\,1.68\times10^{5}$
K_R	$2.62\times10^{12}+i\,2.80\times10^{11}$

TABLE III DAMPING RATIOS
by MATERIAL *and* STRUCTURE TYPE

MATERIAL *and* STRUCTURE TYPE		DAMPING RATIO (h) CASE 1	CASE 2	CASE 3
for REINFORCED CONCRETE STRUCTURE		0.05	0.05	0.05
for STEEL STRUCTURE		0.01	0.01	0.01
for INTERACTION BETWEEN FOUNDATION *and* GROUND	LATERAL MOTION *of* SIDE GROUND		0.05	0.05
	LATERAL MOTION *of* BASE GROUND		0.05	0.259
	VERTICAL MOTION *of* BASE GROUND		0.05	0.051

TABLE IV DAMPING RATIOS *of* GROUPS

GROUP NAME	STRUCTVRE NAMES *of* NVCLEAR POWER PLANT	DAMPING RATIO (h)
G−0	FOUNDATION	*shown in* TAB. III
G−1	REACTOR BUILDING	0.05
G−2	DRY WELL	0.01
G−3	SHIELD WALL	0.05
G−4	TRASS	0.01
G−5	SKIRT	0.01
G−6	REACTOR PRESSURE VESSEL	0.01
G−7	STABILIZER	0.01

TABLE V COMPARISON TABLE :
PERIODS *and* DAMPING RATLOS *for* REACTOR BUILDING

* MODE No.	CASE 1			CASE 2			CASE 3		
	** MODE No.	PERIOD (SEC)	DAMPING RATIO (h)	** MODE No.	PERIOD (SEC)	DAMPING RATIO (h)	** MODE No.	PERIOD (SEC)	DAMPING RATIO (h)
1	1	0.1927	0.0498	1	0.2488	0.0498	1	0.2421	0.0799
2	2	0.1032	0.0495	2	0.1223	0.0498	2	0.1191	0.0934
3	5	0.0690	0.0489	4	0.0835	0.0491	4	0.0812	0.1232
4	14	0.0483	0.0478	6	0.0761	0.0482	11	0.0595	0.0738
5	16	0.0349	0.0499	11	0.0597	0.0485	16	0.0466	0.0556

* MODE NUMBER OF REACTOR BUILDING

** MODE NUMBER OF THE ENTIRE STRUCTURE

TABLE VI DOMPARISON TABLE :
PERIODS *and* DAMPING RATIOS *for* THE ENTIRE STRUCTURE

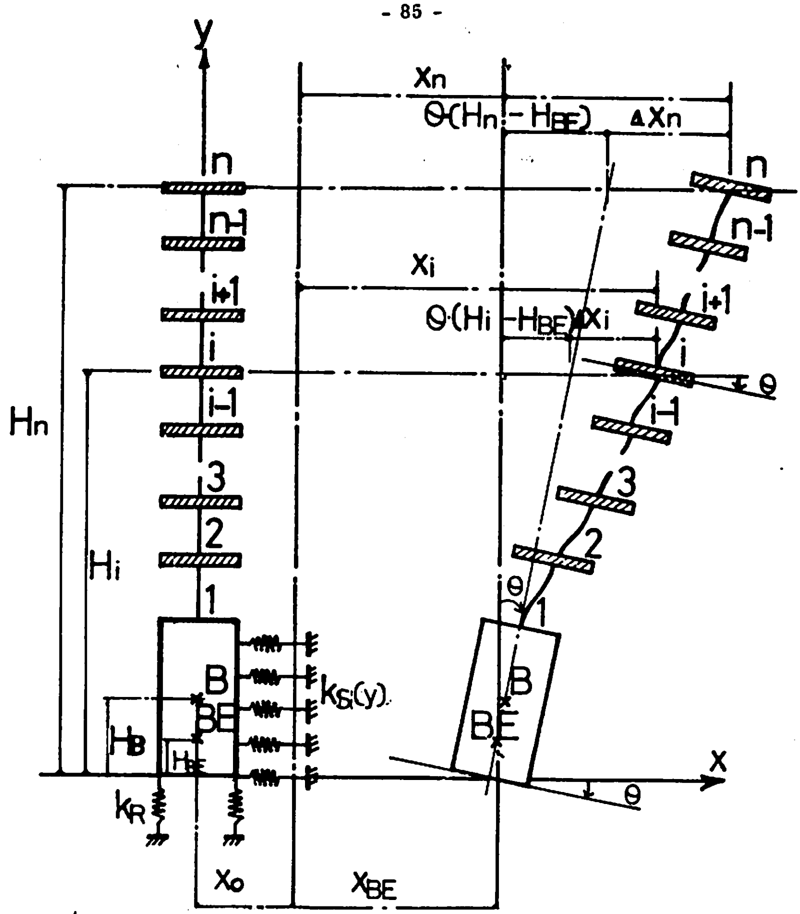
* MODE k	CASE 1		CASE 2		CASE 3	
	PERIOD (SEC)	DAMPING RATIO (h)	PERIOD (SEC)	DAMPING RATIO (h)	PERIOD (SEC)	DAMPING RATIO (h)
1	0.1927	0.0498	0.2488	0.0498	0.2421	0.0799
2	0.1032	0.0495	0.1223	0.0498	0.1191	0.0934
3	0.0850	0.0359	0.0852	0.0364	0.0851	0.0362
4	0.0799	0.0101	0.0835	0.0491	0.0812	0.1232
5	0.0690	0.0488	0.0799	0.0104	0.0799	0.0103

* MODE NUMBER OF THE ENTIRE STRUCTURE

TABLE VII
SUPERIOR GROUP *of* EACH MODE *in* CASE 1 *and* CASE 3
for THE ENTIRE STRUCTURE

MODE	CASE 1		SUPERIOR GROUP							CASE 3		SUPERIOR GROUP						
k	T (SEC)	h	1	2	3	4	5	6	7	T (SEC)	h	1	2	3	4	5	6	7
1	0.1927	0.0498	●	○	●	○	○	○	○	0.2421	0.0799	●	○	●	○	○	○	○
2	0.1032	0.0495	●	○	●	○	○	○	○	0.1191	0.0934	●	○	●	○	○	○	○
3	0.0850	0.0359			●		○	○	○	0.0851	0.0362			●		○	○	○
4	0.0799	0.0101		●						0.0812	0.1232	●	○	●	○	○	○	○
5*	0.0690	0.0488	●	○	●	○	○	○	○	0.0799	0.0103		○					

* *First five modes of all modes are shown*



2 ~ n	:	MASS NUMBERS *of* THE STRUCTURE
BE	:	ORIGIN *of* COORDINATE *for* ROCKING MOTION
H_{BE} (m)	:	HEIGHT *of* THE POINT BE
K_R (t m$/rad$)	:	ROCKING STIFFNESS *of* GROUND
θ (rad)	:	ANGULAR DISPLACEMENT *of* ROCKING MOTION
x_0 (cm)	:	EARTHQUAKE DISPLACEMENT
B	:	GRAVITY CENTER *of* FOUNDATION
H_B (m)	:	HEIGHT *of* THE POINT B
K_S (y) (t/cm)	:	SWAYING STIFFNESS *of* GROUND
x_{BE} (cm)	:	DISPLACEMENT *of* THE POINT BE
H_i (m)	:	HIGHT *of* THE I-*th* MASS POINT

FIG. 1 COORDINATE and PARAMETERS of MOTION SYSTEM

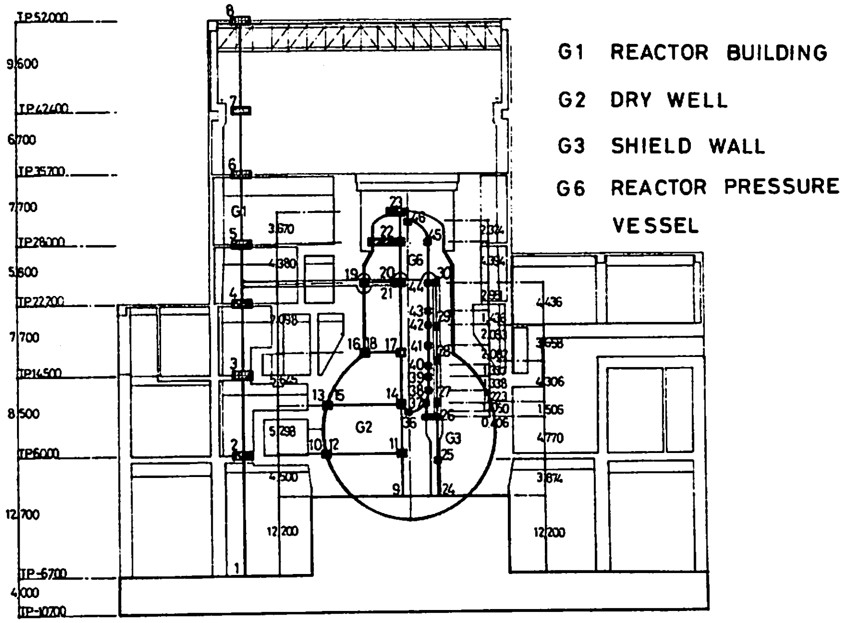


FIG. 2 TRANSVERSE SECTION (X-X SECTION) of ENTIRE STRUCTURE

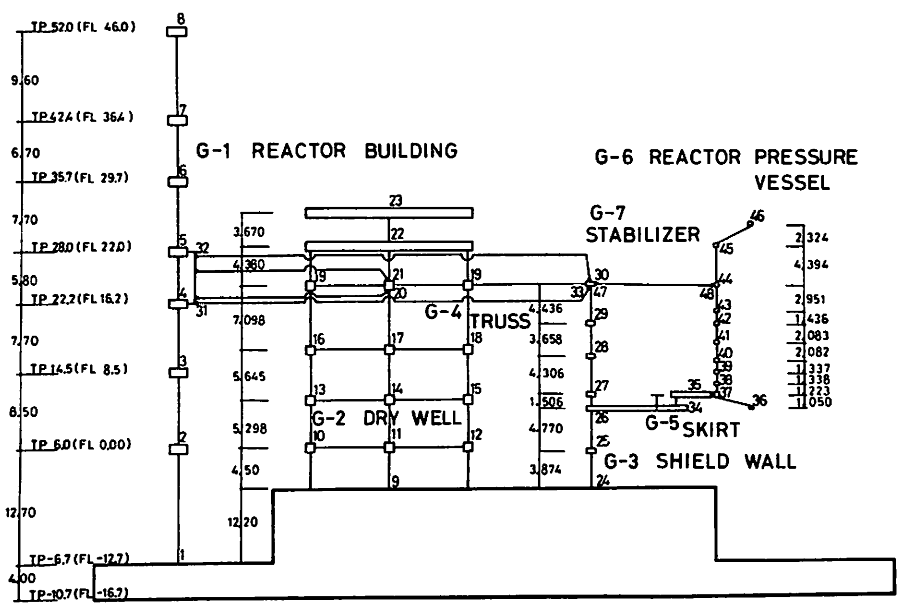


FIG. 3 EQUIVALENT LUMPED MASS SPRING SYSTEM of ENTIRE STRUCTURE

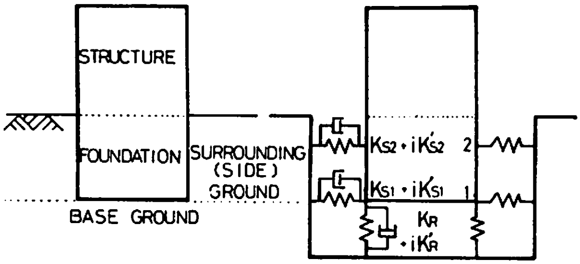


FIG. 4 EQUIVALENT FOUNDATION-GROUND SYSTEM

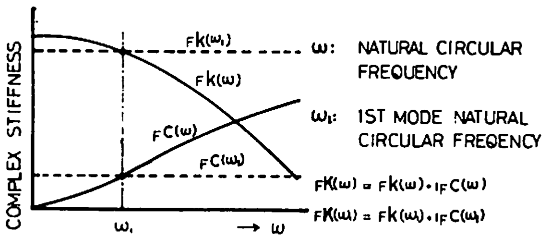


FIG. 5 COMPLEX STIFFNESS of GROUND

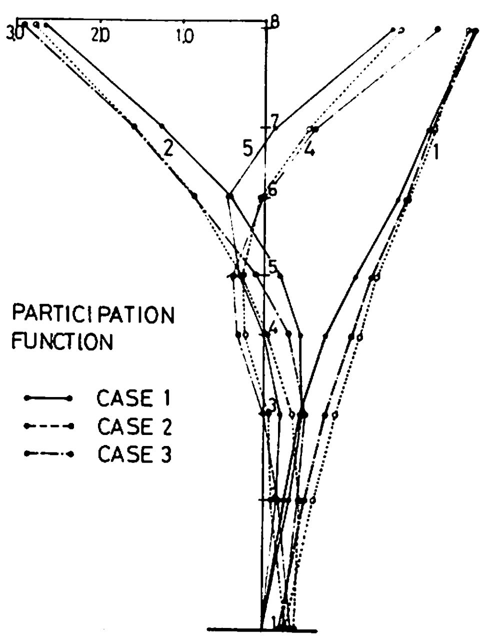


FIG. 6 COMPARISON FIGURE: PARTICIPATION FUNCTIONS of THREE CASES for REACTOR BUILDING

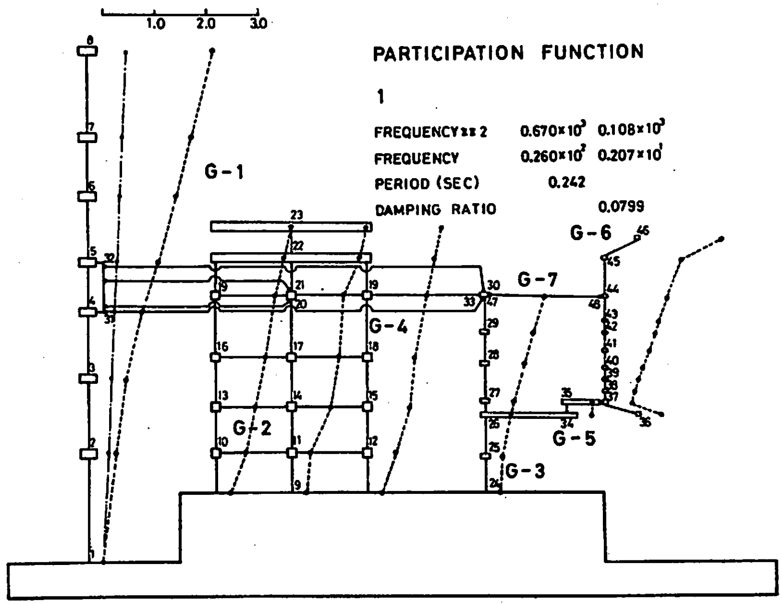


FIG. 7 1ST MODE PARTICIPATION FUNCTION of CASES for ENTIRE STRUCTURE

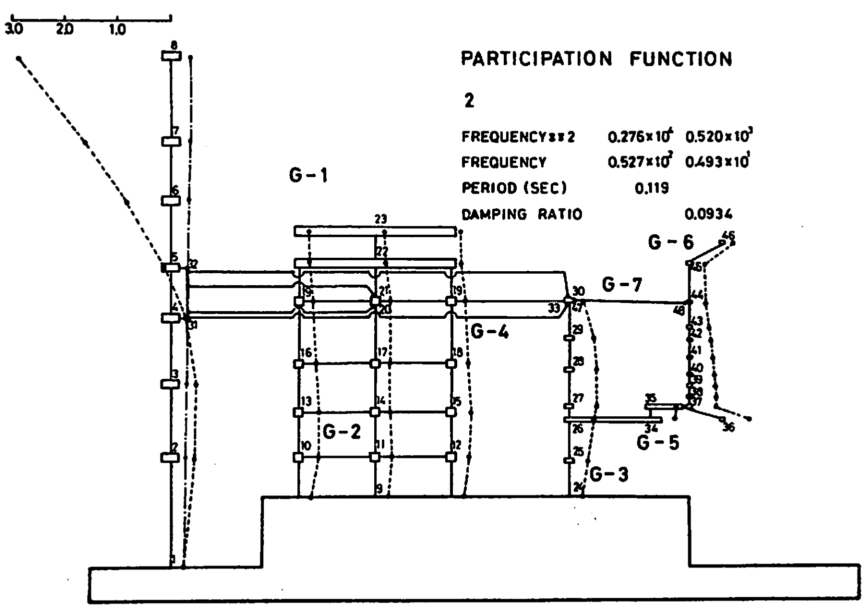


FIG. 8 2ND MODE PARTICIPATION FUNCTION of CASE 3 for ENTIRE STRUCTURE

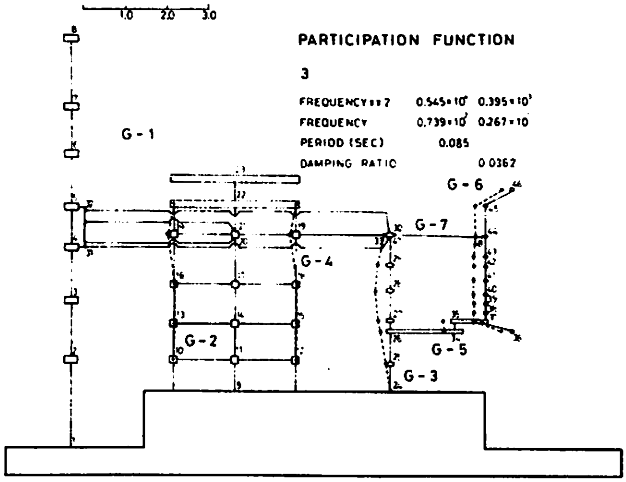


FIG. 9 3RD MODE PARTICIPATION FUNCTION of CASE 3 for ENTIRE STRUCTURE

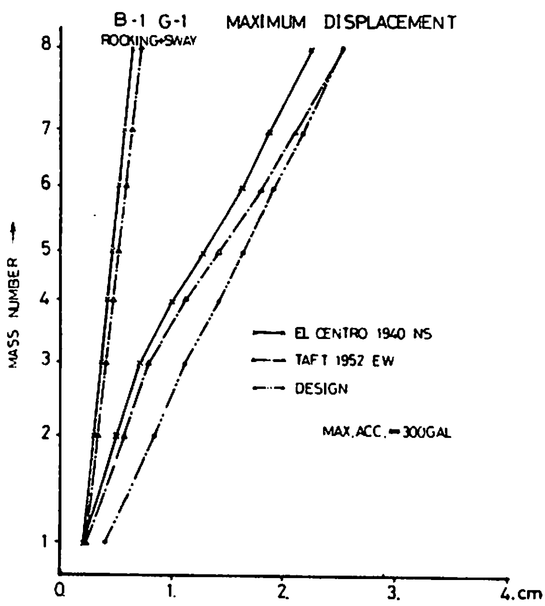


FIG. 10 RESPONSE VALUE (1) of CASE 3 for REACTOR BUILDING

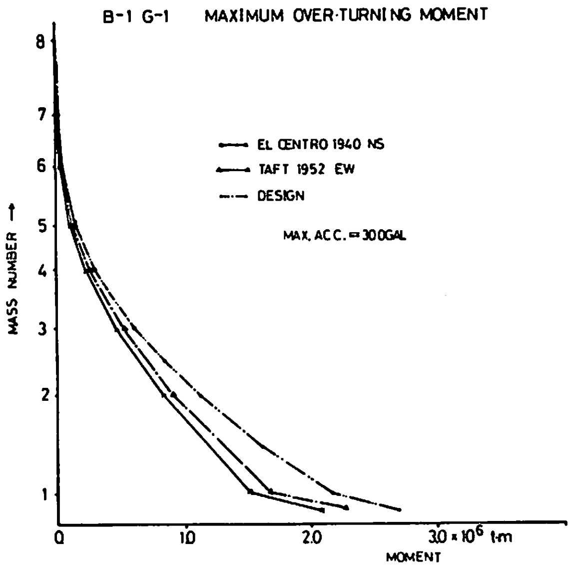


FIG. 11 RESPONSE VALUE (2) of CASE 3 for REACTOR BUILDING

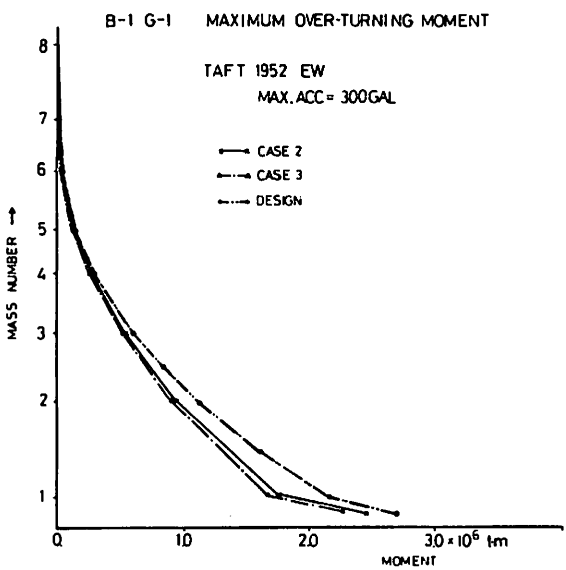


FIG. 12 RESPONSE VALUE (3) of CASE 2 and CASE 3 for REACTOR BUILDING

DISCUSSION

Q T. H. LEE, U. S. A.

In the soil-structure interaction, the radiation damping is a function of the exciting frequency. In your Table III, constant values were given for radiation damping. Are these values the average values over the frequency range or the maximum values ?

A J. JIDO, Japan

Table III in my paper shows the damping ratios to this particular structure in Japan. I suppose the other structures would have different damping ratios from this example.

Q K. UCHIDA, Japan

You use the absolute values as the expression of displacements in your paper. I consider it better to make the displacement projected on the real axis. What do you think of my consideration ?

A J. JIDO, Japan

I am studying whether it is better to consider the actual response displacement as the real part of complex displacement or as the absolute displacement.

Q L. ESTEVA, Mexico

You mentioned that you wanted to take into account hysteretic damping in terms not of an equivalent viscous damping, but rather in terms of a hysteretic, variable stiffness analysis. However, your equations do not show that stiffness is actually varied from step to step. How did you treat the hysteretic damping then ?

A J. JIDO, Japan

I can treat the hysteretic damping by expressing the stiffness of structure by complex number ($K_R + iK_I$).
Because by expressing the stiffness of structure by a complex number, the damping ratios become constant to each mode and as a result this damping can be considered as hysteretic damping.