# MEMBRANE VERSUS SHELL TYPE ELEMENTS IN F.E. ANALYSIS OF BOX TYPE BUILDINGS

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SUMMARY

Finite element analysis of box-type buildings is discussed under typical loading conditions - gravity, seismic and temperature loads. The computation effort is recognized to be noticeably different, according to whether membrane or shell type elements are used.
The relevance of membrane and bending stress components to the total stress distribution is outlined in the table below; the different role of the typical members under the various loading conditions is emphasized.

| LOADING CONDITIONS | STRESSES | | | | | |
|---|---|---|---|---|---|---|
| | MEMBRANE COMPONENTS | | | BENDING COMPONENTS | | |
| | WALLS | SLABS | FOUND.MAT. | WALLS | SLABS | FOUND.MAT |
| GRAVITY | LARGE | NEGLIGIBLE | NEGLIGIBLE | SMALL | LARGE | LARGE |
| EARTHQUAKE,WIND | LARGE | SMALL | SMALL | SMALL | SMALL | LARGE |
| TEMPERATURE | SMALL | SMALL | SMALL | LARGE | LARGE | LARGE |

The adequacy of a F.E. model based on shell type elements for the foundation mat, and membrane type elements for walls and floor slabs, is discussed. It is recognized, through sample models and case histories, that the overall displacements of the structure relative to the foundation slab differ no more than a few percent, in the general case, from the respective values computed on the basis of a more sophisticated model, accounting for the bending stiffness; besides, the wall and slab design only requires a further local modeling under the loads directly applied to them.

## 1. INTRODUCTION

The finite element technology, at the actual state of the art, offers three models for box-type buildings analysis :

- MODEL "A" - where both bending and membrane stiffnesses are represented for floor slabs and walls, by means of plate elements;
- MODEL "B" - where the bending properties of slabs and walls are simulated by equivalent beams and columns, respectively, while membrane properties are represented with plane-stress membrane elements;
- MODEL "C" - where slabs and walls are only represented by plane-stress membrane elements, thus neglecting their bending stiffness, while the foundation mat is represented by plate elements.

Model A is supposed to represent correctly the theoretical behaviour of a box-type structure, as long as the finite element mesh is fine enough. Bending moments in fact can vary meaningfully within the single span dimension, and besides they are only related to the displacement curvature. Thus a bad convergence occurs of numerical to theoretical results; this in practice restricts the use of A - type models to small structures. In the following, reference is made to A - type models under the implicit hypothesis that the relevant mesh is fine enough, so that their response leads very close to the theory solution.

B-type models are in wide practice employed, due to their substantial completeness. Their field of application is primarily concerned with frame-and-shear-walls-type buildings and mixed-frame-and-box-type buildings, and to a lesser extent with box-type buildings. In fact, for large box-type buildings, they become difficult to handle, due to the intrinsic uncertainties in modeling walls and slabs, and to the computational effort connected with 5 or 6 degrees of freedom per nodal point.

The field of application of C -type models is close to previous ones, but they carry over 2 or 3 degrees of freedom per node instead of 5 or 6, as in models B . Provided that their efficiency is demonstrated, for a same problem the number of equations and the bandwidth of the stiffness matrix are about one half of those of a B model, thus reducing the matrix inversion time to about 1/8. Also the input data and output processing are simplified, at the expenses of lack of information on the flexural state of stress in slabs and walls.

C-type models are therefore suitable for large box-type building analysis, provided the following conditions are met :

a) the overall response of the model is adequate; i.e.:
   - nodal displacements

- membrane stresses in walls and slabs
- membrane and bending stresses in the mat

are recognized to be in close agreement with other more complete approaches.

b) a method is found which permits the evaluation of the bending components of stress in walls and slabs;

note that in a wide category of cases such components are of secondary importance, and therefore an approximate, conservative solution is in general satisfactory.

Although the problem is of a kind that quite frequently arises in modeling of structures, no precise reference has been found in literature by the author.

## 2. COMPARISONS BETWEEN THE VARIOUS MODELS

Hereafter a typical comparison will be presented on the basis of several items: state of stress in the mat, membrane actions in slabs and walls, flexural actions directly computed or likely to be evaluated by the nodal displacements in the various models. It is concerned with B and C models and shows a satisfactory compliance with items a, b, above. Then, 2 further comparisons will be briefly presented: 1) A versus B models, on a case history basis, and 2) A versus C models, for a simple cell structure.

### 2.1 "B" versus "C" comparison

The comparison between B and C models is carried out on a simple 3-d structure.

For B-type model, Fig. 1 represents the wall tributary area of the typical beam element. The width of the wall has been choosen to be either one-half the span dimension, or the effective width according to ACI 318 Building Code, (results are quite insensitive to this latter choice). For bending about X-axis, slab in x-y plane and walls in x-z plane only are considered.

Table 1 summarizes the comparison: a satisfactory agreement is apparent as to nodal displacements, membrane stresses in the vertical panels and axial forces in the columns simulating transversal walls.

However a meaningful disagreement is shown as to the mat bending moments. Further observation suggests that the amount of moment lost in C modeling corresponds to the value of the bending moment at the basis of the walls, which is in fact precisely lost in model C. Therefore for both wall and mat design, a numerical procedure has been devised in order to estimate the wall bending moments, on the basis of the building deformation pattern described by model C. More sophisticated procedures are available, to improve the quality of the response. However in this kind of problem such accuracy is useless and misleading for the following two reasons:

a) moments are in most cases very close to the fixed end moments due to the span-applied loads, or otherwise small; in fact the fraction of fixed-end moments which can be redistributed by nodal rotations is small, because of the great stiffness of box structures; therefore an error in the evaluation of the redistributed moments has a weak influence on the accuracy of the total moments.

b) uncertainties in thermal and reological stresses in very hyperstatic structures don't justify a high degree of accuracy in the stress calculation.

The following procedure evaluates moments in slabs and walls due to relative displacements of the end-nodes, and to the span-applied loads (in the C-type model all loads are applied at the nodes).

First a hypothesis is made that nodes can not rotate; moments are computed due to span-applied load and node translations; for a node "o":

$$M_i = (u_i - u_o)\,\frac{6E_iJ_i}{l_i^2} + M_{oi} \qquad i = 1,\dots n \text{ number of elements connected to the node}$$

$\frac{6E_iJ_i}{l_i^2}$ = stiffness of the i-th panel, represented as a beam, for end translation

$u_i - u_o$ = relative displacement of the two ends of the i-th beam

$M_i$ = moment on node o due to end-translation of the i-th beam

$M_{oi}$ = fixed-end moment due to span-applied load in the i-th beam.

Obviously in most cases $\Sigma M_i \neq 0$, so that a rotation $\phi$ occurs at node O; with a simplified approach,

$$\phi = \frac{\Sigma M_i}{\Sigma \frac{4E_iJ_i}{l_i}}$$

$\frac{4E_iJ_i}{l_i}$ = stiffness of the i-th panel, represented as a beam, for end rotation.

Then the end moment in the i-th panel is reduced of the quantity

$$\frac{4E_iJ_i}{l_i}\,\phi$$

This procedure might be iterated until complete moment redistribution is obtained (Cross's method); however the quantity $\frac{4E_iJ_i}{l_i}\phi$ is in most cases small so that further iteration is not required. Moments are evaluated with this method in all the walls and slabs, and a modification occurs in the moments in the mat.

TABLE 1

| TYPICAL COMPARISON ITEMS | | MODEL 1 (X-Z) | | MODEL 2 (Y-Z) | |
|---|---|---|---|---|---|
| | | TYPE B | TYPE C | TYPE B | TYPE C |
| MAX. VERTICAL DISPL. | (DEAD LOAD) | 1.156 | 1.156 | 1.127 | 1.128 |
| MAX. HORIZONTAL DISPL. | (EARTHQUAKE) | 1.787 | 1.788 | 1.599 | 1.600 |
| MAX. HORIZONTAL DISPL. | (TEMP.LOAD) | 9.689 | 9.791 | 0.752 | 0.759 |
| MAX. VERTICAL STRESS | (DEAD LOAD) | 60.23 | 60.23 | 46.66 | 46.68 |
| MAX. VERTICAL STRESS | (EARTHQUAKE) | 20.10 | 20.18 | 15.45 | 15.41 |
| MAX. SHEAR STRESS | (EARTHQUAKE) | 23.21 | 24.16 | 18.40 | 18.60 |
| MAX. HORIZONTAL STRESS | (TEMP. LOAD) | 557.3 | 559.5 | 49.90 | 55.76 |
| MAX. SHEAR STRESS | (TEMP. LOAD) | 123.2 | 132.2 | 49.99 | 53.48 |
| MAX. AXIAL FORCE | (DEAD LOAD) | 355.7 | 356.4 | 452.3 | 452.8 |
| MAX. AXIAL FORCE | (EARTHQUAKE) | 122.3 | 122.9 | 155.0 | 155.7 |
| MAX. AXIAL FORCE | (TEMP. LOAD) | 947.4 | 954.9 | 245.8 | 273.7 |
| MAX BENDING MOMENT | (DEAD LOAD) | 14.81 | 12.97 | 18.58 | 16.39 |
| MAX. BENDING MOMENT | (TEMP.LOAD) | 72.88 | 30.08 | 84.54 | 0.00 |
| MAX. BENDING MOMENT | (EARTHQUAKE) | 10.31 | 8.736 | 12.18 | 4.419 |

For example, for model 2 (see tab. 1) we have, for the maximum moment in the mat, due to temperature load :

MODEL 'B'  M = 84.54
MODEL 'C'  M = 0.00  (node A)

Considering the displacements of the nodes of the lowest story in model 'C', and redistributing the unbalanced moments that arise at the nodes, (see fig. 2), a moment is obtained at node A, whose value is well close to the B model's; we have in fact 92.1 instead of 84.54; moreover the estimated value is on the safe side.

## 2.2 'A' versus 'B' and 'A' versus 'C' comparisons

A simple mixed frame and box type building was analysed with different models to check the discretization procedure in Caorso Nuclear Power Plant Auxiliary Building.
Different models were compared to investigate the behaviour of the building and to compare the responses of various modeling schemes.

An A-type model was obtained with the finest mesh compatible with actual computer capabilities (fig. 3).
Each floor panel was discretized in 32 triangular plate elements, having 9 internal nodes and 16 boundary nodes each; analogous discretization was employed for the vertical walls; the complete model consisted in 659 free nodes, 1344 plate elements, 117 beam elements (representing stiffening beams and real columns).

Three different type models were analyzed besides A-type.
1) Simple frame model, which represents all stiffness properties by means of

beam elements having appropriate cross-sections; in this model the shear stiff ness of the walls was added to the shear area of columns and beams.

2) Frame with bracing trusses; flexural stiffness is given to beams and columns, while the stiffness of shear walls is represented by bracing trusses.

3) Frame with membrane panels. This is properly the B-type model; flexural stiffness of walls and slabs it represented by means of beam elements, while shear and axial stiffness is supported by coarse-mesh membrane panels (fig.4).

The loading conditions considered are dead load and seismic load.

The comparison of the results of the 4 analyses leads to the following observations :

1) The simple frame scheme fails in representing the in-plane behaviour of the floor slabs, which behave as a Vierendeel girder; under this point of view the frame with bracing trusses model is better; truss actions are in fact coherent with principal stresses given by A-type model.

2) The frame with coarse-mesh membrane panels gives good results; though its accuracy is worse than the A-type's, the substantial response is quite correct, and results are very satisfactory also considering the reduced computational burden with respect to A-type's.

As an example let us consider the shear force at the base of two shear walls for seismic load. Table 2 below summarizes results for the 4 models.

TABLE 2

| MODEL | SHEAR (FILE 1) | | SHEAR (FILE 2) | |
|---|---|---|---|---|
| | VALUE | % DIFF. RESP.A | VALUE | % DIFF. RESP.A |
| MODEL A | 486 | - | 460 | - |
| Simple frame | - | - | 559 | 21 |
| Frame with bracing trusses | 518 | 6,5 | 530 | 15 |
| MODEL B | 513 | 5,5 | 515 | 12 |

A direct comparison between A and C models is under development. For a first two-cell model under horizontal forces, results in a vertical wall are reported in figures 5(a), (b), (c). Agreement between the two models is self-evi dent.

## 3. CONCLUSIONS

Comparison between typical membrane+beams and pure membrane models (resp. B and C) shows that agreement is close; flexure actions in C models can be calculated in a separate stage as non-independent variables. The response of B models is checked through comparison with a full-plate fine mesh model (A); it is shown that the accuracy of B models is sufficient for design purposes; this implies, on the basis of the first comparison, that also C models are satisfactory for design purposes; a direct comparison between simple A and C models, confirms the former results.

The use of a C-type model for the stress-analysis of a nuclear fuel pool bulding has given good results with a great economy in all the phases of computation; figures 6 and 7 show typical results for the foundation mat.

ACKNOWLEDGEMENT

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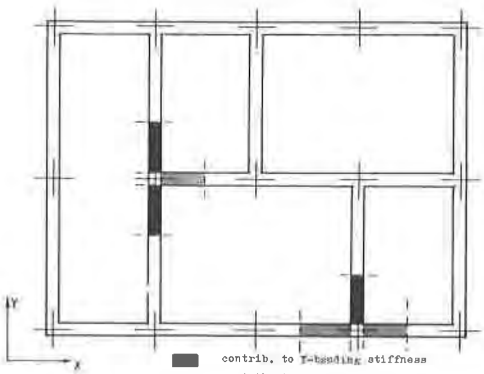


Figure 1. Typical horizontal section

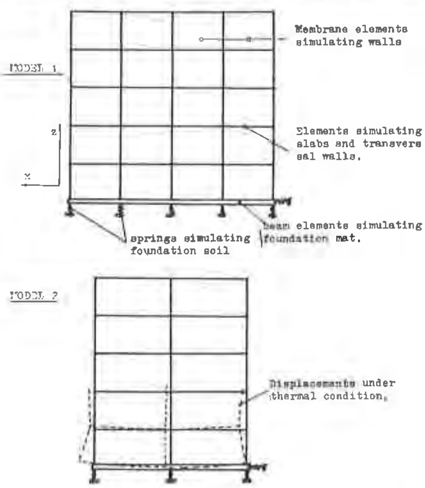


Figure 2. Vertical sections of the model

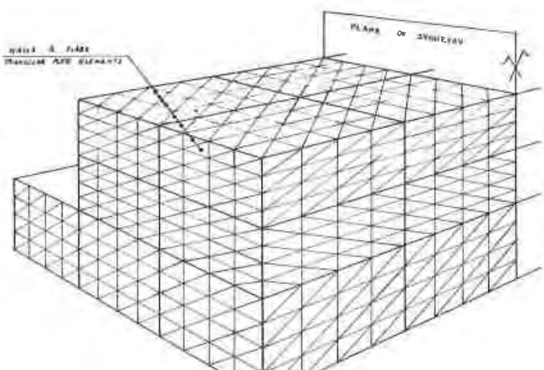

Figure 3. A-type model

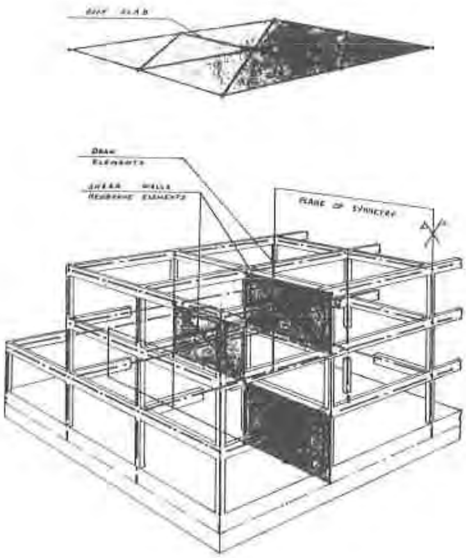

Figure 4. B-type model

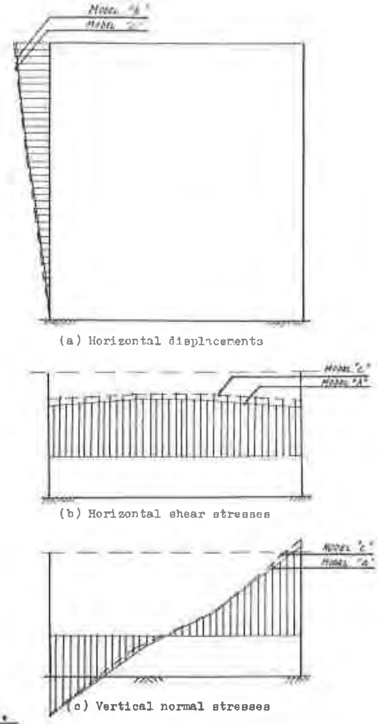


Figure 5.
Typical results of A – C comparison

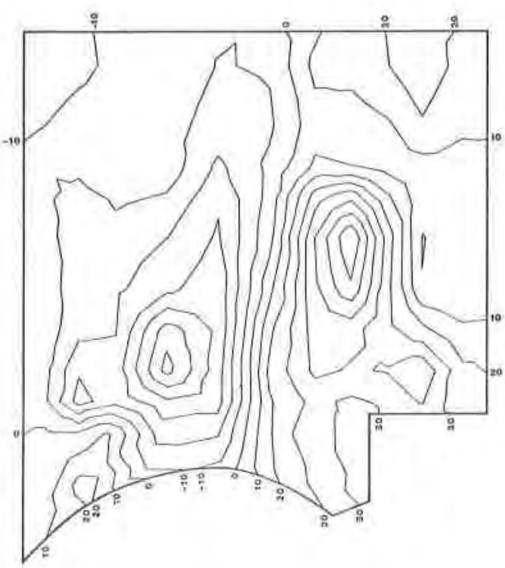

Figure 6. X-bending moment
(dead load )

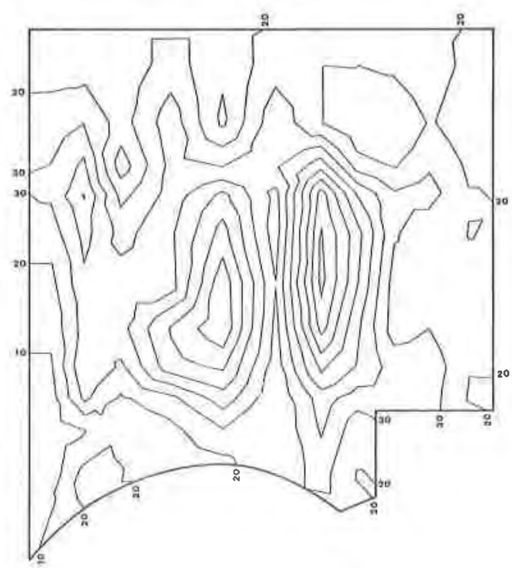

Figure 7. X-bending moment
(earthquake)