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Division V

DEFINITION OF LOCAL COMPLETENESS CRITERIA OF A MODAL BASIS – CONCEPT OF LOCAL EFFECTIVE MASS

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ABSTRACT

For the seismic study of structures using the modal decomposition technique (both with spectral or time-history analysis), the completeness of the modal basis has to be checked. Regarding the global seismic response, the usual completeness criterion is based on the effective mass concept: the truncated modal basis is complete enough if the cumulated effective mass is greater than 90% of the total mass. Regarding the local response of a part of the structure, this global criterion is not relevant and has to be replaced by local or semi-local criterion. This paper proposes the definition of a local criterion based on the residual mode, and a semi-local criterion based on the local effective mass (generalisation of the effective mass concept). The paper presents some theoretical reminders of multi-modal dynamic analysis principles (participation factor, effective mass, and their properties), the definition of the two proposed criteria, their properties, and two cases studies.

INTRODUCTION

The modal decomposition technique is widely used by engineers to study the dynamic linear elastic response of a large variety of structures. In earthquake engineering, this is used for modal spectral and time history analysis using modal basis.

Usually the modal basis is truncated and therefore never fully complete. One should bear in mind that the quality of the results depends on the completeness of the modal basis. Regarding the global seismic response of a structure, the completeness criterion is usually based on the concept of effective mass: it is common practice to consider that the modal basis is complete when the cumulated effective mass is greater than 90% of the total mass of the model.

When interested by the local response of an area of a structure, the global effective mass criterion is no longer relevant. This happens especially for complex structure constituted of multiple sub-structures with masses different by orders of magnitude (e.g. a light steel frame or an equipment supported by a concrete structure). It is then essential to use a local criterion to check the completeness of the modal basis.

This paper offers the use of two local criteria indicating the completeness of the modal basis, both derived from the properties specific to the eigenmodes of the model. The first local criterion allows to examine the situation at the level of each node of a finite element model. It is elaborated from the residue of the decomposition of the unity vector in the modal basis. This is quite similar to the well know concept of residual mode frequently used in modal spectral analysis.

The second is a semi-local criterion obtained by the generalisation of the effective mass concept, at the level of a sub-structure.

After a brief recall of the basis of the dynamic multi-modal analysis of structures (participation factor, effective mass, and their associated properties), the paper presents the definition of these two criteria and their properties, before illustrating their interest through two case studies.

EIGENMODES PROPERTIES: PARTICIPATION FACTOR AND EFFECTIVE MASS - A BRIEF RECALL

By applying the finite element theory to study the dynamic behaviour of structures, the system is represented by its stiffness matrix K, its mass matrix M, and its damping matrix C. The movements of the structure are then determined by the dynamic equation of motion:

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = -M\Delta_k \Gamma(t) \quad [1]$$

Where:

$U(t)$ = relative displacement vector

Δ_k = unity vector for each direction of ground motion (k= x, y, z). That is, each term of the vector is equal to 0 except the component corresponding to the considered direction

$\Gamma(t)$ = ground acceleration time history

Undamped eigenmodes are obtained by the resolution of the following equation of free movements:

$$M\ddot{U} + KU = 0 \quad [2]$$

Considering solutions of the type $U = \Phi \cos \omega t$, this equation can be written as:

$$(K - \omega^2 M)\Phi = 0 \quad [3]$$

Solutions of this equation are defined by the eigenvectors Φ_i and eigen angular frequencies ω_i (i=1 to n, the total number of degrees of freedom of the system). The participation factor p_{ik} and effective mass m_{ik} of the eigenmode i along the k direction are:

$$p_{ik} = \frac{\Phi_i^t M \Delta_k}{\Phi_i^t M \Phi_i} \quad [4]$$

$$m_{ik} = \frac{\left(\Phi_i^t M \Delta_k\right)^2}{\Phi_i^t M \Phi_i} = p_{ik} \Phi_i^t M \Delta_k \quad [5]$$

Eigenvectors are of great interest, since the dynamic response of a system can be studied by means of more efficient numerical techniques. If the modal basis constituted of the eigenvectors is complete, the properties of the participation factors and effective masses are as follows:

- Participation factors are then the coefficients of the unity vector Δ_k decomposed in the modal basis:

$$\Delta_k = \sum_{i=1}^{n} p_{ik} \Phi_i \quad [6]$$

In other words, considering dij the movement of a given node of the model within eigenmode i, for the direction j (=x, y or z), then:

$$\sum_{i=1}^{n} p_{ik} d_{ij} = 1 \text{ if } j = k \text{, else } 0 \quad [6a]$$

- For each direction, the sum of all effective modal masses is equal to the total active mass of the model:

$$\sum_{i=1}^{n} m_{ik} = m_k \quad [7]$$

In modal spectral analysis, the maximum response of each mode i, in terms of deflection, acceleration and the equivalent force vector created by the deflection shape (i.e. inertia forces created by the response due to mode i), are calculated as indicated below:

$$U_{ik} = \frac{S_a(\omega_i, \xi_i)}{\omega_i^2} p_{ik} \Phi_i \quad [8]$$

with $S_a(\omega_i, \xi_i)$ = spectral acceleration at ω_i angular frequency and ξ_i damping

$$A_{ik} = S_a(\omega_i, \xi_i) p_{ik} \Phi_i \quad [9]$$

$$F_{ik} = M A_{ik} = S_a(\omega_i, \xi_i) p_{ik} M \Phi_i \quad [10]$$

Furthermore, the effective mass is an indicator of the "total driven mass" within each eigenmode. Therefore, in modal spectral analysis, the global reaction force created by the response of mode i is obtained by the product of the effective mass with the spectral acceleration:

$$f_{ik} = \Delta_k^t F_{ik} = S_a(\omega_i, \xi_i) p_{ik} \Delta_k^t M \Phi_i = m_{ik} S_a(\omega_i, \xi_i) \quad [11]$$

For the general case where the modal basis is truncated, the cumulated effective mass constitutes a good indicator of the completeness of the modal basis regarding the global seismic response of the structure. That is why, it is commonly agreed that the modal basis is complete when the cumulated effective mass is greater than 90% of total mass.

LOCAL COMPLETENESS CRITERIA OF MODAL BASIS

Here we consider the usual case where the modal basis is truncated. Equalities [6] and [7] are then not verified any more, and the difference between the left and right hand side of the equation can be used as an indicator of the modal basis completeness. Therefore, equation [7] can be used to elaborate a global indicator based on the cumulated effective mass.

Local criterion based on the residual mode

Equation [6] is used to determine the residue of the unity vector Δ_k decomposition in the modal basis:

$$R_k = \Delta_k - \sum_{i=1}^{n_{mods}} p_{ik} \Phi_i \quad [12]$$

This residue vector is nothing else than the pseudo-mode or the residual mode which is used to complete the modal basis in a modal spectral analysis with pseudo-mode.

Therefore, a low residue indicates a good completeness of the modal basis. The residue being calculated for each DOF, the advantage of this approach is that this leads to a local indicator.

Considering equations [6], [6a] and [12], it is the comparison between the value of a residue vector and the unity vector Δ_k that determines its importance. If at each node of the model, all components are small with respect to 1 (i.e. lower than 0.1) then the truncated modal basis is sufficient to well represent the movements of this node. On the other hand, higher residue values (i.e. greater than 0.1 - 0.2) indicate that the truncated modal basis is insufficiently complete to well predict the movements of the node. Also the pseudo-mode will have a stronger contribution in the seismic response of all those areas concerned by high residue values.

Semi-local criterion based on local effective mass of sub-structure

Considering a part of an entire structure, designated as G. the concept of effective mass can be generalized over the subset G. For this to make sense, the local effective mass must verify similar properties as the global effective mass, especially for the following points:

- The cumulated effective masses of all modes must be equal to the total active mass of the sub-structure G;

- The local effective mass must be an indicator of the seismic response of the sub-structure. That is, the global reaction force within the sub-structure due to mode I must be equal to the product of the modal effective mass by the spectral acceleration.

The equation of the local effective mass satisfying these criteria is obtained from the last part of the equation [5], by replacing the mass matrix M by the partial mass matrix M^G reduced to the sub-structure:

$$m_{ik}^{G} = p_{ik}\,\Phi_i^{t} M^{G} \Delta_k \qquad [13]$$

We can verify that the two required properties are fulfilled. However, two other properties of the global effective mass are not satisfied for the local effective mass:

- The global effective mass of a given mode is always positive, while the local effective mass of a mode can be negative within the sub-structure. That means the movements of the sub-structure are out of phase to the rest of the model;
- The global effective mass of a mode is always lower than the total mass, however the local effective mass of a mode can be higher than the total mass of the sub-structure. That means that for the given mode, the movements of the sub-structure are amplified compared to the rest of the structure. This characteristic is then used to identify and quantify the effect of acceleration amplification that can occur in areas of a model (i.e. the “bullwhip effect”).

This concept of local effective mass allows better understanding the movements of masses corresponding to each eigenmode, by identifying and quantifying the masses in movements at each area of a structure.

CASE STUDIES

Through the following examples, we demonstrate how the two criteria defined in this paper are interesting for the seismic study of structures with specific geometrical configuration. The first example concerns a building constituted of separated blocs for which it is interesting to have their individual effective modal mass. The second example is a building with a secondary structure where local effective modal mass constitutes a useful indicator.

For each of them, are presented the finite element modelling, a table of their eigenmodes with the indication of local effective masses, the deflected eigenmode shapes and the residue results.

Building constituted of several blocks

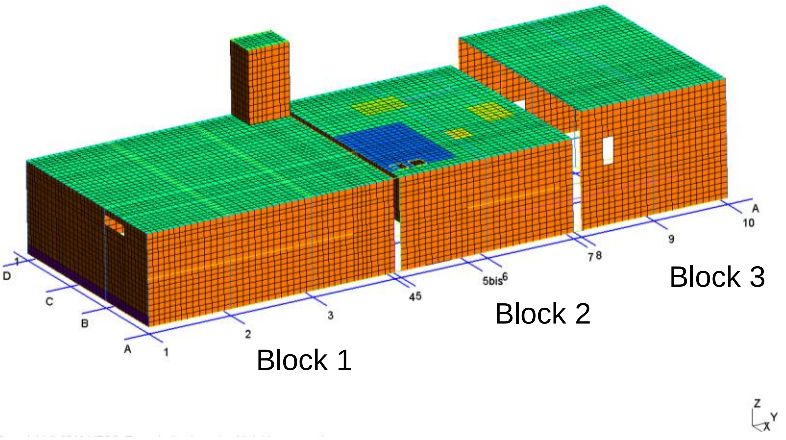

Figure 1. Overall view of the FE model

As one can see, the local effective masses of each block indicated in table 1 constitute a more efficient indicator in identifying the most significant eigenmodes of the structure rather the qualitative examination of eigenmode shapes. This indicator is also very useful in verifying that the cumulated effective mass reaches the 90% completeness criterion.

Table 2. Eigenmodes with global and local effective masses

n° mode	f (Hz)	T (s)	Global effective mass (%)			Local effective mass per block (%)								
						Block 1			Block 2			Block 3		
			dir. X	dir. Y	dir. Z	dir. X	dir. Y	dir. Z	dir. X	dir. Y	dir. Z	dir. X	dir. Y	dir. Z
1	4.84	0.21	26.5	0.0	0.1	**66.3**	0.0	0.2	-0.3	0.0	0.0	0.0	0.0	0.0
2	5.10	0.20	16.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**64.3**	0.0	0.0
3	6.34	0.16	0.0	6.9	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**27.0**	1.1
4	6.64	0.15	0.1	0.1	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.5	3.1
5	6.97	0.14	0.1	0.2	7.3	0.2	0.4	**18.1**	0.0	0.0	0.0	0.0	0.0	0.0
6	7.62	0.13	0.0	0.5	2.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.2	**11.3**
7	7.82	0.13	0.0	2.1	2.8	0.0	0.0	0.0	0.0	0.0	0.0	0.1	8.3	**10.8**
8	7.87	0.13	0.0	0.1	2.2	0.1	0.3	5.5	0.0	0.0	0.0	0.0	0.0	0.0
9	7.89	0.13	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.7
10	7.94	0.13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2
11	8.08	0.12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	8.29	0.12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	8.45	0.12	0.0	4.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**15.8**	0.6
14	8.49	0.12	11.3	0.0	0.1	1.1	-0.2	0.0	**30.8**	0.3	0.2	0.0	0.0	0.0
15	8.50	0.12	2.1	0.8	0.0	-0.1	1.4	0.0	6.0	0.6	0.0	0.0	0.0	0.0
16	8.54	0.12	0.1	5.7	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.3	**22.3**	1.0
17	8.62	0.12	14.8	0.3	0.1	1.2	0.0	0.0	**40.6**	0.9	0.3	0.0	0.0	0.0
18	8.77	0.11	0.0	4.7	0.8	0.2	**12.1**	1.6	-0.1	0.1	0.5	0.0	0.0	0.0
...														
190	35.00	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cumulated effective mass			93.1	94.5	98.0	91.6	92.9	97.4	94.0	95.9	98.3	92.2	93.8	97.9

Considering the residue vector, we notice from figure 3 that in some areas, mostly close to the foundations, the values are not negligible. This is mainly due to the strong stiffness of the soil constraining the movements of the foundation. This means that the truncated modal basis is not enough to completely encounter for the seismic movements of the structure in these areas, Therefore, the pseudo-mode will have an influence on the response of the structure in these areas.

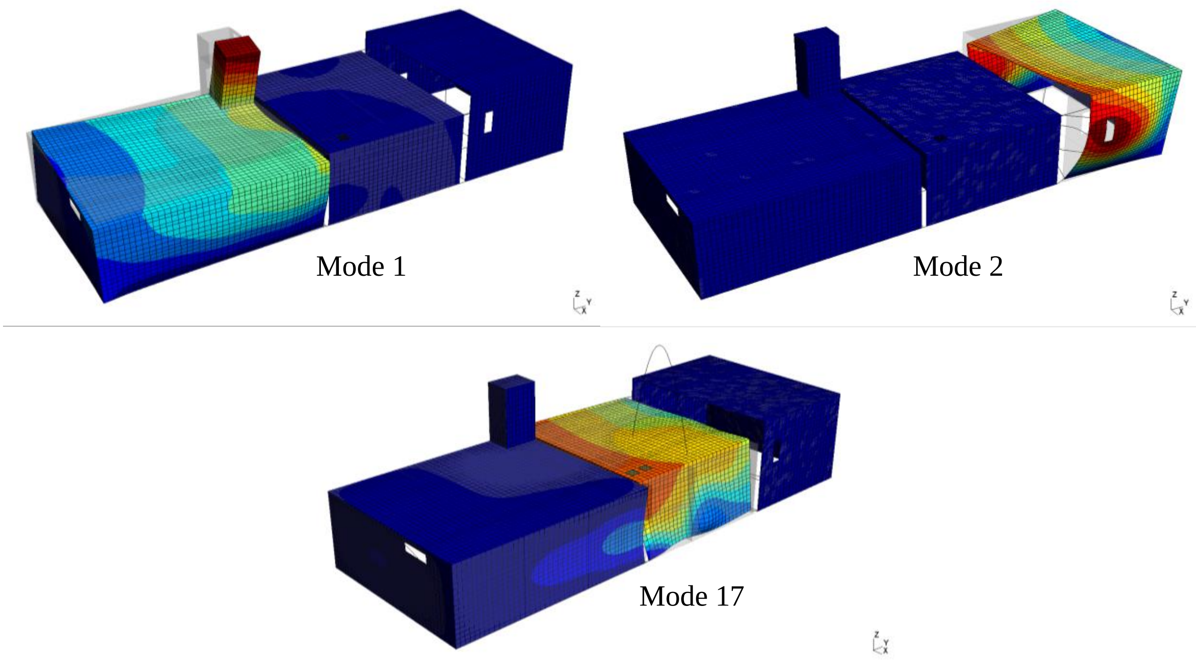

Figure 2. Deflected shapes of some significant modes

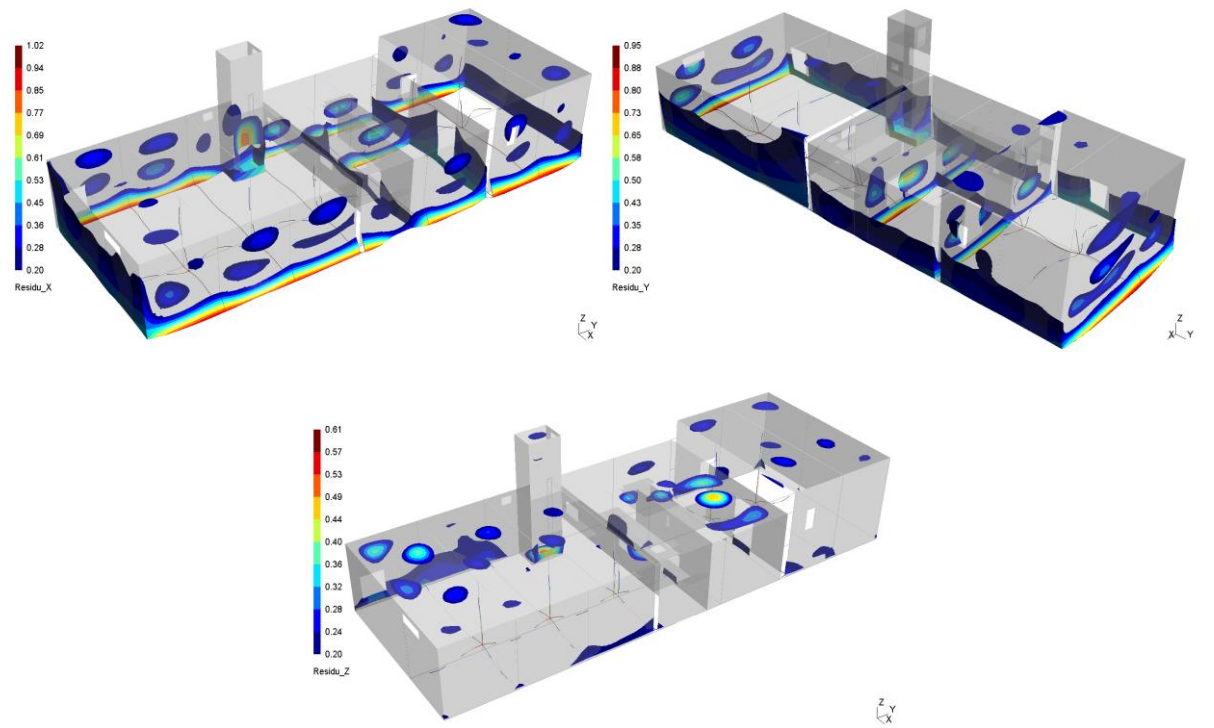

Figure 3. Residual mode for each direction of ground motion (X, Y, Z) – Only areas with the norm $(=\sqrt{R_X^2+R_Y^2+R_Z^2})$ greater that 0.2 are indicated – The results are illustrated by means of deflected mesh indicating their sign

Building supporting a secondary structure

The studied building has a double wall containment vessel; the internal wall is built of reinforced concrete and the external wall is made of steel. The total mass is 54 000 tons, with 900 tons for the external containment vessel. In this study the aim is to simulate the seismic response of the external vessel, for which we examine the completeness of the eigenmode basis through the residue vector and the local effective mass of the external vessel.

The eigenmode table indicates that for the X direction, the first principal vibration mode of the overall structure (mode number 6 with 74% of the global mass) also corresponds to the principal mode of the external vessel with a local effective mass of 109%. This value greater than 100% indicates an amplification of the external vessel movements. The second principal vibration mode is the number 27 with 20% of the global mass and a local effective mass of -17% for the external vessel. This means that the external vessel vibrates oppositely to the rest of the structure.

At the global scale, it should be noticed that total effective modal mass is close to 100%, both for the overall structure and the external vessel. This is consistent with the calculated low residue values (figure 6).

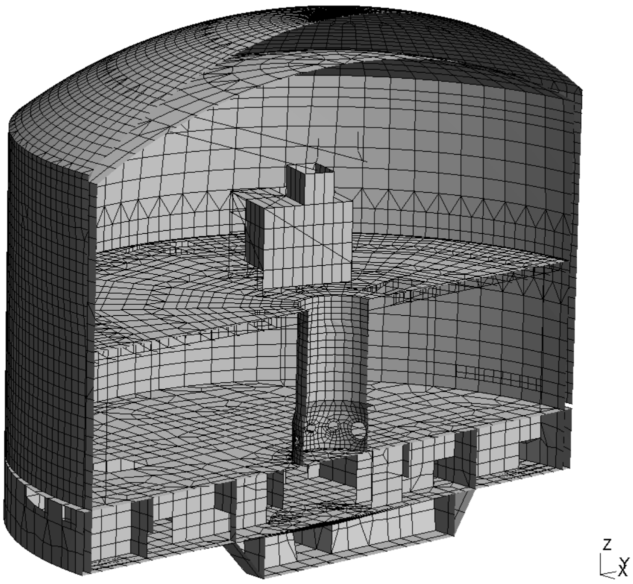

Figure 4. Overall view of the FE modelling

N° mode	f (Hz)	Global effective mass (%)			External vessel local effective mass (%)		
		dir. X	dir. Y	dir. Z	dir. X	dir. Y	dir. Z
1	1.36	0.0	0.0	0.0	0.0	0.0	0.0
2	1.47	0.1	0.0	99.6	0.2	0.0	98.6
3	1.72	0.0	1.5	0.0	0.0	1.6	0.0
4	1.84	0.2	0.1	0.3	0.3	0.1	0.5
5	2.09	3.7	**59.2**	0.0	5.6	**84.7**	0.0
6	2.10	**74.1**	2.9	0.2	**109.4**	4.1	0.4
7	2.12	0.0	**18.4**	0.0	0.0	**26.9**	0.0
8	2.36	0.7	0.1	0.0	1.3	0.1	0.0
9	2.71	0.0	0.0	0.0	0.0	0.0	0.0
10	2.81	0.0	0.0	0.0	-0.1	0.0	0.0
…	…	…	…	…	…	…	…
26	3.52	0.0	0.0	0.0	0.0	0.0	0.0
27	3.55	**20.2**	0.0	0.0	**-16.7**	0.0	0.2
28	3.55	0.0	0.0	0.0	0.1	0.0	0.0
29	3.65	0.0	**11.7**	0.0	0.0	**-18.3**	0.0
30	3.66	0.0	0.1	0.0	0.1	0.1	0.0
…	…	…	…	…	…	…	…
267	6.98	0.0	0.0	0.0	0.0	0.0	0.0
Cumulated effective mass (%)		99.9	99.9	100.0	99.2	99.6	100.2

Table 2. Eigenmodes with global and local effective masses

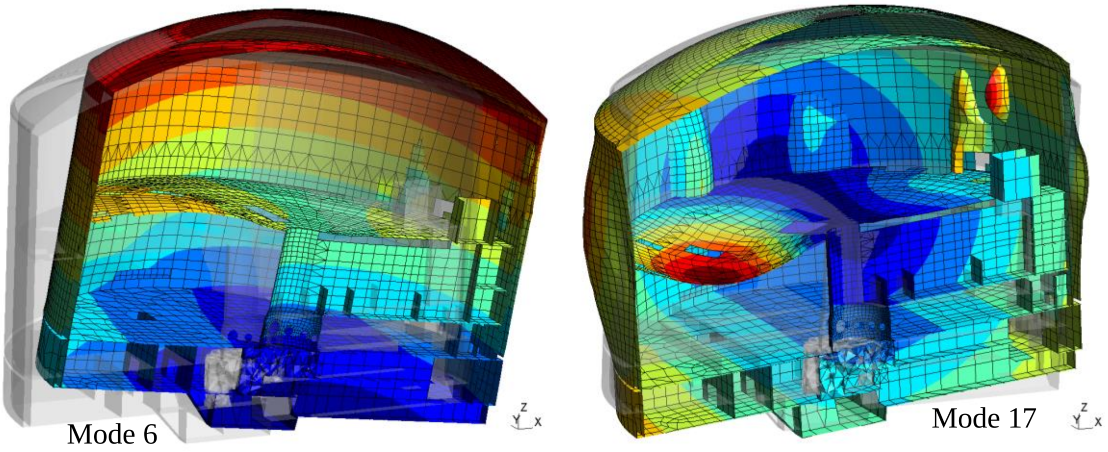

Figure 5. Deflected shapes of some significant modes

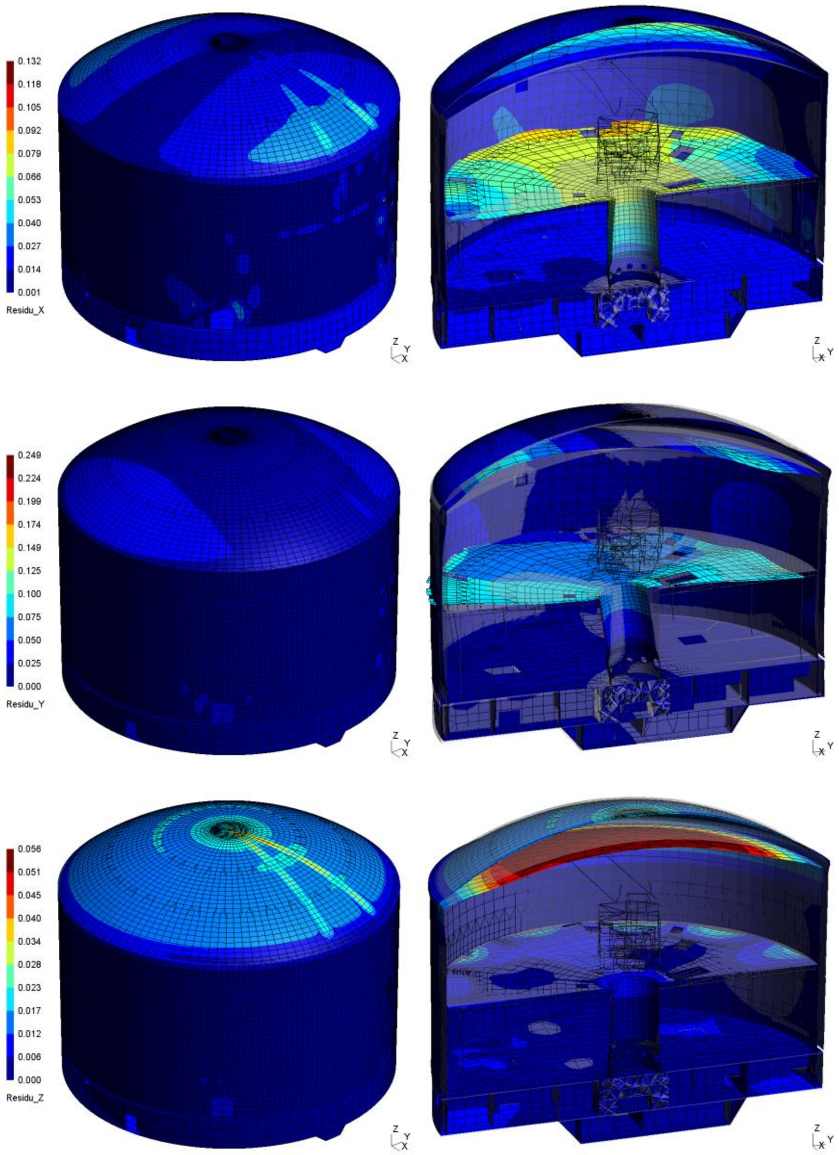

Figure 6. Residual mode for each ground motion direction (X, Y, Z) – The results are illustrated by means of deflected mesh indicating their sign – Values correspond to the norm of the residue vector at each node ($= \sqrt{R_X^2+R_Y^2+R_Z^2}$)

CONCLUSION

To study the local completeness of the modal basis, two criteria have been proposed: a local criterion based on the residual mode, computed at each node of the FE model, and a semi-local criterion based on the concept of local effective mass, computed for a subset of the structure.

These studies indicate that the most interesting criterion is the local effective mass. It is of a better help in identifying the most significant eigenmodes participating in the seismic response of a given area of the structure and verify the necessary completeness of the considered modal basis, regarding the cumulated local effective mass.

REFERENCES

Clough & Penzien, *Dynamics of Structures*, Third Edition, Computers & Structures Inc., 1995.