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Stress Intensity Factor of Partly Circumferential Surface Cracks at the Inside of a Pipe Loaded Arbitrarily

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Abstract

Surface cracks play an important role in reactor technology. In order to predict fatigue and the stability behaviour, knowledge of the stress-intensity factor in the actual case of loading is necessary. In this paper the weight function for this crack problem is derived by finite elements. In this way the stress-intensity factor for constant and linear load distributions in the direction of thickness of the pipe wall is calculated and fitted into simple analytical expressions.

1. Introduction

The knowledge of stress intensity factors is necessary for the theoretical description of fatigue crack growth. Especially in connection with the "leak before break philosophy", it is important to predict the size and shape of a surface crack after a defined number of loading steps. Whilst for axially orientated cracks Newmann and Raju /1/ have given a satisfactory solution only approximate solutions exist for the circumferential surface crack. Lim et al. /2/ proposed a weight function solution for not too long semi-elliptical cracks. For larger crack lengths on the circumference the cracks are not longer semi-elliptical. Therefore, in this investigation a curved rectangular crack shape (fig. 1) was used. Only the stress-intensity factor at the deepest point A of the crack will be calculated here.

2. Determination of the weight function

The weight function method /3,4,5/ starts from a reference solution of the crack geometry of interest. As the latter is not known for the present case, it will be determined from finite element calculations. The crack is loaded by uniform axial tension and the maximum reference crack opening displacement u_{max} obtained from FE-calculations is fitted as follows /6,7/:

$$u_{max} = \frac{\sigma_o \; R_i}{H} \sum_{j=1}^{4} C_j \; (a/t)^j \qquad (1)$$

C_j will be determined by FE-calculations lateron. $H=E/(1-\nu^2)$ for plane strain. R_i is the inner radius and t the wall thickness of the pipe. For the displacements along the small semi-axis (x-axis) of the surface crack a modified near-tip displacement field is used:

$$u_r(x,a) = u_{max}(a) \quad \sqrt{1-\frac{x}{a}} \tag{2}$$

The surface crack is treated in this sense as a plane problem. This method has already been used successfully in /6,7/. The reference stress-intensity factor follows from basic equations of the weigth function method:

$$K(a) = \int_0^a \sigma(x)\ h(x,a)\ dx \tag{3}$$

where h(x,a) is the weigth function

$$h(x,a) = \frac{H}{K_r} \ \frac{\partial u_r}{\partial a} \tag{4}$$

If $\sigma(x) = \sigma_r = \sigma_o$ = const., $K = K_r$ follows and the reference stress-intensity factor is

$$K_r(a) = \left[H\ \sigma_o \int_0^a \frac{\partial u_r}{\partial a}\ dx\right]^{1/2} \tag{5}$$

The constants C_j in eq(1) are determined from FE-calculations and are given in the appendix. Applying eqs.(1) and (2) $K_r(a)$ can be expressed as

$$K_r = \sigma_o \ \sqrt{\pi a}\left(\frac{R}{t}\ 0.424\ C_1 + 0.637\ C_2\beta + 0.849\ C_3\beta^2 + 1.061\ C_4\beta^3\right)^{1/2} \tag{6}$$

with $\beta = a/t$. Eq. (6) is valid for $0.2 \leq \beta \leq 0.7$ and $R/t = 10$.
With eqs. /1,2,4,6/ the weight function is known.

Figure 2 shows one of the FE-structures. As only the maximum crack opening displacement is used, a very refined mesh at the crack tip is not necessary.

In fig. 3 the normalized stress intensity factor $F = K/\sigma_o\sqrt{\pi a}$ for constant stress σ_o is plotted versus a/t for the different crack angles. The curves converge in a plausible manner on the solution for the complete circumferential crack /8/.

In fig. 4 the comparison is shown with the solution from /1/ for the semi-elliptical crack in a plate. Although the rectangular crack envelops the semi-ellipse with equal semi-axis, the stress intensity factor of the rectangular crack is smaller for large crack length. This can be understood by looking at the support of the opposite uncracked side of the pipe circumference. For smaller crack lengths the rectangular crack may give higher K-values than the semi-elliptical one because of its larger crack face. This is the case especially for deep cracks.

3. Application to bending stresses

Figure 5 shows the normalized stress-intensity factor for a bending stress across the pipe wall. The tension side of bending is on the inner side of the wall leading to $\sigma(x) = \sigma_o \,(1 - \frac{2x}{t})$. The curves show the typical decrease with increasing crack length. Again the curves are below the solution for the complete circumferential crack /6/.

Figure 6 is a comparison with the stress-intensity factor for a semi-elliptical surface crack in a plate /1/ for the pure bending loading case. Again for large crack lengths (2α) the semi-elliptical crack gives higher stress-intensity factors than the rectangular crack.

4. Conclusions

The weight function for the deepest point of a rectangular circumferential surface crack in a pipe is calculated. Applying this weight function the stress-intensity factor for any stress varying across the pipe wall can be calculated.

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Appendix A

	α = 10°	α = 20°	α = 30°	α = 40°
C_1	0.228	0.3035	0.1824	0.1712
C_2	0.333	-0.2763	0.6411	0.7931
C_3	-0.231	1.3196	-0.6118	-1.0752
C_4	0.094	-0.5354	0.9697	1.5136

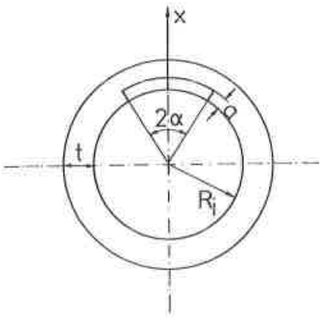

Fig. 1: Sketch of the geometry considered (R/t = 10)

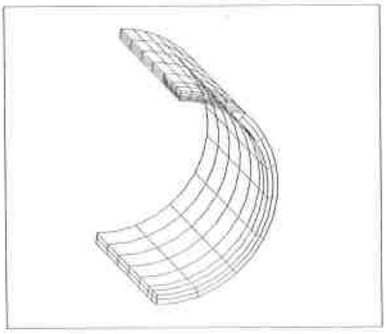
Fig. 2: One of the FEM structures used (α = 20°, a/t = 6,5).

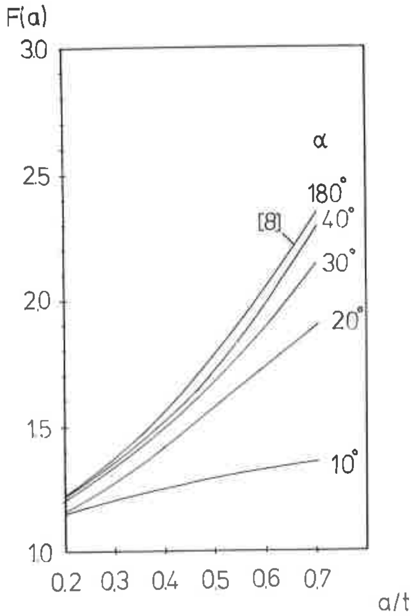

Fig. 3: Normalized stress-intensity factors $F = K/\sigma_o\sqrt{\pi a}$ for constant axial stress σ_o.

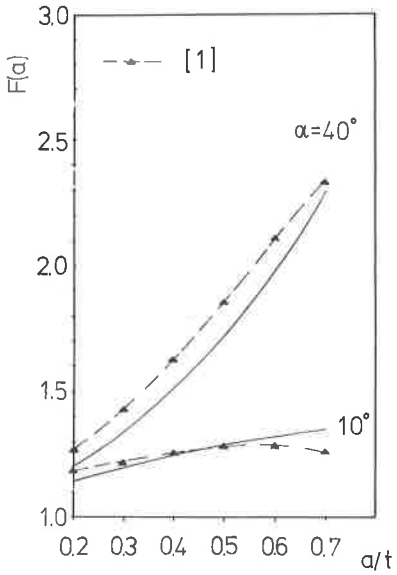

Fig. 4: Comparison of rectangular circumferential cracks with the solution for the axial semi-elliptical crack of the same semi-axis loaded by constant axial stress σ_o ($\alpha = 10°$ and $\alpha = 40°$).

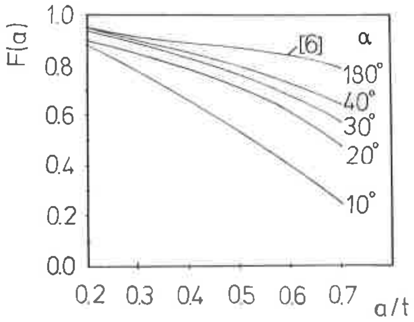

Fig. 5: Normalized stress-intensity factor $F = K\ \sigma_o\ \sqrt{\pi a}$ for a linearly varying bending stress versus the pipe thickness.

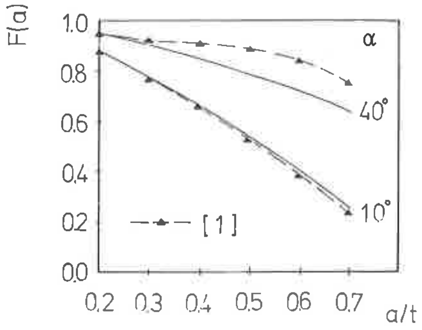

Fig. 6: Comparison of rectangular circumferential cracks with the solution for the axial semi-elliptical crack of the same semi-axis exclusively exposed to bending stress acting over the pipe wall ($\alpha = 10°$, $\alpha = 40°$).