Methods of Analysis of Piping Systems with Nonlinear Vibration Absorbers

Takeru IGUSA, Ravi SINHA
Northwestern University, Evanston, IL USA
E. KOKUBO
Mitsubishi Heavy Industries, Ltd., Kobe, Japan
S. FURUKAWA
Toshiba Corporation, Yokohama, Japan
J. KAWAHATA
Hitachi, Ltd., Hitachi, Japan

ABSTRACT

A response spectra-based method of analysis for piping with nonlinear supports is presented. The required data is the hysteresis loops of the supports, the linear properties of the piping and the linear response spectra. The method was developed to be as simple and versatile as possible, yet accurate enough to model the essential nonlinear behavior of the supports. Comparison with full-scale experimental results show good agreement.

1 INTRODUCTION

A research group in Japan, under the leadership of Prof. H. Shibata, has been conducting experimental tests of piping with high-damping supports. The new concept is the use of supports with hysteretic nonlinear behavior. These supports allow thermal deflections under ordinary operating conditions, and provide large amounts of energy dissipation during seismic or other severe dynamic events. Details of the experimental results are presented elsewhere in the proceedings.

In this paper, a simplified nonlinear response spectrum method is presented for piping systems with nonlinear supports. The assumptions are: the piping remains linear; the supports are the only source of nonlinear behavior; the response is narrow band; and the floor motion at all supports are the same. The third assumption is usually satisfied because the piping and reactor structures act as filters for the dynamic excitation.

The only data required are the hysteresis loops of the nonlinear supports, linear floor response spectra, and the properties of the linear piping without nonlinear supports. This data is readily available, and makes the method accessible to a wide variety of problems.

The analysis consists of three-steps. First, equivalent linear properties of the supports are determined from hysteresis loops. These properties depend on the amplitude and frequency of the support motion. Next, the modal properties are determined for the piping with equivalent linear supports. Then, a response spectrum mode combination rule is used to determine the response. Since the problem is inherently nonlinear, these three steps are solved by a nonlinear equation solver.

The method gives responses at the supports as well as in the piping. It also gives equivalent natural frequencies and damping ratios which show how the supports affect the dynamic properties of the piping.

SMiRT 11 Transactions Vol. K (August 1991) Tokyo, Japan, © 1991

The analysis method is compared with experimental results of a three-dimensional piping with multiple nonlinear supports.

2 EQUIVALENT LINEAR ANALYSIS OF THE NONLINEAR SUPPORTS

One type of support that is being investigated is a friction device (Nomura et al., 1989). A typical force-displacement relationship for the support is shown in Fig. 1. This relationship is modeled by a bilinear hysteresis loop. By curve fitting, it is found that the lower bound of the pre-yield stiffness is k_o = 562.5 kgf/mm and the yield force is 100 kgf. The bilinear hysteresis model is shown in Fig. 2. The loop consists of an upper curve, $F_U(x)$, and a lower curve, $F_L(x)$.

Equivalent linear stiffness and damping for the nonlinear support are obtained by a variation of the method of slowly varying parameters (Caughey, 1960). The basic assumption of the method is that the displacement is narrow band. Let A be the amplitude of the displacement and let ω be the average circular frequency. Then, the equivalent linear stiffness, k^e, and damping, c^e, are functions of A and ω which are given by

$$k^e = k_o + 2\frac{C(A)}{A^2}, \qquad c^e = \frac{2}{\omega}\frac{S(A)}{A^2} \tag{1}$$

where

$$C(A) = \frac{A}{2\pi}\left\{\int_0^{\pi} F_L[A\cos\theta]\cos\theta\, d\theta + \int_{\pi}^{2\pi} F_U[A\cos\theta]\cos\theta\, d\theta\right\} \tag{2}$$

$$S(A) = \frac{-A}{2\pi}\left\{\int_0^{\pi} F_L[A\cos\theta]\sin\theta\, d\theta + \int_{\pi}^{2\pi} F_U[A\cos\theta]\sin\theta\, d\theta\right\} \tag{3}$$

are integrals of the hysteresis loops. The limits of integration correspond to integration around one hysteresis loop. The product $2\pi S(A)$ has a physical meaning: it is the energy dissipated by the nonlinear member in one displacement cycle (the area of the hysteresis loop).

The equivalent stiffness and damping for the friction-type support is shown in Figs. 3 and 4 for frequency $\omega/2\pi$ = 8.27 Hz. The equivalent stiffness approaches zero with increasing amplitude. The equivalent damping initially increases but also decreases at large amplitudes. This behavior is usual for hysteretic supports (Sinha and Igusa, 1990).

3 EIGENVALUE ANALYSIS OF THE PIPING WITH SUPPORTS

To obtain modal properties for the piping with nonlinear supports, the supports are modeled using the equivalent linear support properties. The equivalent linear stiffness and damping of the supports are included in the stiffness matrix $\mathbb{K}$ and the damping matrix $\mathbb{C}$. The main difficulty in the eigenvalue analysis is the determination of the correct modal damping ratios, ζ_i. The simplest approximation is

$$\zeta_i = \frac{\Phi_i^T \mathbb{C} \Phi_i}{2\omega_i \Phi_i^T \mathbb{M} \Phi_i} \tag{4}$$

where Φ_i are the undamped mode shapes and ω_i are the undamped natural frequencies for i = 1, ..., n, in which n is the number of modes.

If the equivalent damping terms are large, which is possible for nonlinear supports, then the approximation in Eq. 4 may not be accurate. In some cases, Eq. 4 can overestimate the damping ratios, which means that the response will be underestimated (Igusa and Xu).

To obtain more accurate damping ratios, the eigenvalue analysis should contain the damping matrix. A state-vector approach (Hurty and Rubinstein, 1964) is used and the eigenvalue problem is given by

$$\mathbb{A}\widehat{\Psi}_i = \lambda_i \widehat{\Psi}_i , \qquad \mathbb{A} = \begin{bmatrix} 0 & \mathbb{I} \\ -\mathbb{M}^{-1}\,\mathbb{K} & -\mathbb{M}^{-1}\,\mathbb{C} \end{bmatrix} \tag{5}$$

The natural frequencies and modal damping ratios are given by

$$\omega_i = |\lambda_i| , \qquad \zeta_i = \frac{-\mathrm{Re}\lambda_i}{|\lambda_i|} \tag{6}$$

It is shown later how these eigenvalue analysis results are used in the response spectrum method.

4 LINEAR MODE COMBINATION

Let $S(\omega, \zeta)$ be the response spectrum for frequency ω and damping ratio ζ. A mode combinations rule for the simple modal damping approximation in Eq. 4 is shown first. A common response spectrum method is the square-root-of-sums-squared (SRSS) rule. The expression for the maximum response, R, is

$$R = \sqrt{\sum_{i=1}^{n} a_i^2\, S_i^2(\omega_i, \zeta_i)} \quad , \qquad a_i \equiv \frac{\left(\Phi_i^T \mathbb{q}\right)\left(\Phi_i^T \mathbb{M}\mathbb{r}\right)}{\Phi_i^T \mathbb{M} \Phi_i} \tag{7}$$

where $\mathbb{r}$ and $\mathbb{q}$ are vectors of input and output coefficients.

A mode combination rule which more accurately models the support damping effects uses the modal properties in Eqs. 5 and 6. One rule, which is based on Igusa, et al. (1984) is

$$R = \sqrt{\sum_{i=1}^{n} |b_i|^2\, S_i^2(\omega_i, \zeta_i)} \quad , \qquad b_i \equiv \frac{2\omega_i\left(\Psi_i^T \mathbb{q}\right)\left(\Psi_i^T \mathbb{M}\mathbb{r}\right)}{\widehat{\Psi}_i^T \mathbb{A} \widehat{\Psi}_i} \tag{8}$$

where Ψ_i is the vector consisting of the lower half of the mode shape $\widehat{\Psi}_i$.

The mode combination rules in Eqs. 7 and 8 are used for structures with natural frequencies which are widely spaced. If the structure has closely spaced natural frequencies, then the mode combination rules which include mode correlations are more accurate. Any of the rules in Der Kiureghian (1981), Igusa, et al. (1984), and Singh (1980) can be used.

5 NONLINEAR EQUATIONS FOR THE RESPONSES

For a given input, piping, and nonlinear supports, the only variable parameters are the equivalent linear properties of the supports. Therefore, the response of the piping can be expressed as a function of these variable properties:

$$R = f(K_1^e, C_1^e, \ldots, K_p^e, C_p^e, \mathbb{q}) \tag{9}$$

where K_j^e and C_j^e are the equivalent linear stiffness and damping of support j, p is the number of supports, and f is a function representing the modal analysis and mode combination rules presented in sections 3 and 4. For convenience, the vector $\mathbb{q}$ is retained in the function.

The response R_j at support j is obtained by letting $\mathbf{q} = \mathbf{q}_j$ be the vector corresponding to the response of support j. This response can be expressed in terms of the response function f:

$$R_j = f(K_1^e, C_1^e, \ldots, K_p^e, C_p^e, \mathbf{q}_j) \tag{10}$$

The analysis in section 2 shows that the equivalent linear properties for each support is not a constant, but is a function which depends on the amplitude A and average frequency ω of the response at that support. The response R at support j is given by R_j. The frequency ω can be set to either the fundamental frequency of the piping or the central frequency of the input response spectra. Substituting these results into Eq. 10 yields

$$R_j = f\left[K_1^e(R_1), C_1^e(R_1,\omega), \ldots, K_p^e(R_p), C_p^e(R_p,\omega), \mathbf{q}_j\right] \tag{11}$$

Eq. 11 is a set of p algebraic nonlinear equations with p nonlinear variables, R_1, ..., R_p. These can be solved using standard nonlinear solvers. After the solution is obtained, a final set of equivalent linear properties are obtained for all supports. Therefore, the final model for the piping system is linear. This linear model can be used to obtain response at all other parts of the piping. In addition, the modal properties of this model will indicate how the nonlinear supports will change the dynamic properties of the piping.

6 EXAMPLE ANALYSIS

The three-dimensional piping system shown in Fig. 5 is analyzed using the proposed method. The piping has three friction-type nonlinear supports which were analyzed in Figs. 1-4. The floor response spectra for the ground acceleration is given in Fig. 6. The initial modal damping ratios of the piping are 0.6%.

The results of the analyses with the modal damping approximation (Eq. 7) and the complete damping analysis (Eq. 8) are given in Table 1 and are compared with actual experimental data. It can be seen that Eq. 8 gives excellent results. The analysis using Eq. 7 gives conservative results.

Table 1. Comparison of analytical results with experimental observations.

Displacements (mm)	Support 1	Support 2	Support 3
Experimental	12.50	0.50	6.50
Equation 7	30.37	1.98	13.78
Equation 8	15.59	0.55	6.78

To see how the supports affect the modal properties of the piping, the fundamental frequency and damping ratio are determined at small response amplitudes before the supports yield, and are evaluated again at the peak response levels in Table 1. The fundamental frequency decreases from 8.28 Hz to 6.06 Hz. This is because once the supports yield, they have zero stiffness. The damping ratio increases from 0.6% to 2.79%. This is due to the friction energy dissipation of the supports. The increase in damping gives a clear physical measure of the effectiveness of the supports.

7 CONCLUSIONS

A method is described for analysis of piping with nonlinear supports. The method is versatile because it can use any response spectrum method. It is also simple to use because the required data are the hysteresis loops for the nonlinear supports and linear response spectra for the input.

Comparison with experimental results show good agreement. Excellent agreement is not always expected, however, the combination of simplicity and versatility makes the method suitable for practical use.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. H. Shibata of University of Tokyo for many enlightening discussions. Numerous discussions with Dr. M. Kunieda of Meisei University, Dr. F. Hara of Science University of Tokyo, Dr. K. Suzuki of Tokyo Metropolitan University and Mr. I. Ichihashi of the Kansai Electric Power Company were also very helpful.

REFERENCES

Caughey, T. K. (1960). "Sinusoidal Excitation of a System with Bilinear Hysteresis", ASME *J. Appl. Mech.*, Vol. 27, pp. 640-643.

Der Kiureghian, A. (1981). "A Response Spectrum Method for Random Vibration Analysis of MDF Systems", *Eqk. Engrg. Str. Dyn.*, Vol. 9, pp. 419-435.

Hurty, W. C. and Rubinstein, M. F. (1964). *Dynamics of Structures*, Prentice-Hall, Inc., Englewood Cliffs, N. J.

Igusa, T., Der Kiureghian, A. and Sackman, J. L. (1984). "Modal Decomposition Method for Stationary Response of Non-Classically Damped Systems", *Eq. Engrg. Str. Dyn.*, Vol. 12, pp. 121-136.

Igusa, T. and Der Kiureghian, A. (1985). "Dynamic Response of Multiply Supported Secondary Systems", *ASCE J. Engrg. Mech*, Vol. 111, pp. 20-41.

Igusa, T. and Xu, K. "Dynamic Characteristics of Non-Classically Damped Structures", Submitted to *Eqk. Engrg. Str. Dyn.*

Nomura, T. et al. (1989). "Study of Friction Damper as a Seismic Support", *Proc. SMiRT 10*, Vol. K(b), pp. 727-732.

Singh, M. P. (1980). "Seismic Response by SRSS for Nonproportional Damping", *ASCE J. Engrg. Mech.*, Vol 106, pp.1405-1419.

Sinha, R. and Igusa, T. (1990). *Analysis Methods for Piping Systems with Nonlinear High-Damping Supports*, Report to Mitsubishi Heavy Industries, Ltd., Japan.

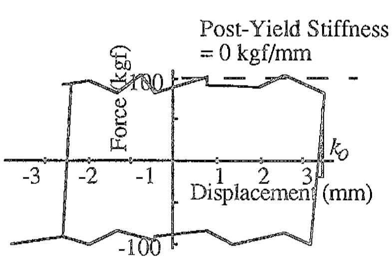

Fig. 1. Experimental support hysteresis curves.

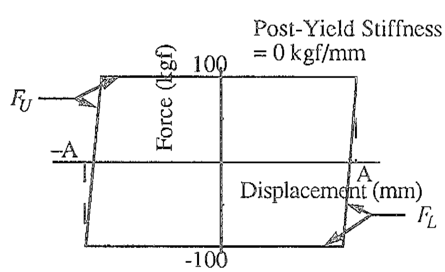

Fig. 2. Analytical Model of hysteresis curves in Fig. 1.

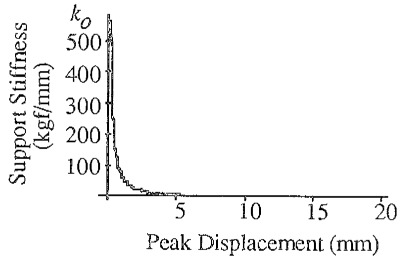

Fig. 3. Equivalent support stiffness.

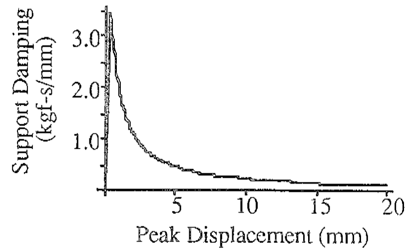

Fig. 4. Equivalent support damping.

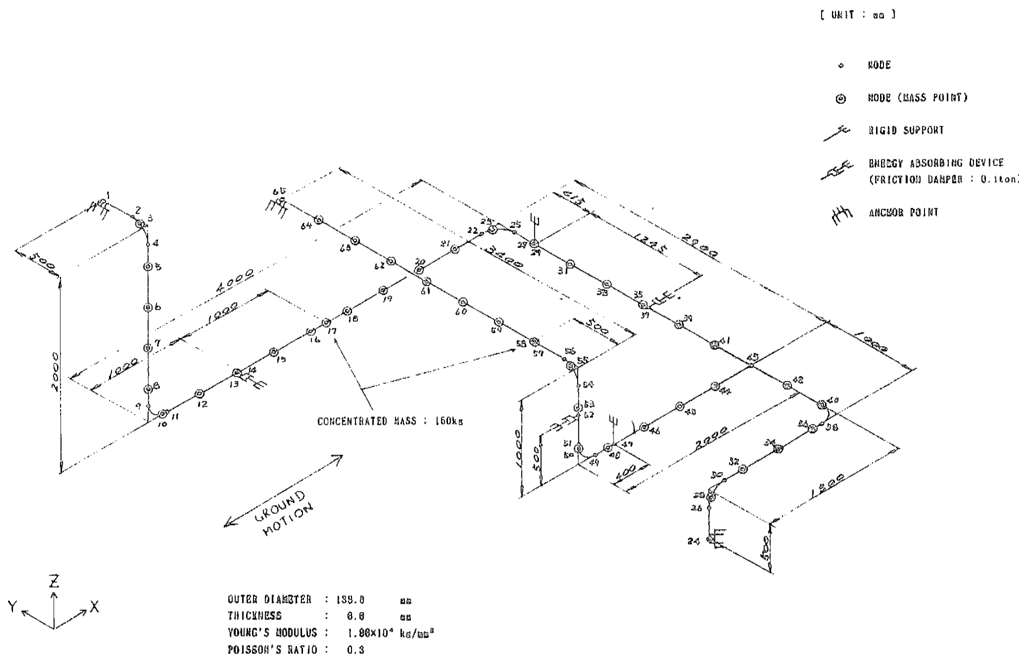

Fig. 5. Example piping system.

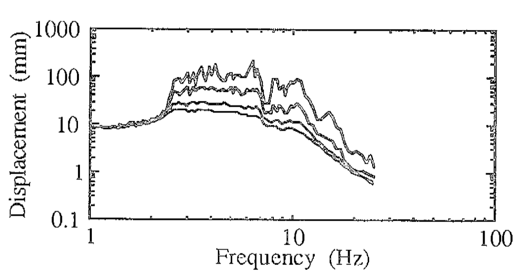

Fig. 6. Floor response spectra of ground acceleration. (——— 0% Damping, ------ 1% Damping, — — - 5% Damping, .—.—. 10% Damping)