Predicting Forces and Margins of Safety of Pile Foundations

P.P. Christiano

Department of Civil Engineering, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213, U.S.A.

J.-N. Levadoux

D'Appolonia S.A.R.L., 35-41 rue Capitaine Guynemer, F-92090 Paris la Défense, Cedex 28, France

F. Henning

Tractionel, 31 rue de la Science, B-1040 Bruxelles, Belgium

SUMMARY

The prediction of forces and the associated margins of safety for foundations subjected to combinations of dead loads and seismic excitation presents an important problem for nuclear power plant structures. It is the purpose of this paper to:

- Describe a practical and sufficiently accurate method, using a linear analysis, for determining the forces in individual piles within a group,
- Present a methodology for estimating the margin of safety of the pile group against either sliding or overturning; the methodology takes into account the interaction among axial load, shear force and bending moment, each of which may attain different values in each pile, and
- Illustrate the analytical approach as it applies to a particular reactor building founded on more than 650 piles.

The margins of safety against shear failure and overturning of the entire foundation are related to the load-carrying capacities of the piles acting as an aggregate; i.e., the capacity of the system is governed by the combination of all individual pile capacities. However, because the strength of a concrete member in combined compression and bending is dependent on the combination of axial forces and bending moment, the capacity of an individual pile in the presence of other piles (some of which may have already realized their capacity) may be determined either by performing a complete nonlinear analysis or by invoking certain assumptions regarding the loading history of each pile. The latter approach is employed herein whereby, based on the results of the linear analysis described above, a conservative load path (on the ultimate axial-load/bending-moment curve for reinforced concrete piles) is established and is used to derive global margins of safety for the foundation against shear failure and overturning.

1. Introduction

Evaluating the margin of safety for large groups of reinforced concrete piles poses a difficult problem; indeed, a rigorous analysis is impracticable when considering time-dependent loading. Beyond the usual difficulties associated with determining internal forces due to static and seismic loads in complex soil-structure systems, such as a reactor building founded on a large number of piles, the problem is further complicated by one's inability to know at the outset the theoretical ultimate resistance of the piles. More specifically, because of the interdependence that exists among ultimate axial resistance, ultimate bending moment, and (to a somewhat lesser extent) ultimate shear resistance, the load-carrying capacity of a reinforced concrete pile may assume an infinite number of values, depending on the relative magnitudes of the components of internal stress resultants.

If, in an actual situation, one were assured that all piles would reach their capacity at the same time (i.e., at the same stage of loading), then it would be appropriate to compare the combinations of force components (say axial load and bending moment) related to the applied load with the same combination of the components pertaining to the ultimate state, where for both combinations the ratio of force components would be the same. The "proportional-load" assumption is, of course, implied in most rational designs that are based on a single loading condition. However, where multiple loading conditions are to be considered such that upon continual increase of the loads the capacities of different piles are reached at different stages of loading, only a nonlinear approach will yield a theoretically exact assessment of ultimate resistance.

For the type of situation considered herein, in which combinations of static loads and seismically induced loads exist, one may choose to treat the problem through either an "exact" nonlinear analysis or an approximate linear analysis. The former approach, which is generally considered impracticable, would include a time-step analysis in which the nonlinear effects created by different piles yielding at different times (as well as any other nonlinear effects) are taken into account. A margin of safety could be determined on various bases, one of which would require multiple analyses incorporating increasingly large acceleration levels until overall failure is achieved, say by shearing or by overturning; a factor of safety might then be defined as the ratio of the peak acceleration required for failure to the design-basis acceleration.

In a linear analysis, the computations are greatly simplified and are far less expensive than those for a nonlinear treatment. However, the above cited difficulties associated with determining the ultimate capacity of the foundation then exist. Yet, because a linear analysis is the only one practically feasible, it becomes essential to estimate the ultimate capacity in a rational (and conservative) manner. In the following paragraphs, an approach is presented first for analyzing the interaction of a structure founded on piles and the surrounding soil; it is followed by a description of the rational use of interrelationships among force components in concrete members for determining the margin of safety for the foundation.

2. Method for Determining Forced and Moments in Piles

2.1 Illustrative Case Study

The method of analysis is, for the sake of illustration, described briefly as it applies to a particular reactor building founded on reinforced concrete piles. The foundation shown in Figure 1, consists of an almost circular, three-meters-thick slab supported by 658 piles. At the center of the foundation, a pit has been constructed at a level approximately eight meters below the surrounding slab. Two types of reinforced concrete piles support the mat. The majority of the piles (487 piles shown outside the shaded area in Figure 1) are 66-centimeters in diameter, while those piles in the vicinity of the pit (171 piles shown in the shaded area in Figure 1) have a 70-centimeter diameter and are more heavily reinforced. The spacing between adjacent piles varies from 1.6 to 2.9 meters. The piles outside the pit are 18.4 meters long, whereas the short piles under the pit are 10.4 meters long.

The soil profile beneath the upper mat level comprises a 2.5-meter-thick layer of silty clay, underlain by 2.0 meters of loose to medium dense sand, 4.8 meters of peaty clay, 35.1 meters of medium dense to very dense sand, and finally by 76 meters of stiff clay. The piles are thus founded approximately nine meters into the lower sand layer.

2.2 Analytical Model

Owing to the multiple layers comprising the soil profile and the presence of the foundation pit, it was considered essential to establish a three-dimensional finite element model to determine both the overall stiffness of the pile group and the distribution of loading, laterally and with depth.

However, in order to keep the problem to a manageable size, the following simplifications were made:

- The protrusion on the southeast side of the circular mat (see Figure 1) was neglected and the piles beneath it were included in the main foundation,
- The pit was idealized as an equal-area, circular pit, and
- The piles were lumped in groups of four and modelled by means of equivalent piles. The total number of piles analyzed was, therefore, reduced by a factor of four.

2.3 Analytical Procedure

A flow chart is shown in Figure 2, indicating the steps in the analysis. These steps are described briefly below:

2.3.1 Development of Artificial Time Histories

The seismic input was defined in terms of artificial time histories of acceleration whose response spectra match those specified by the USNRC Regulatory Guide 1.60 [1] for the site SSE. For the particular site considered here, the horizontal record was scaled to a maximum of 0.10 g and the vertical record to a maximum of 0.067 g and applied at a rock outcrop, in the free-field, away from the reactor building.

2.3.2 Wave Propagation Analyses

The response of the subgrade to SSE excitation in the absence of both the piles and the structure was determined by establishing a one-dimensional free-field analytical SHAKE [2] model incorporating shear and dilational wave velocities obtained from field cross-hole and down-hole measurements and relationships of both shear modulus and damping values with strain that are typical of the site soils. The SSE acceleration time histories were input to the base of the model to determine the accelerations throughout the depth.

2.3.3 Determination of Static Stiffness of Foundation

The static stiffness of the foundation was determined, using the computer code SAP V [3], by subjecting the idealized mat-pile-soil system to unit forces and moments applied to the top of the mat in each of the three coordinate directions. The resulting six-by-six flexibility matrix was then inverted to obtain the stiffness matrix for the global foundation. In addition, influence coefficients were determined for relating the internal loads in each pile to the loads applied at mat level. As mentioned previously, each of the lumped piles uaed in this analysis represented a group of four actual piles. To calibrate the stiffness properties of the lumped piles, it was therefore necessary to conduct closed-form and numerical analyses on a group of four actual piles prior to conducting the stiffness analysis of the global foundation.

2.3.4 Computation of Foundation Impedance

The impedance matrix (dynamic stiffness matrix) was determined by correcting the aforementioned static stiffness matrix for frequency dependence. This was done in an approximate manner by calculating the complex impedance matrix of the mat in the absence of the piles, using a computer program based on work by Kausel [4] which treats a rigid foundation bearing on the surface of a visco-elastic, horizontally layered soil profile. The correction coefficients, which provide equivalent damping as well as stiffness, were then applied to the static stiffness matrix that accounted for the presence of the piles. The slight effect of embedment was also included using the method of Johnson et al. [5].

2.3.5 Dynamic Analysis of Foundation

With the knowledge of the foundation impedance and the free-field SSE response at the elevation of the mat, displacement time histories of the structure were found. This structure-foundation interaction was determined through modal synthesis and a time-integration scheme, in which both structural (hysteretic) and composite soil (radiation and hysteretic) damping were taken into account, using the computer code DAPSYS [6].

The forces and bending moments that develop at the top of the piles as a result of the structure-foundation interaction were computed by multiplying the foundation impedance coefficients by the difference between the free-field response and the seismic response of the structure. These interaction forces were then used as input to the finite element model for the pile system to compute forces and moments over the lengths of the piles.

2.3.6 Dynamic Analysis of Single Pile in the Free Field

The forces and moments incurred by the piles due to the passage of vertically propagating shear and compressional waves were obtained by applying the free-field ground motion obtained from the SHAKE analysis to the vicinity of a finite element representation (DAPSYS) for an individual pile. The compliance of the soil was taken into account using a modification of the computer code DEFPIG [7]. Thus, the computed pile stresses are associated with the passage of modified seismic waves through the piles.

2.3.7 Combined Pile Stresses

The stresses incurred by the piles are the sum of those related to the dead loads and the structure-foundation interaction forces due to wave passage. Perhaps the only practical way of determining stresses due to combined static loading and seismic excitation in multiple directions is that suggested by Hall et al. [8]. This procedure requires that 100 percent of the response due to one component of excitation be combined with 40 percent of the response due to each of the remaining two components of excitation. The resultant seismic force is then combined with all other loads that act on the structure. Each component of excitation is taken in turn as providing the primary (100 percent) motion and analysis is based on the envelope of results. This procedure requires that only the peak response for each component of excitation be used to evaluate internal forces and moments. Typical results are shown in Figure 3.

2.3.8 Evaluation of Pile Strength

The ultimate bending moment that can be developed by a pile is a function of the axial load acting on the pile. Similarly, the shearing resistance that can be mobilized also depends on the axial force and on the bending moment as well. These relationships, which follow the Belgian and United States design codes [9, 10], may be represented as follows. The interaction between the ultimate axial compression (N_u) and the ultimate moment (M_u) may be expressed most conveniently in graphical form as shown in Figure 4, where it can be seen that the maximum value of N_u is developed when M_u equals zero. The maximum value of M_u is developed at a positive value of N_u, and M_u decreases to a lower value when N_u equals zero. If, as is usually the case in typical design situations, the design values of axial load and bending moment are prescribed then the designer is required to choose a cross section for the structural member whose ultimate capacity envelope lies above the design point (N_d, M_d). Generally, the closer the point (N_d, M_d) is to the envelope, the more efficient is the design.

However, the problem of determining the capacity of the piles, each of which interacts with the other as part of the foundation system, is considerably more complicated than the foregoing approach. It should also be mentioned that the shear strength provided by the concrete is a function of the axial force and bending moment; for the sake of brevity, however, this point is not discussed further. The envelopes of ultimate capacity in terms of axial-force/bending-moment were computed for the long and short piles, and are presented in Figure 5. This figure also presents the initial dead load conditions for each pile, as well as the most critical level (the base of the mat). The combined loads remain well inside the ultimate capacity envelope for Quadrant I, although in Quadrant II (not shown) a few piles reach more than 90 percent of their capacities under SSE loading conditions.

The computed shear capacity and loading conditions of the piles are presented schematically in Figure 6. The highest shear forces would be experienced by the short piles under the reactor building pit, whereby the most severely loaded pile in shear is at 97 percent of its capacity.

3. Design Phylosophy and Margins of Safety

In assessing the safety of the structure-foundation system, it is necessary to establish an appropriate comparison between the forces that the piles must resist under the postulated load combinations and the resistance which the piles are able to provide at the limit of translational and rotational stability of the overall foundation. It is noted at the outset that, owing to the high redundancy of the pile foundation, the proximity of the load in the single most severely stressed pile to the ultimate capacity of that pile is an erroneous measure of the margin of safety of the foundation system against overall yielding. Rather, should the magnitude of loading be increased beyond the postulated values, yielding of one or more piles would lead to the additonal applied load being carried by piles whose capacity had not yet been realized. Accordingly, it is appropriate to establish margins of safety for the foundation system which account for the aggregate capacity of the piles. Without conducting an analysis of progressive nonlinear failure, it is not possible to establish a unique set of ultimate axial-force/bending-moment values which are associated with a prescribed failure mechanism. It is feasible, however, to estimate the ultimate shear and moment capacity of each pile by invoking a reasonable assumption on the path of loading that the pile follows towards its ultimate stress state. With the knowledge of the load capacity of each pile, it is a comparatively straightforward matter to determine the overall capacity of the foundation system. A factor of safety which may then be defined as the global resisting force (or moment) divided by the overall driving force (or moment), can be evaluated along the following lines.

One may view the progression of loading in a single pile in terms of a "loading path" on the axial-force/bending-moment interaction diagram shown in Figure 4. Suppose that, initially, due to the dead load, the combination of axial force and resultant bending moment are represented by (N_i, M_i), on the diagram. If the loading, considered to be quasi-static, increases to a design value (one of the postulated load combinations in the present case), the force and moment would change along a linear load path to (N_d, M_d), assuming linear behavior of the steel and concrete. The same type of change would occur in all piles except, of course, that the particular path may be different for each pile. If all piles were so proportioned as to allow the ultimate capacity to be reached simultaneously, then the ultimate capacity of the overall foundation would be directly related to (N_u, M_u) which represents for each pile the combination of ultimate compression and bending moment defined by intersection of the linear loading path with the ultimate capacity envelope.

In reality, however, the loading path for each elastic pile would change as certain other piles attain their ultimate capacity. Additional applied loading would thus be apportioned differently among all remaining elastic piles as additional piles continue to realize their capacity. Thus, for any generic pile, the actual loading path would be curved rather than straight, and it could be determined exactly only by a nonlinear analysis of the foundation system over the complete loading history.

It appears, however, that for reasons described below, the use of (N_u, M_u) associated with the linear loading path for each pile provides a reasonable, yet conservative, estimate of the global factor of safety for the foundation. Here, one factor of safety for the foundation is defined as the ratio of overall resisting moment furnished by the piles to the overall driving moment. (A similar factor of safety is defined subsequently for shear.) Because the system derives its rotational stability primarily from axial forces in the piles acting through moment arms, it is reasonable to expect that most of those piles which might experience increased compression in the range between (N_i, M_i) and (N_d, M_d) would continue to incur at least the same rate of change in compression, as neighbouring piles yield and more load is carried by the elastic piles. Similarly, if the load would be reduced and all piles would remain elastic, the linear loading path would have a negative rather than the positive slope shown in Figure 4. Accordingly, upon yielding of some piles, most of those which were initially unloading would continue to unload at an increased rate. Thus, the actual change from the design axial force (N_d) to the ultimate force (N_u) would be underestimated by the change from N_d to N_u.

Since, for most piles, the ultimate compressive force plays a much larger role than does the ultimate moment in resisting overturning of the foundation mat, an underestimate of the change in axial force would probably lead to an underestimate of the factor of safety against overturning. One could obviously claim that the most conservative estimate of the factor of safety would be associated with the ultimate moment capacity of the piles at the design compression load, i.e. considering $N_u' = N_d$. Such an assertion would imply, however, that as loading increases and the foundation approaches its limiting load-carrying capacity, the increased capacity beyond the design value would be attributable solely to the bending moment capacity of the piles. For the foundation geometry considered herein, such a position would be contrary to sound engineering judgement.

The global factor of safety against shear failure is defined as the resultant shear capacity of all piles in the direction of applied resultant horizontal force divided by the magnitude of the resultant horizontal force. As noted above, the shear capacity of a pile is a function of the state of loading, particularly the axial load developed in the pile. The shearing capacity of each pile was determined with the knowledge of (N_d, M_d).

For the illustrative case described above, it was determined that the global factors of safety against overturning and shear were 2.0 and 4.0, respectively, under the most severe loading combination. These factors of safety are considered to be entirely adequate, despite the fact that some piles evidently would reach their ultimate state under loads just a few percent greater than those postulated for SSE conditions.

4. Conclusions

A method based on conditions of linearity has been presented for predicting the forces and moments in pile foundations subjected to combined static and seismic loading. A global margin of safety pertaining to overall overturning is then determined by assuming that the direction of the load path on the axial-force/bending-moment interaction diagram is maintained during progressive failure of the piles. A similar assumption is made as regards the margin of safety against sliding. The assumption on load path is considered to be conservative; the degree of conservatism is presently being studied by comparing, for some simplified cases having relatively few piles, results associated with the present method to those based on a nonlinear analysis.

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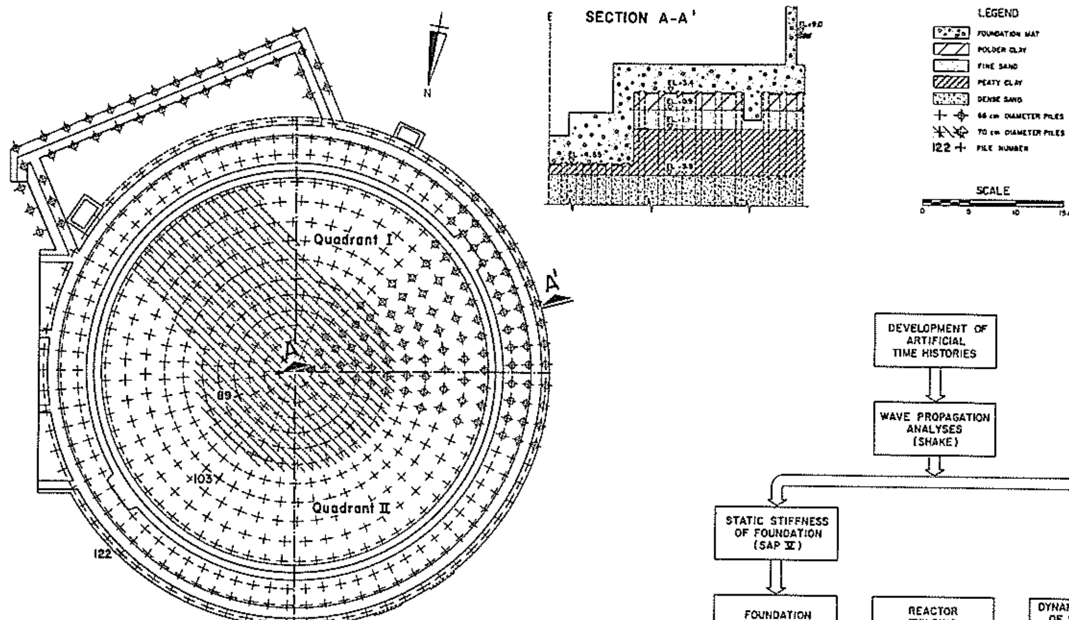


FIGURE I
PLAN AND SECTION
OF REACTOR BUILDING FOUNDATION

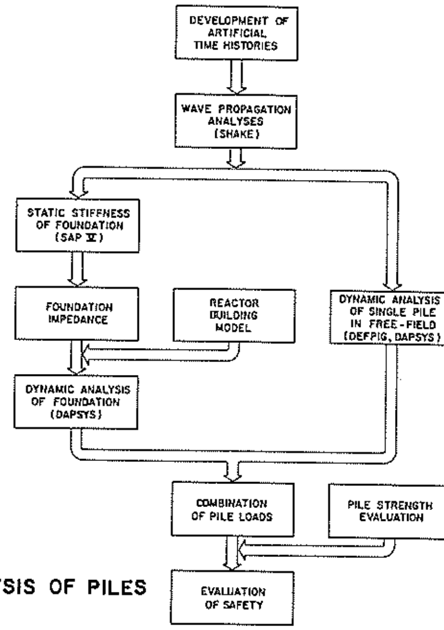
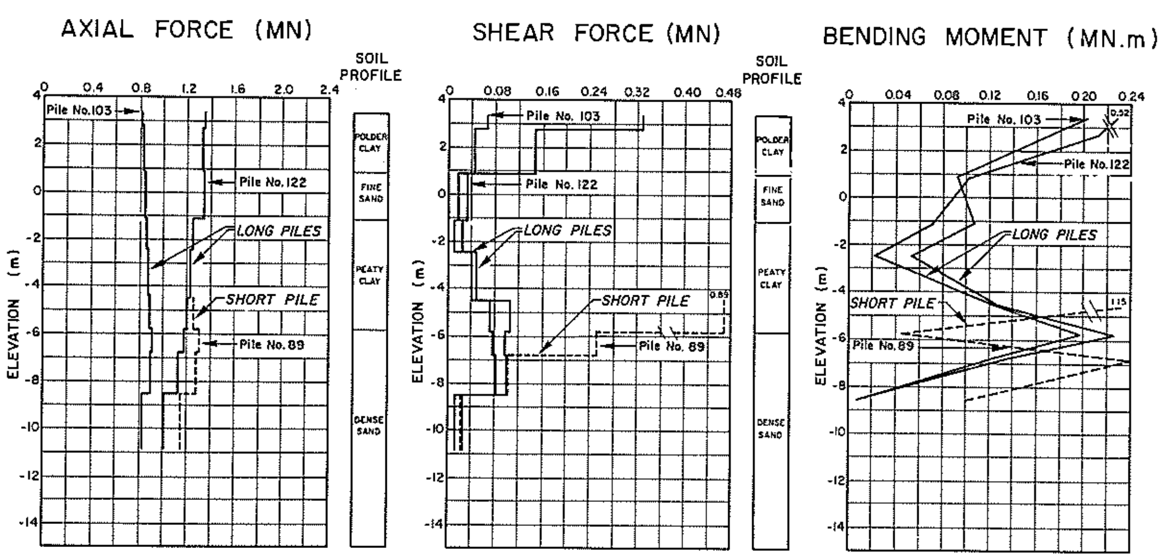


FIGURE 2
FLOW CHART FOR SEISMIC ANALYSIS OF PILES



NOTE
SEE FIGURE I FOR LOCATIONS OF PILES 89, 103 AND 122

FIGURE 3
TYPICAL FORCE PATTERNS IN PILES

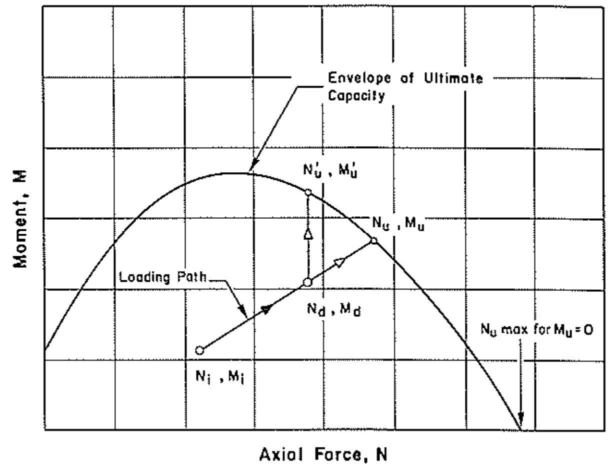
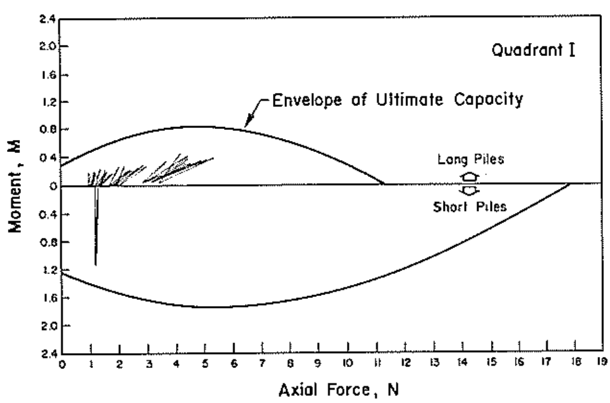


FIGURE 4

GENERIC AXIAL-FORCE / BENDING-MOMENT INTERACTION DIAGRAM

LEGEND

N_i, M_i	Initial Dead Loads
N_d, M_d	Dead Loads + SSE Loads
N_u, M_u N_u', M_u'	Ultimate Capacities



NOTES

1. LOADS ARE EXPRESSED IN MEGANEWTONS AND METERS
2. VALUES OF LOADS PERTAIN TO THE CRITICAL LOAD COMBINATION AT BASE OF MAT
3. SEE FIGURE 1 FOR LOCATION OF QUADRANT I

FIGURE 5

LOADING PATHS ON INTERACTION DIAGRAM

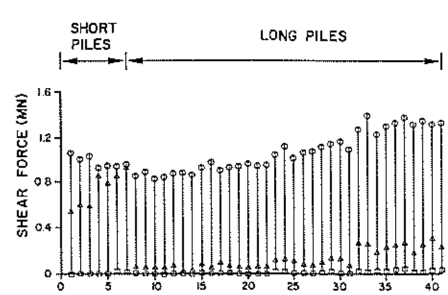


FIGURE 6

PILE SHEAR LOADS AND ULTIMATE SHEAR CAPACITIES

LEGEND

- o ULTIMATE CAPACITY
- △ DEAD LOAD + SSE
- □ DEAD LOAD

NOTE

VALUES OF LOADS PERTAIN TO THE CRITICAL LOAD COMBINATION AT A DEPTH OF ONE PILE DIAMETER BELOW THE MAT.