Dynamic Buckling of Viscoplastic Spherical Shell

R. Bukowski, W. Wojewodzki
Politechnika Warszawska, Instytut Mechaniki Konstrukcji Inzynierskich,
Al. Armii Ludowej 16, PL-00-637 Warsaw, Poland

Summary

General case of buckling of a complete thin spherical shell is presented. The shell is loaded uniformly by a pressure impulse and subjected to a given temperature. The problem is formulated according to the Kirchhoff-Love's theory of small deformations. The influence of strain rate and temperature on material response is described by the Perzyna's equations P.Perzyna "The constitutive equations for rate sensitive plastic materials", Quart.Appl.Math.,

$$\dot{\varepsilon}_{ij} = \frac{\gamma}{2} \langle \phi(F) \rangle \frac{s_{ij}}{J_2^{1/2}}, \quad F = \frac{J_2^{1/2}}{K} - 1, \quad \langle \phi(F) \rangle = \begin{cases} \phi(F) & \text{for } F > 0 \\ 0 & \text{for } F \leq 0 \end{cases} \qquad (1)$$

where $\dot{\varepsilon}_{ij}$ is the strain rate tensor, s_{ij} denotes the stress deviator, $J_2 = \frac{1}{2} s_{ij} s_{ij}$, $K = \sigma_0 / 3^{1/2}$, σ_0 is the static yield stress in simple tension and γ stands for the viscocity coefficient of the material. The power function $\phi(F) = F^N$ is assumed, where N denotes the material constant. Experimental results confirmed a good description of the behaviour of metals with only two quantities γ and F dependent on the temperature and ϕ itself independent of it.

Analytically, the problem is formulated as a superposition of small perturbations $w_p(\varphi, \vartheta, t)$, $u_p(\varphi, \vartheta, t)$ and $v_p(\varphi, \vartheta, t)$ on the basic unperturbed motion $w_0(t)$, $u_0 = v_0 = 0$. The amplitudes of perturbed motion are restricted to be so small that the homogeneous compressive deformation is predominant over the local bending. This condition permits the constitutive equations to be linearized by the expension of the Eqs.(1)into Taylor's series of three variables in the vicinity of unperturbed motion and by retaining only two terms. As a result, a set of three linear partial differential equations of the fifth order is obtained for describing the perturbed motion of the shell. The time - dependent coefficients of this governing set of equations are determined by the solution for the unperturbed motion. The governing set of equations is solved using the series of spherical functions. As a result, for the time - dependent amplitudes $u_{mn}(t)$, $v_{mn}(t)$, $w_{mn}(t)$ in the series, the set of the three ordinary equations with variable coefficients is arrived at. This set of equations is solved numerically. It turns out certain harmonics grow very rapidly and cause the shell to buckle. This property of the amplitudes is used to determine the threshold impulse.

Unlike in the existing solutions (W.Wojewódzki, P.Lewiński,"Viscoplastic axisymmetrical buckling of spherical shell subjected to radial pressure impulse", 5th Int.Conf.SMiRT, L 8/4, Berlin, (1979)), the present paper takes into account the asymmetrical mode of buckling and the non - linear function $\phi(F), N \neq 1$.

The solution of the problem in the displacements, the influence of viscosity effects, the temperature and the initial imperfections of the geometry and loading are shown. Also, the influence of the tangential displacements on the magnitude of the radial displacements, buckling mode and critical impulse is investigated.

1. Introduction

Dynamic buckling of cylindrical and spherical shells made of an elastic--plastic and a rigid-plastic material with linear hardening has recently been investigated in many papers: Abrahamson, Goodier[1], Lindberg[2], Vaughan, Florence[3], Jones, Ahn[4,5]and others. The viscosity effects of the material are taken into account in the papers: Perrone[6], Florence[7], Florence, Abrahamson[8], Wojewódzki[9,10], Wojewódzki, Lewiński[11]. In most of the above mentioned papers only the radial displacement was considered. The buckling mode and the threshold impulse were determined.

In the paper a general case of buckling of a complete thin spherical shell is presented. The shell is loaded uniformly by a pressure impulse and subjected to a given temperature.

2. Basic equations

The problem is formulated according to the Kirchhoff-Love's theory of small deformations. The strain rate relations and the equilibrium equations can be written in the following form,[12], Fig.1:

$$\dot{\varepsilon}_\varphi=\frac{1}{a}\left(\frac{\partial \dot{u}}{\partial \varphi}-\dot{w}\right)-\frac{z}{a^2}\left(\frac{\partial^2 \dot{w}}{\partial \varphi^2}+\frac{\partial \dot{u}}{\partial \varphi}\right),$$

$$\dot{\varepsilon}_\vartheta=\frac{1}{a}\left(\frac{1}{\sin\varphi}\frac{\partial \dot{v}}{\partial \vartheta}+\dot{u}\operatorname{ctg}\varphi-\dot{w}\right)-\frac{z}{a^2}\left(\frac{1}{\sin^2\varphi}\frac{\partial^2 \dot{w}}{\partial \vartheta^2}+\frac{1}{\sin\varphi}\frac{\partial \dot{v}}{\partial \vartheta}+\operatorname{ctg}\varphi\frac{\partial \dot{w}}{\partial \varphi}+\dot{u}\operatorname{ctg}\varphi\right), \tag{2.1}$$

$$\dot{\varepsilon}_{\varphi\vartheta}=\frac{1}{2a}\left(\frac{\partial \dot{v}}{\partial \varphi}+\frac{1}{\sin\varphi}\frac{\partial \dot{u}}{\partial \vartheta}-\dot{v}\operatorname{ctg}\varphi\right)-\frac{z}{a^2}\left(\frac{1}{\sin\varphi}\frac{\partial^2 \dot{w}}{\partial \varphi\partial \vartheta}-\frac{\cos\varphi}{\sin^2\varphi}\frac{\partial \dot{w}}{\partial \vartheta}+\frac{\sin\varphi}{2}\frac{\partial}{\partial \varphi}\left(\frac{\dot{v}}{\sin\varphi}\right)+\frac{1}{2\sin\varphi}\frac{\partial \dot{u}}{\partial \vartheta}\right),$$

$$a\cos\varphi N_\varphi+a\sin\varphi\frac{\partial N_\varphi}{\partial \varphi}+a\frac{\partial N_{\vartheta\varphi}}{\partial \vartheta}-\tilde{r}_2N_\vartheta-\tilde{r}_1N_{\varphi\vartheta}+\tilde{q}_1Q_\varphi+\tilde{p}_2Q_\vartheta+a^2\sin\varphi P_1=0,$$

$$a\cos\varphi N_{\vartheta\varphi}+a\sin\varphi\frac{\partial N_{\vartheta\varphi}}{\partial \varphi}+a\frac{\partial N_\vartheta}{\partial \vartheta}+\tilde{r}_1N_\varphi+\tilde{r}_2N_{\vartheta\varphi}-\tilde{q}_2Q_\vartheta-\tilde{p}_1Q_\varphi+a^2\sin\varphi P_2=0,$$

$$a\cos\varphi Q_\varphi+a\sin\varphi\frac{\partial Q_\varphi}{\partial \varphi}+a\frac{\partial Q_\vartheta}{\partial \vartheta}-\tilde{q}_1N_\varphi-\tilde{p}_2N_{\vartheta\varphi}+\tilde{p}_1N_{\varphi\vartheta}+\tilde{q}_2N_\vartheta+a^2\sin\varphi P_3=0, \tag{2.2}$$

$$a\cos\varphi M_{\varphi\vartheta}+a\sin\varphi\frac{\partial M_{\varphi\vartheta}}{\partial \varphi}-a\frac{\partial M_\vartheta}{\partial \vartheta}-\tilde{r}_1M_\varphi-\tilde{r}_2M_{\vartheta\varphi}+a^2\sin\varphi Q_\vartheta=0,$$

$$a\cos\varphi M_\varphi+a\sin\varphi\frac{\partial M_\varphi}{\partial \varphi}+a\frac{\partial M_{\vartheta\varphi}}{\partial \vartheta}+\tilde{r}_1M_{\varphi\vartheta}-\tilde{r}_2M_\vartheta-a^2\sin\varphi Q_\varphi=0,$$

$$\tilde{p}_1M_\varphi+\tilde{p}_2M_\vartheta-\tilde{q}_1M_{\varphi\vartheta}+\tilde{q}_2M_{\vartheta\varphi}+a^2\sin\varphi(N_{\varphi\vartheta}-N_{\vartheta\varphi})=0$$

where

$$\tilde{p}_1=\frac{\partial^2 w}{\partial\varphi\,\partial\vartheta}-\operatorname{ctg}\varphi\,\frac{\partial w}{\partial\vartheta}\,,\qquad \tilde{p}_2=-\frac{\partial}{\partial\vartheta}\left(\frac{\partial w}{\partial\varphi}+u\right)+\left(\frac{\partial w}{\partial\vartheta}+v\sin\varphi\right)\operatorname{ctg}\varphi-\sin\varphi\,\frac{\partial v}{\partial\varphi}\,,$$

$$\tilde{q}_1=-\sin\varphi\left(a+\frac{\partial}{\partial\varphi}\left(\frac{\partial w}{\partial\varphi}+u\right)\right)\,,\qquad \tilde{q}_2=a\sin\varphi+\frac{\partial}{\partial\vartheta}\left(\frac{1}{\sin\varphi}\,\frac{\partial w}{\partial\vartheta}+v\right)+\cos\varphi\left(\frac{\partial w}{\partial\varphi}+u\right),$$

$$\tilde{r}_1=\sin\varphi\,\frac{\partial^2 v}{\partial\varphi^2}+\frac{\partial w}{\partial\vartheta}+v\sin\varphi\,,\qquad \tilde{r}_2=a\cos\varphi+\frac{\partial^2 v}{\partial\varphi\,\partial\vartheta}-\sin\varphi\left(\frac{\partial w}{\partial\varphi}+u\right),$$

$P_1=q_x-\varrho h\ddot{u}$, $P_2=q_y-\varrho h\ddot{v}$, $P_3=q_z-\varrho h\ddot{w}$, h denotes the thickness of the shell, ϱ is the density.

The constitutive equations. The influence of work hardening, strain rate and temperature on material response can be described by the Perzyna's equations, [13]:

$$\dot{\varepsilon}_{ij}=\frac{\gamma}{2}\left\langle\phi(F)\right\rangle\frac{s_{ij}}{J_2^{1/2}}\,,\qquad F=\frac{J_2^{1/2}}{K}-1\,,\qquad \left\langle\phi(F)\right\rangle=\begin{cases}\phi(F) & \text{for } F>0\\ 0 & \text{for } F\leqslant 0\end{cases}\tag{2.3}$$

where $\dot{\varepsilon}_{ij}$ is the strain rate tensor, s_{ij} denotes the stress deviator, $J_2=\frac{1}{2}s_{ij}s_{ij}$ $K=\sigma_0/3^{1/2}$, σ_0 is the static yield stress in simple tension and γ stands for the viscocity coefficient of the material, $i,j=1,2,3$. The non-linear function $\phi(F)=F^{\delta}$ is assumed, where δ denotes the material constant. Experimental results confirmed a good description of the behaviour of metals with only two quantities γ and F dependent on the temperature and ϕ itself independent of it, see Perzyna, Wierzbicki [14]. For the mild steel we have $\delta=5$,

$$K(\Theta)=146{,}98\exp\left[0{,}45\left(\frac{288}{\Theta}-1\right)\right]\,,\qquad \gamma(\Theta)=34{,}78\left[1+2{,}6\left(\frac{220-\Theta}{273}\right)^2\right]\tag{2.4}$$

where Θ denotes the temperature in K.

According to the Kirchhoff-Love theory will be assumed $\sigma_z=\sigma_{33}=0$, $\dot{\varepsilon}_{13}=\dot{\varepsilon}_{23}=0$.

In the state of compressive plastic flow the shell buckles due to imperfections. A characteristic feature of dynamic buckling is the significance of inertial effect in restraining the growth of buckling mode amplitudes at an early stage of the motion. This effect results in the yielding of the shell before the instabilities can become dominant. Analytically, the problem is formulated as a superposition of small perturbations $u_p(\varphi,\vartheta,t)$, $v_p(\varphi,\vartheta,t)$ and $w_p(\varphi,\vartheta,t)$ on the basic unperturbed motion $w_o(t)$, $u_o=v_o=0$. The amplitudes of perturbed motion are restricted to be so small that the homogeneous compre-

ssive deformation is predominant over the local bending. This condition permits the constitutive equations to be linearized by the expension into Taylor's series of three variables in the vicinity of unperturbed motion and by retaining two terms only. We get:

$$\sigma_{ij} = A^*(\dot{\varepsilon}_{ij} + \dot{\varepsilon}_{tt}\delta_{ij}) + B^*[(\dot{\varepsilon}^0_{\tau s} + \dot{\varepsilon}^0_{ll}\delta_{\tau s})(\dot{\varepsilon}_{\tau s} - \dot{\varepsilon}^0_{\tau s})](\dot{\varepsilon}^0_{ij} + \dot{\varepsilon}^0_{tt}\delta_{ij}) \qquad (2.5)$$

where

$$A^* = A^p + A^v, \quad B^* = \frac{1}{2I_2^0}\left(-A^p + \frac{1-\delta}{\delta}A^v\right), \quad A^p = K I_2^{0\,-1/2}, \quad A^v = K\left(\frac{2}{\sqrt{3}}\right)^{1/\delta} I_2^{0\,(1-\delta)/2\delta}, \quad I_2^0 = \frac{1}{2}(\dot{\varepsilon}^0_{ij} + \dot{\varepsilon}^0_{ll}\delta_{ij})\dot{\varepsilon}^0_{ij},$$

$\dot{\varepsilon}^0_{ij}$ denotes the strain rate tensor of the unperturbed motion and all the indices $i, j, l, \ldots$ can take on 1,2 only.

3. Equations of viscoplastic flow buckling

Integrating the components (2.5) over the thickness of the shell and using the relation (2.1) the resultant forces and moments can be expressed in terms of displacement velocity field. Eliminating shear forces from the equilibrium equations and accouting for the resultant forces and moments in eqs. (2.2) we obtain the following equations of the viscoplastic flow buckling:

$$\frac{C_0}{2}\left(L\dot{\mu} - \frac{6\dot{w}_0}{a}\mu\right) + aq_\mu - a\rho h\ddot{\mu} = 0,$$

$$C_1[L\dot{\eta} + c^*(LL\dot{w}_p - 2L\dot{w}_p)] + C_2(\dot{\eta} + L\dot{w}_p - 2\dot{w}_p) - 3C_0\frac{\dot{w}_0}{a}\left(\eta + Lw_p - 2w_p\right) + aq_\eta - a\rho h\ddot{\eta} = 0, \qquad (3.1)$$

$$-C_1 c^*(L\dot{\eta} + LL\dot{w}_p - 2L\dot{w}_p) - C_2(\dot{\eta} - 2\dot{w}_p + c^*L\dot{w}_p) - 3C_0\frac{\dot{w}_0}{a}\left(\eta + Lw_p - 2w_p\right) + aq_z^p - a\rho h\ddot{w}_p = 0,$$

$$\ddot{w}_0 + c\dot{w}_0^{1/\delta} = \alpha \qquad (3.2)$$

where

$$C_0 = \frac{h}{a}A^*, \quad C_1 = \frac{h}{2a}\left(A^p + \frac{3+\delta}{\delta}A^v\right), \quad C_2 = -\frac{3h}{2a}A^v, \quad c^* = \frac{h^2}{12a^2}, \quad c = \beta\left(\frac{2\sqrt{3}}{\gamma a}\right)^{1/\delta}, \quad \beta = \frac{2\sqrt{3}K}{\rho a}, \quad \alpha = \frac{q_z^0}{\rho h} - \beta,$$

$$L = \frac{\partial^2}{\partial\varphi^2} + \operatorname{ctg}\varphi\frac{\partial}{\partial\varphi} + \frac{1}{\sin^2\varphi}\frac{\partial^2}{\partial\vartheta^2} + 2$$

$$\eta = \frac{\partial u_p}{\partial\varphi} + u_p\operatorname{ctg}\varphi + \frac{1}{\sin\varphi}\frac{\partial v_p}{\partial\vartheta}, \qquad \mu = \frac{1}{\sin\varphi}\frac{\partial u_p}{\partial\vartheta} - \frac{\partial v_p}{\partial\varphi} - v_p\operatorname{ctg}\varphi,$$

$$q_\eta = \frac{\partial q_x^p}{\partial\varphi} + q_x^p\operatorname{ctg}\varphi + \frac{1}{\sin\varphi}\frac{\partial q_y^p}{\partial\vartheta}, \qquad q_\mu = \frac{1}{\sin\varphi}\frac{\partial q_x^p}{\partial\vartheta} - \frac{\partial q_y^p}{\partial\varphi} - q_y^p\operatorname{ctg}\varphi.$$

The general solution of eqs. (3.1) for the buckling mode may be obtained by using the infinite series of spherical functions,

$$w_p = \sum_{n=0}^{\infty}\sum_{m=0}^{n} w_{mn}(t)\,P_n^m(\cos\varphi)\cos m\vartheta,$$

$$u_p = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left[u_{mn}(t) \frac{dP_n^m(\cos\varphi)}{d\varphi} + \frac{m v_{mn}(t)}{\sin\varphi} P_n^m(\cos\varphi) \right] \cos m\vartheta \ , \qquad (3.3)$$

$$v_p = -\sum_{n=0}^{\infty} \sum_{m=0}^{n} \left[\frac{m u_{mn}(t)}{\sin\varphi} P_n^m(\cos\varphi) + v_{mn}(t) \frac{dP_n^m(\cos\varphi)}{d\varphi} \right] \sin m\vartheta$$

where P_n^m are associated Legendre polynomials of degree n and order m.

The initial imperfections of the displacements and the perturbations of the loading are assumed to be distributed in the same form as equations (3.3).

Substituting (3.3) in eqs.(3.1) we find the following equations for the amplitudes $v_{mn}(t)$, $u_{mn}(t)$, $w_{mn}(t)$:

$$\ddot{v}_{mn} + \frac{1}{2a\rho h} C_0 (\lambda_n \dot{v}_{mn} + \frac{6\dot{w}_0}{a} v_{mn}) - \frac{q^y_{mn}}{\rho h} = 0 \ ,$$

$$\ddot{u}_{mn} + \frac{1}{a\rho h} \left[C_1 \lambda_n (\dot{u}_{mn} + c^* \dot{w}_{mn}) - C_2 (\dot{u}_{mn} + \dot{w}_{mn}) + \frac{3\dot{w}_0}{a} C_0 (u_{mn} + w_{mn}) - a q^x_{mn} \right] = 0 \ , \qquad (3.4)$$

$$\ddot{w}_{mn} - \frac{1}{a\rho h} \left[(\lambda_n + 2)(C_2 - \lambda_n c^* C_1) \dot{u}_{mn} + (\lambda_n + 2) c^* (C_2 - \lambda_n C_1) \dot{w}_{mn} + 2C_2 \dot{w}_{mn} + 3(\lambda_n + 2) C_0 \frac{\dot{w}_0}{a} (u_{mn} + w_{mn}) + a q^z_{mn} \right] = 0 \ ,$$

where $\lambda_n = n(n+1) - 2$.

The easiest way to solve eq. (3.2) for the unperturbed motion and eqs.(3.4) for the amplitudes, complying with the given initial conditions, is by numerical integration. Buckling stems from the growth of small imperfections in the initial displacement and loading fields. It turns out that certain harmonics grow rapidly and cause the shell exhibit a characteristic wrinkled shape which is characterized by the critical mode numbers. This property of the amplitudes is used to determine the threshold impulse that the shell can tolerate without excessive deformation.

4. Numerical results

The equations (3.2) and (3.4) have been solved numerically for several values of material and geometrical parameters. Some of the results for the shell made of mild steel and for the data: $a = 10$ cm, $h = 0{,}3$ cm, $\rho = 7{,}8$ kg/m^3, $\delta = 1;3;5;7$, σ_0 and γ according to eqs.(2.4) are presented here diagrammatically. Loading is assumed as an ideal radial impulse V_0 ($q_x = q_y = q_z = 0$) together with the following initial conditions: $w_0(0) = 0$, $\dot{w}_0(0) = V_0$, $u_{mn}(0) = v_{mn}(0) = 0$, $\dot{u}_{mn}(0) = \dot{v}_{mn}(0) = 0$, $w_{mn}(0) = a_{mn}$, $\dot{w}_{mn}(0) = b_{mn}$; in particular, $a_{mn} = 0{,}1\,h\,; 0{,}001\,h$ and $b_{mn} = 0{,}1\,V_0\,; 0{,}01\,V_0$.

In Figs. 2 and 3 the results (eq.(3.2)) for the unperturbed displacement $w_o(t_f)$ are shown, t_f denoting the instant $t=t_f$ when $\dot{w}_o(t_f)=0$. Substantial influence of the material constant δ and the temperature Θ can be seen.

In Figs. 4,5,6 and 7 the amplitudes of perturbed displacement w_{mn} (eqs.(3.4)) are presented for different values of δ, γ and Θ. Again, the pronounced influence of these parameters is readily observed.

In Fig.8 the shape of buckled shell is given at $t=t_f=63.1\ \mu s$. It is seen that the u_p is many times smaller than w_p; it is the growth of normal displacement that mainly causes the instability of the shell. In this type of buckling the loss of stability is not quite instantaneous, the process need the increment of loading and some time to develop. The functions $w_{mn}(t_f)$ reach large values in a certain narrow interval of the impulse variation Fig.6; hence it is natural to determine the critical value of the impulse graphically as the abcissa of such a point on the curve at which a small increment of the pulse begins to produce considerable increments of the deflection amplitude.

It can be concluded that accounting for the non-linearity of function $\Phi(F)$, $\delta \neq 1$, particularly for smaller values of γ (see Figs.4 and 5) as well as elevated temperature causes a decrease of the buckling resistance of the shell.

References

[1] ABRAHAMSON, G.R., GOODIER, J.N., "Dynamic plastic flow buckling of a cylindrical shell from uniform radial impulse", Proc.Fourth U.S.Nat.Congr. Appl.Mech., (June, 1962)

[2] LINDBERG, H.E., "Buckling of a very thin cylindrical shell due to an impulsive pressure", J.Appl.Mech., 31, (1964).

[3] VAUGHAN, H., FLORENCE, A.L., "Plastic flow buckling of cylindrical shells due to impulsive loading", J.Appl.Mech., 92, (1970).

[4] JONES, N., AHN, C.S., "Dynamic buckling of complete rigid-plastic spherical shells", J.Appl. Mech., 43, 3, (1974).

[5] JONES, N., AHN, C.S., "Dynamic elastic and plastic buckling of complete

spherical shells", Int.J.Solids Structures, 10, 1957-1374, (1974).

[6] PERRONE, N., "On a simplified method for solving impulsively loaded structures of rate-sensitive materials", J.Appl.Mech, 32, (1965).

[7] FLORENCE, A.L., "Dynamic buckling of viscoplastic cylindrical shells", Inelastic behavior of solids, ed. by Kanninen, Adler, Rosenfield and Jaffee, McGraw-Hill, (1970).

[8] FLORENCE, A.L., ABRAHAMSON, G.R., "Critical velocity for collapse of viscoplastic cylindrical shell without buckling", J.Appl. Mech., 44, 89-94, (1977).

[9] WOJEWÓDZKI, W., "Buckling of viscoplastic cylindrical shells subjected to radial impulse and temperature", Trans.of the 2nd Inter.Conf. SMiRT,L 6/7, (1973).

[10] WOJEWÓDZKI, W., "Dynamic symmetric buckling of viscoplastic shells of revolution", in polish, Eng. Trans., 26, 2, (1978).

[11] WOJEWÓDZKI, W., LEWIŃSKI, P., "Viscoplastic axisymmetrical buckling of spherical shell subjected to radial pressure impulse", Trans. of the 5th Inter.Conf. SMiRT, L8/4,(1979).

[12] SANDERS, J.L., "Nonlinear theories for thin shells" Quart.Appl.Math.,XXI, 1,(1963).

[13] PERZYNA, P., "The constitutive equations for rate sensitive plastic materials", Quart.Appl.Math., 20, (1963).

[14] PERZYNA, P., WIERZBICKI, T., "Temperature dependent and strain rate sensitive plastic materials", Arch.Mech., 16, 1, (1964).

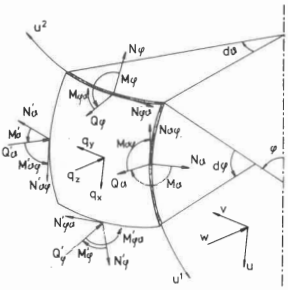

Fig.1 Shell element - notation

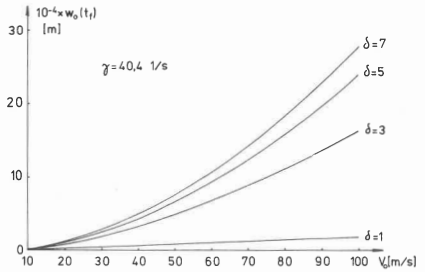

Fig.2 Unperturbed final displacement w_o vs. impulse V_o for a few values of material coefficient δ and $\Theta = 288$ K

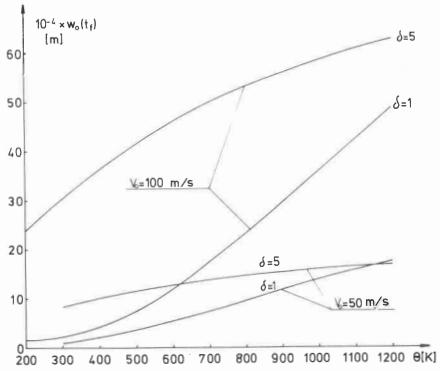

Fig.3 Unperturbed final displacement w_o vs. temperature for $\delta = 1; 5$ and $V_o = 50; 100$ m/s

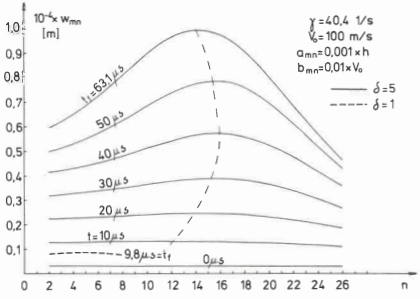

Fig.4 Amplitudes of perturbed displacement w_{mn} vs. the mode numbers n and time for $\delta = 1; 5$ and small value of $\gamma = 40,4$ 1/s ($\Theta = 288$ K)

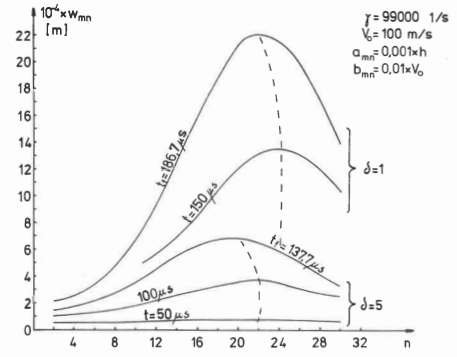

Fig.5 Amplitudes of perturbed displacement w_{mn} vs.the mode numbers n and time for $\delta = 1;5$, $\sigma_o = 206{,}9$ MPa and large value of $\gamma = 99000$ 1/s

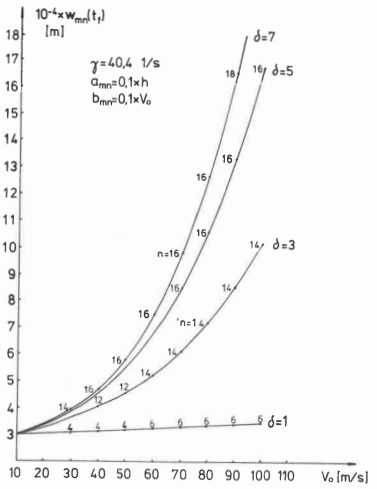

Fig.6 Maximum amplitudes of perturbed displacement w_{mn} and the mode numbers n vs. applied impulse V_o for a few values of δ and $\theta = 288$ K

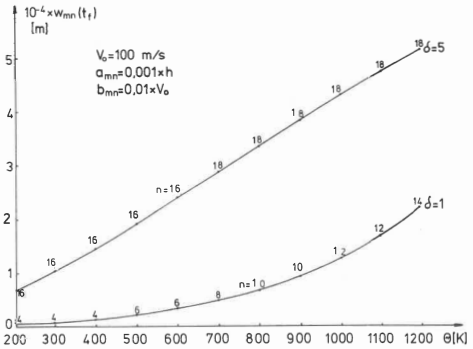

Fig.7 Amplitudes of perturbed final displacement w_{mn} vs. temperature for $\delta = 1;5$ and $V_o = 100$ m/s

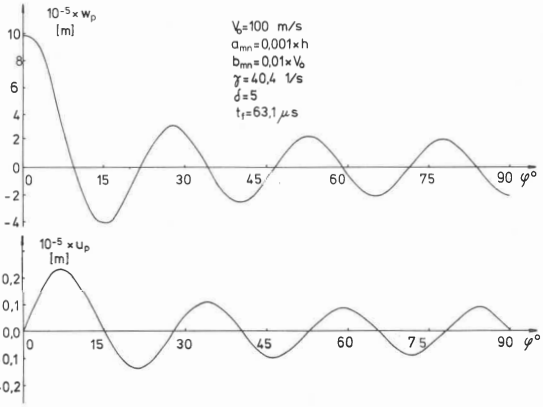

Fig.8 Perturbed final displacements w_p and u_p for $\delta = 5$ and $\gamma = 40{,}4$ 1/s ($\theta = 288$ K)