Optimal Nozzle Design to Minimize Stress Concentrations

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Summary

The problem of minimizing stress concentration factors occurring in the region of a nozzle-vessel transition is considered. The geometrical shape of the transition is defined by a set of design variables and in this treatment balanced design will be studied.

A sequential modelling technique is formed which describes the maximum stress concentration in the body as a function of the design variables. The stress gradients required to form the system model are calculated numerically by the finite element method and the system is optimized by using a penalty function approach combined with a variable metric unconstrained minimization technique.

Applications of the method are considered for a nozzle under pressure loading, nozzle thrust and thermal conditions and plots of initial and final optimized shapes are given showing the change in stress concentration factor.

Introduction

The optimal shapes of components such as pressure vessel nozzle intersections is frequently concerned with minimizing stress concentration effects thereby increasing the fatigue life of the component.

In this paper the problem presented is the optimal design of the reinforcement required at the nozzle to head connection of the component shown in fig. 1. The geometry of reinforcement is defined by a set of design variables $\bar{x}_n$ and the objective is to determine the vector $\bar{x}_n^{OPT}$ which gives the minimum-maximum value of the elastic stress concentration factor under a given set of loading conditions.

Although much information is available on predicting values for stress concentration factors[1,2] it is only recently that the problem of shape optimization to minimize stress concentration effects has been closely studied[3,4,5], and reasonable solutions obtained. In these studies various methods have been employed with the most successful attempts being founded on the classical optimization approach of minimizing some function Z which is approximated by some numerical technique.

Here the penalty function approach of sequential unconstrained minimization[6] (SUMT) will be used for optimization while the stress gradients required to form the objective function will be calculated numerically by the finite element method[7] in which an eight noded isoparametric formulation is employed. This procedure is of a general nature since the SUMT algorithm can effectively deal with nonlinear optimization problems such as considered herein and furthermore the finite element technique is well suited to the requirement of analysing shapes of arbitrary geometry.

The optimization requirement that the maximum stress concentration factor is minimized implies that the usual concept of weight or cost optimization is precluded. In this application geometric and technological constraints on such dimensions as the vessel and nozzle diameters and thicknesses are such that minimum weight considerations do not apply. Here the area to be reinforced is constrained within prescribed limits thus the amount of reinforcement is effectively controlled and will contribute little to the total amount of material used in the complete fabrication of the vessel.

Stress Concentrations: Calculation and Accuracy

Component shapes such as nozzle intersections give rise to high local stresses and it is important that the magnitude and position of these stress peaks are calculated. This information allows the engineer to predict when initial yielding takes place and also since high stress intensities can cause the onset of fatigue failure some estimate of fatigue life can be made.

In order to predict when yielding will first occur some failure theory must be postulated and in this study the maximum shear stress or Tresca theory will be adopted. This may be stated as, if σ_1, σ_2 and σ_3 are the three principal stresses at a point with $\sigma_1 > \sigma_2 > \sigma_3$ then the maximum shear stress is equal to half the difference of the maximum and minimum principal stresses and is defined as

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \qquad (1)$$

This failure theory has been shown to be in good agreement with experimental results on ductile material under steady state conditions and furthermore Morrison et al[8] have shown

that the fatigue life of pressurised components can also be predicted by the maximum shear stress theory.

In order to accurately determine the stress field at the nozzle junction the method of analysis employed must be capable of dealing with arbitrary component shapes together with prescribed mechanical and thermal loading conditions. To meet these requirements the finite element technique will be used together with an isoparametric parabolic type element. Test results using an eight noded element with a 2x2 Gauss point rule have shown agreement within some 2% of photoelastic studies and this order of accuracy is acceptable to form the basis of the optimization algorithm presented.

Optimization

The failure criterion discussed in the previous section has been accepted by the ASME, BS and other pressure vessel committees as a basis for design. Therefore the objective function will be formulated in terms of the maximum shear stress theory and then minimized to produce an optimal shape. Thus the objective function can be defined in mathematical form as

$$Z(\bar{x}_n) = \min_{\bar{x}_n} \left| \max_{\Omega} \left(\frac{\tau}{P} \right) \right| \qquad (2)$$

where $\left(\frac{\tau}{P}\right)$ is the maximum stress concentration factor occurring at some point within the domain of the body Ω and the terms $\bar{x}_n$, n=1, N are the design variables describing the boundary of the component, N being the number of variables.

In equation 2, the term P is a normalised or standard stress with respect to which the stress concentration factor is defined. The definition given above is however unsuitable in its present form for optimization based on a discrete element formulation and therefore modification is required such that only a finite number of stress sampling points are considered. Thus the reformulation of equation 2 may be written as

$$Z(\bar{x}_n) = \min_{\bar{x}_n} \left| \max_{i} \left(\frac{\tau_i}{P} \right) \right| \qquad (3)$$

where i are the stress sampling points.

The correct choice of the design variables is a critical step in the process of shape optimization. It has been shown[9] that balanced reinforcement gives a minimum stress concentration factor therefore the nozzle geometry defined in fig. 1 has been specified such that the design variables will be effective in placing material inside and outside the vessel thus tending to produce a balanced design. Thus the design variables are chosen as

$$\bar{x}_n = \begin{pmatrix} R_1 \\ \theta \\ P \\ S \\ R_2 \end{pmatrix} \qquad (4)$$

since these control the reinforcement at the nozzle junction.

Here it should be noted that the dimensions R_3 and W of fig. 1 will depend on the choice of the design vector $\bar{x}_n$, thus

$$R_3 = R_1 - \left| \frac{(D - (D-R_1)x \cos\beta - P}{SIN\left(\frac{\pi}{2} - \theta - \beta\right)} \right| \qquad (5)$$

$$W = \{D - |D - P - (D-R_2)\cos\beta| - P\} \tan\beta$$
$$- |D - (D-R_1)\cos\beta - P| \tan\gamma - \frac{d}{2} - R_3 \qquad (6)$$

Equations (5) and (6) ensure that R_1, R_3 and D are tangential and therefore prevent distortion of the nozzle protrusion into the vessel.

Objective Function Formulation

The configuration described in fig. 1 consists of an axisymmetric solid of revolution subject to axisymmetric loading and as mentioned previously analysis consists of subdividing the body into elements as shown in fig. 2. Here it should be noted that the mesh consists of regions A-E as shown in figure 2 and in each of the regions nodal point densities can be selected to control mesh fineness. This allows the subregions at the boundary to be controlled such that mesh densities are higher in the regions of greatest stress and furthermore a finer mesh can be specified near the optimal solution thus allowing greater accuracy of analysis.

With such a problem as presented herein the effort involved at the analysis stage will invariably predominate over that required to optimize the objective function, defined by equation 3. Therefore it becomes mandatory to reduce this numerical work as much as possible and here the method of sequential mathematical models will be applied[10] to control the accuracy of the system to be optimized.

This sequential modelling consists of placing restrictions on the design vector $\bar{x}_n$ such that the step length taken between successive design steps is limited between upper and lower values $\Delta\bar{x}_n^U$ and $\Delta\bar{x}_n^L$ respectively. These limits are then simply incorporated as additional side constraints on the design space and can be written as

$$P(\bar{x}_n) = \bar{x}_n^j - |\Delta\bar{x}_n^L| \leq \bar{x}_n^{j+1} \leq \bar{x}_n^j + |\Delta\bar{x}_n^j|, \quad n=1,N \qquad (7)$$

The objective function is then formed initially from a linear approximation

$$Z(\bar{x}_n) = \sum_{n=1}^{N} \{(a_n x_n) + a_{N+1}\} \qquad (8)$$

which is subsequently extended to a second order expression by a fitting procedure[4] which makes use of the design derivatives at the previous design point $\bar{x}_n^{j-1}$. The updated function can then be written as

$$Z(\bar{x}_n) = \sum_{n=1}^{N} (a_n x_n^2) + \sum_{n=1}^{N} \{(a_{n+N} x_n) + a_{2N+1}\} \qquad (9)$$

In equations 8 and 9 the unknown stress gradients a_n, n=1, N are determined numerically via a sensitivity analysis thus

$$a_n = \frac{\partial Z(\bar{x}_n)}{\partial \bar{x}_n}, \quad n=1, N \qquad (10)$$

Constraints

The optimization criterion defined by equation 3 requires that the maximum stress concentration factor occurring in the component under a given set of load conditions is brought to a minimum value. This implies that no behavicural constraints are required to control the stress field therefore only side constraints need be considered. These side constraints are applied directly to the design variables and hence control the geometry of the component.

In the application considered herein three types of side constraint will be considered and these are described as follows.

1) Constraints required to control the geometry of the component and thus prevent highly distorted shapes which may be unacceptable. These constraints, $\ell(\bar{x}_n)$ are based on the expressions given by equations 5 and 6 where both $R_3 \geq 0$ and $W \geq 0$ must be satisfied to guarantee structural compatibility.

2) Constraints which may be required to restrict the design variables within fixed limits which may be a mandatory design requirement. These constraints are termed technological constraints and can be written as

$$h(\bar{x}_n) = \bar{x}_{MIN} \leq \bar{x}_n \leq \bar{x}_{MAX} \tag{11}$$

3) These constraints are described by equation 7 and as such should have no affect on the optimum shape produced but are required to control the accuracy of the objective function.

Optimization

The optimization problem may now be written fully as

$$\underset{\bar{x}_n}{\text{Min}} \left| \max_i \left(\frac{\tau_1}{P} \right) \right|, \quad \text{s.t.} \quad \ell(\bar{x}_n) \geq 0, \quad h(\bar{x}_n) \geq 0, \quad P(\bar{x}_n) \geq 0 \tag{12}$$

under prescribed loads $\{q\}_i$, i=1, R.

As mentioned previously the technique to be used for the optimization of equations 12 will be that of the penalty function technique[6]. This penalty function approach involves transforming the system of equations given above into an unconstrained form by adding the applied constraints to the objective function through a penalty parameter r_k, thus we may write,

$$\text{Min } \phi(\bar{x}_n, r_k) = Z(\bar{x}_n) - r_k \sum_{m=1}^{m} G(g_m(\bar{x}_n) \tag{13}$$

where m is the number of applied constraints. Here the left hand side of the equation is the penalty term and controls the constraints while the function $(\bar{x}_n, r_k)$ is the penalty function and is minimized as an unconstrained function of $(\bar{x}_n, r_k)$. As the term r_k is reduced sequentially the sequence of minima obtained converged to the constrained minimum of the problem as $r_k \to 0$ and $k \to \infty$.

Numerical Examples

Examples of the design algorithm are shown in fugres 3 and 4. Different loading conditions were applied in each case and the component was discretized into sixty elements with the mesh being similar to that shown in figure 2. A two point Gauss rule was used for numerical integration with the material properties being Poisson's ratio $\nu = 0.3$, Elastic modulus $E = 30 \times 10^6$ and the coefficient of thermal expansion $\alpha = 12 \times 10^{-6}/^{o}C$.

Optimal shapes were generally produced after seven to eight design steps and in all cases studied the maximum stress concentration factor was reduced. In figure 3 the component is subject to internal pressure loading and nozzle thrust. The plot shows that the maximum stress concentration factor occurs on the outside surface in the region of the radius R_3 and after shape optimization substantial reductions in stress are obtained. Figure 4 shows a vessel subject to internal pressure loading and an internal start up temperature of $600^{o}C$. In this example the stress concentration factor peaks on the inside surface of the thickened area of the nozzle due to the combined effect of the applied loads. Again reductions in the maximum stress concentration factor are produced but these are not so pronounced as the previous example due to the type of loading conditions applied to the structure.

Conclusions

The optimal shape design of nozzle reinforcement has been successfully treated by applying an algorithm based on classical optimization with the objective function being modelled numerically by the finite element method.

From the output, as shown in figures 3 and 4, the design engineer is supplied with valuable information on how the stress field behaves within the component due to a change in design variables.

Finally it is concluded that due to the flexibility of the method arbitrary geometries and loading conditions can be easily dealt with allowing the procedure to be extended to a whole range of engineering applications.

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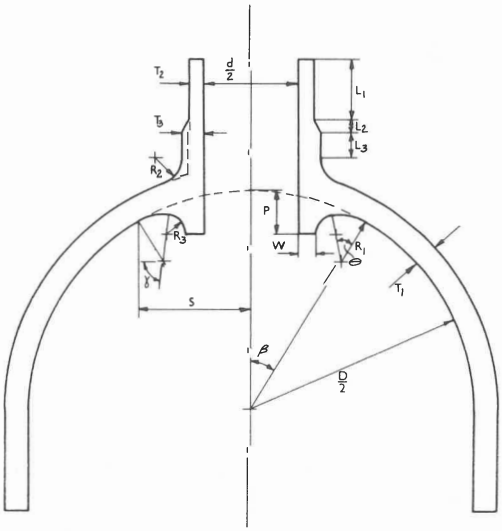

Fig. 1 NOZZLE GEOMETRY AND DESIGN VARIABLES

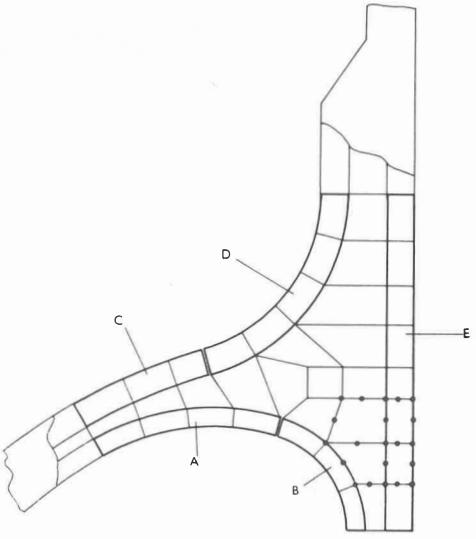

Fig. 2 TYPICAL FINITE ELEMENT MESH FOR NOZZLE INTERSECTION, REGIONS A-E ALLOW CONTROL OF MESH DENSITIES

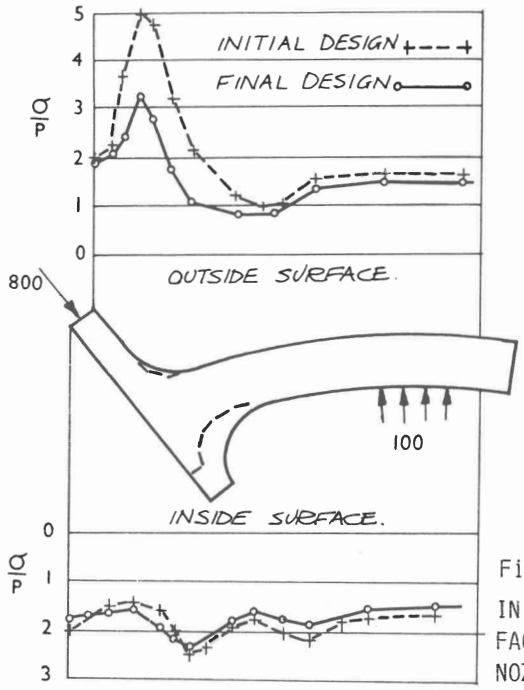

Fig. 3

INITIAL AND FINAL STRESS CONCENTRATION FACTORS FOR INTERNAL PRESSURE AND COMBINED NOZZLE THRUST

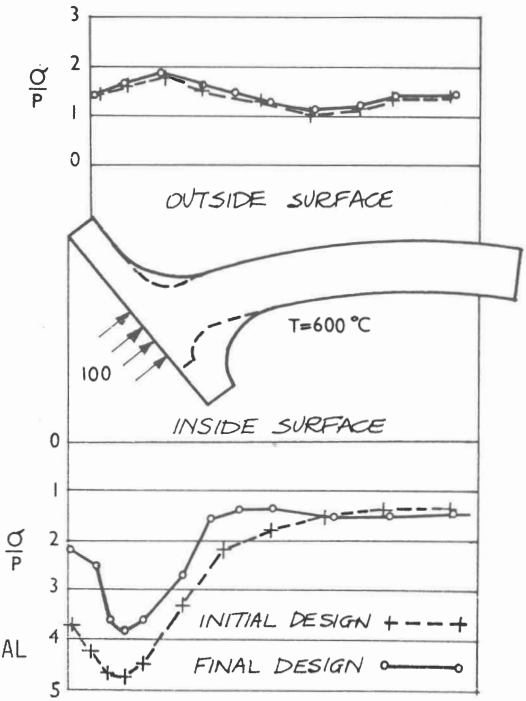

Fig. 4

INITIAL AND FINAL STRESS CONCENTRATION FACTORS FOR INTERNAL PRESSURE AND INTERNAL START UP TEMPERATURE OF 600°C