Numerical modelling of fracture of concrete

P.E.Roelfstra & F.H.Wittmann
Swiss Federal Institute of Technology, Lausanne

1 INTRODUCTION

Soon it has been realized that the application of linear elastic fracture mechanics to concrete must be considered to be a rough approximation of reality. With the aim to overcome this difficulty more complex concepts have been developed. Hillerborg and his co-workers developed the fictitious crack model [1978,1983] and Bazant and Oh used the same basic idea and developed the blunt crack model [1983]. Both concepts are primarely aimed for application in computerized structural analysis.

In the meantime a RILEM recommendation on the determination of fracture energy has been published [1985]. Fracture energy needs to be known in both just mentioned concepts. Later it turned out that fracture energy is necessary but not sufficient for the application of either concept. It has been shown that the tensile strain softening diagram has a major influence on the way of failure of concrete [Sadouki 1987, Roelfstra & Wittmann 1986, Rots 1986].

It is very difficult to determine strain softening directly. Therefore a module called "SOFTFIT" has been developed [Roelfstra 1986]. With this program an optimal bilinear stress-crack width relation can be obtained with which the observed load-displacement diagram is correctly described. The program can be used to analyze experimental results obtained with notched beams, slabs, wedge splitting cubes, CT-specimens, and ring-specimens. The principle of the program and its application to notched beams has been published earlier [Wittmann & al. 1987].

In analyzing specimens of different geometry and differing size a geometry effect has been observed. It is the aim of this contribution to show that it is possible to simulate the fracture process zone numerically and to explain in this way the significance of the fracture process zone for the size effect.

2. FRACTURE ENERGY AND STRAIN SOFTENING OF CONCRETE AS DETERMINED BY MEANS OF CT-SPECIMEN

In an earlier paper we have described experiments and the numerical procedure to determine fracture energy and strain softening of concrete [Wittmann & al. 1987]. Experiments were carried out on notched concrete beams under three point bending according to a recent RILEM-recommendation [1985].

This recommendation is based on the fictitious crack concept as developed by Hillerborg and his co-workers [1978,1983]. In the meantime similar tests have been carried out to determine fracture energy and strain softening by means of a wedge splitting test [Wittmann & al. 1987, Brühwiler & al. 1987]. In this contribution another test series will be briefly described with which fracture energy and strain softening have been determined using CT-specimens.

Three different sizes of test specimens have been prepared :

- Small specimen (S)
- Medium size specimen (M)
- Large specimen (L)

The length of the fractured ligament was a_S = 150 mm, a_M = 300 mm, and a_L = 600 mm respectively.

Mean curves for three different specimen sizes are shown in Fig. 6 by a solid line with dots. Modules "FRACTURE I" and "SOFTFIT" have been modified as to allow the evaluation of experiments with CT-specimens. In Fig. 2 the strain softening obtained in this way is plotted. At the same time all parameters are indicated in the Table in Fig. 2. In the last line of the table fracture energy G_F is given. The calculated load-crack opening displacement-diagrams using the fitted parameter are in addition shown by a simple solid line in Fig. 1. Good agreement between experimental data and theoretical predictions can be observed.

From the Table in Fig. 2 it can be seen that fracture energy increases with the length of the ligament up to 30 cm, then G_F seems to remain constant. It will be tried to explain this experimentally observed effect with an appropriate numerical model.

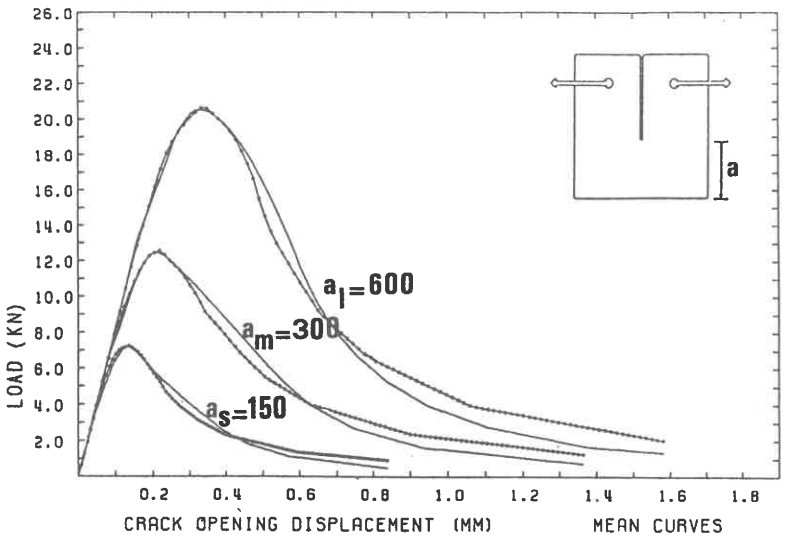
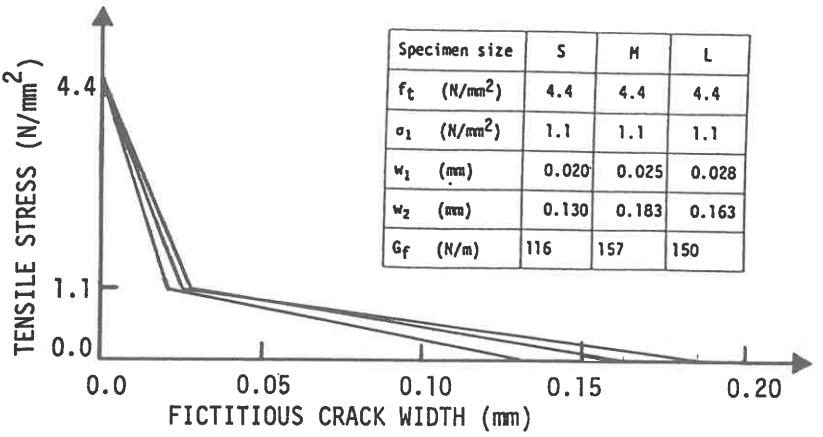


Figure 1 Measured load-crack opening displacement-curves (solid lines with dots) for three specimen sizes and corresponding calculated curves (solid lines).



Specimen size	S	M	L
f_t (N/mm²)	4.4	4.4	4.4
σ_1 (N/mm²)	1.1	1.1	1.1
w_1 (mm)	0.020	0.025	0.028
w_2 (mm)	0.130	0.183	0.163
G_f (N/m)	116	157	150

Figure 2 Strain softening as determined from experimental data shown in Fig. 1. The exact values obtained are indicated in the Table.

3. SOME CONSIDERATIONS CONCERNING THE SIZE OF THE FRACTURE PROCESS ZONE AND THE GEOMETRY EFFECT

The fictitious crack model is based on the idea that the material ahead of a crack tip is gradually damaged until a crack can propagate. After the tensile strength is reached in elements ahead of the crack tip the fictitious crack opens, but stress transfer is still possible until a critical width (w_2 in Fig. 2) is reached. At this stage real crack growth is taking place. In its original version the fictitious crack model is based on the assumption that all fracture energy G_F is consumed in the fracture process zone of the fictitious crack. This tacitly implies that all other parts of a loaded specimen behave ideally elastic. This is not totally true of course. In a loaded specimen there may be parts which are stressed high enough so that some damage is occuring in the structure of the material. This damage energy G_d is not directly linked with the formation of a macroscopic crack and certainly it will depend on the size and the geometry of a specimen. The size of the fracture process zone has been observed experimentally by Cedolin [1987] and Cedolin & al. [1983,1987].

This effect becomes most obvious in a direct tension test. In several areas localized fracture process zones can develop before one fracture process zone becomes dominant and propagates thus causing failure. It is evident that the heterogeneity of the composite structure of concrete has a major influence on the development of localized fracture process zones and as a consequence on the damage energy G_D which is not directly linked with crack formation.

Without having here the possibility of going into further details on this point we can conclude that in a real specimen the total fracture energy has to be subdivided in a part which is really consumed for crack formation G_F and a part which is consumed by damage occuring in highly stressed zones of a specimen G_D. The first componant G_F should be a real material parameter which is independent on size and geometry of a given specimen, while the second component G_D depends on the stress gradient and thus on size and geometry of a specimen.

4. BASIC CONCEPT OF THE NUMERICAL MODEL OF DAMAGE ENERGY AND FRACTURE ENERGY

In a first step energy consumption other than the part used for the widening of the crack width has to be introduced. This is done in assuming a nonlinear stress strain diagram. This is shown in Fig. 3. The area under the polygon in the left part of Fig. 3 is supposed to be the damage energy G_D. All parts in a specimen which are loaded higher than the data point in the bilinear stress strain diagram (f.e. σ = 3.0 N/mm^2 in Fig. 3) are progressively damaged independent on the fact if a crack is created or not. A similar approach has been suggested by Nallathambi & Karihalou [1986] and Cedolin & al. [1987]. Only in those zones where the tensile strength is reached (f.e. σ = 4.0 N/mm^2 in Fig. 3) real fracture energy G_F is consumed to widen the fracture process zone until final failure.

In the finite element program the heterogeneity of the composite structure and the local variability of the material have to be taken into consideration by introducing a statistical distribution of the properties.

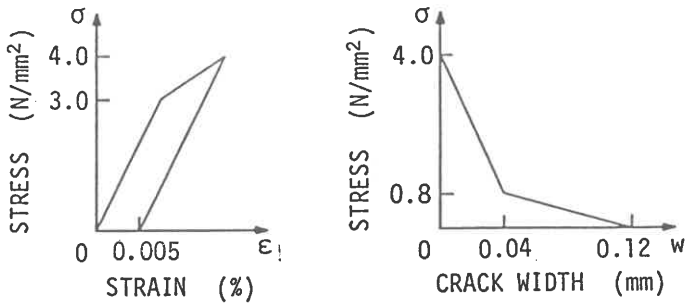


Figure 3 The principles for division of the deformation properties into σ-ε-diagram and σ-w-diagram.

5. AN EXAMPLE

The above mentioned model will now be used to calculate the failure of a CT-specimen. The finite element idealization of the element is shown in Fig. 4.

Four different stages of a CT-specimen under increasing load are shown in Fig. 5. The left column indicates the deformation of the inner part of the specimen. It also shows the progressing of the real crack length. The second column shows the energy which is dissipated in the elements ahead of the crack tip. Circles are drawn in the centre of each element in which energy is dissipated. In the right column finally the total crack length i.e. real plus fictitious crack length is shown.

It can be clearly seen that (a) energy is consumed in the fracture process zone and (b) the fictitious crack length a_f increases considerably before (c) a real crack starts to grow. The real crack length is called a_r. These processes are the major reasons why linear elastic fracture mechanics is not appropriate to describe failure of concrete. It can also be seen that the shape of the fracture process zone changes as the crack advances towards the upper bound.

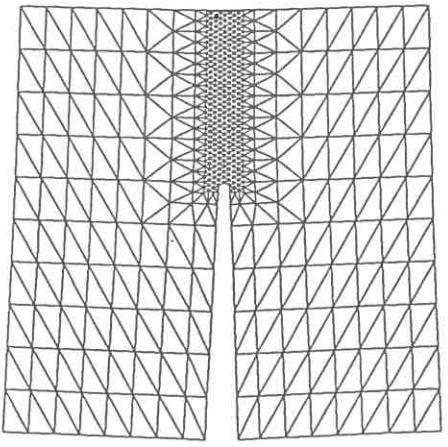


Figure 4 Finite element idealization of a CT-specimen used in this analysis.

Calculations have been carried out for three different sizes and the length of the ligament was chosen to be 150 mm, 450 mm, and 750 mm. The obtained load displacement diagram for the three CT-specimens are shown in Fig. 6a. In the same figure the length of the real crack (stress-free crack) as function of the displacement of the loading point is indicated. As can be seen, the real crack starts to run only after the peak load has been reached. This has also been observed experimentally.

In Fig. 6b the cummulative fracture energy and damage energy as function of the length of the real crack are shown. In addition a straight line which represents the fracture energy G_F as function of the real crack is drawn in this figure.

6. DISCUSSION OF RESULTS

In Fig. 6 it is shown that $G_F \cdot a_r$ as well as $G_F a_r + G_D$ are linearly dependent of the length of the ligament (maximum real crack length). The fracture process zone is defined as the volume in which the break point of the stress-strain diagram shown in Fig. 3 has been exceeded. If these products are linear relation it means that the total fracture energy is independent on the ligament length. In the Table shown in Fig. 2 it is indicated, however, that fracture energy increases initially with ligament length and reaches finally a constant value. This observation is further justified by measurements on different types of specimens such as bending beams and wedge splitting cubes [Rokugo & al. 1987]. The numerical examples shown in Fig. 6 have all a ligament length which is long enough that the total fracture process zone can be developed.

If we consider Fig. 5 (a) to (d) we see that the fracture process zone increases steadily before a real crack starts to propagate. Obviously less energy is needed to split a specimen which is small enough so that the total fracture process zone cannot develop fully. In modifying the stress-strain diagram shown in Fig. 3 the simulated fracture process zone can be adjusted to different types of concrete and other composite materials.

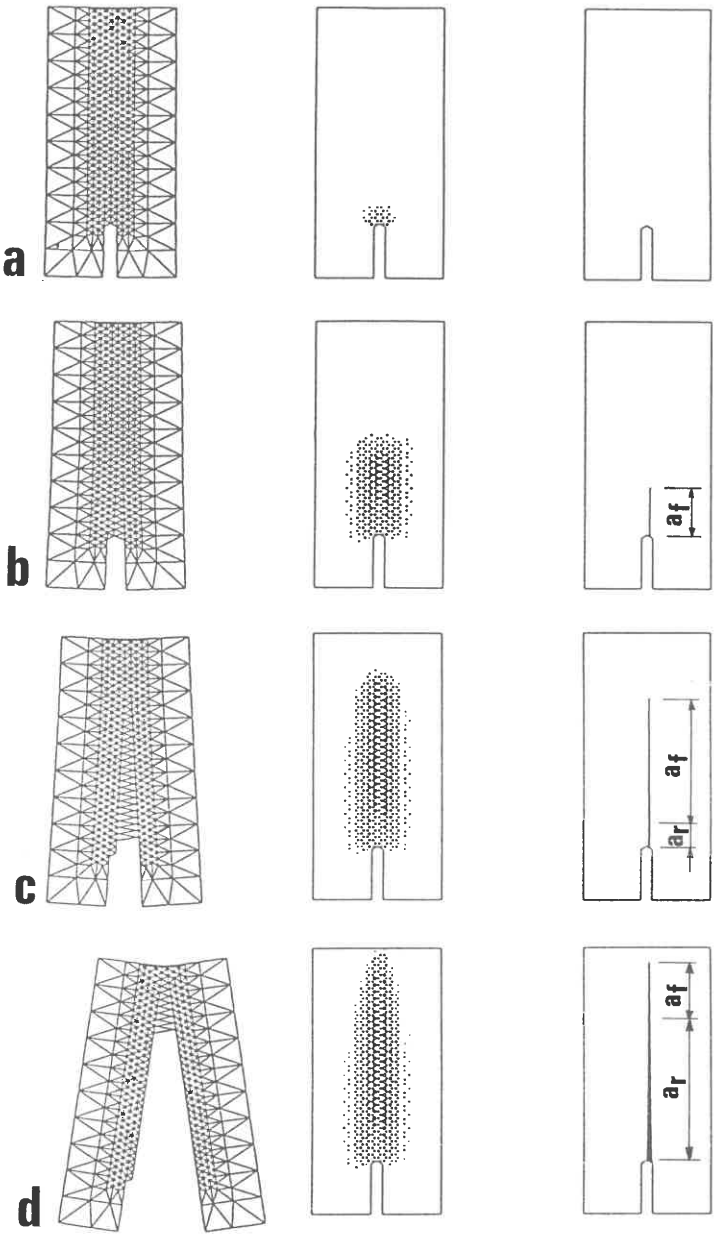


Figure 5 Four different stages of the numerically simulated failure process of a CT-specimen.

The width of the fracture process zone is dependent on the geometry of the specimen and the type of loading and thus on the stress distribution ahead of a crack. As an extreme case we can consider a direct tension bar. In this case there exists no stress concentrations if we consider the material to be homogeneous.

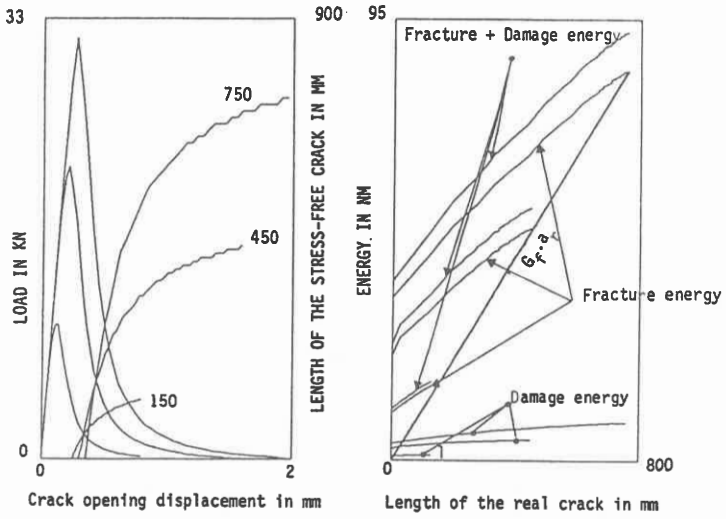


Figure 6 (a) Calculated load-displacement diagrams and length of the real cracks as function of the displacement of the loading point of three CT-specimens with different size.
(b) Cummulative fracture and damage energies G_F and G_D respectively as function of the length of the real crack of three CT-specimens.

In principle the fracture process zone is then smeared out over the total volume. In a real material, however, and even more so in a composite material, material properties vary considerably locally [Wittmann & Roelfstra 1987]. As a consequence several independent fracture process zones may develop, each consuming energy. Finally some of these fracture process zones may grow and coalesce thus causing failure of the element. This means in essence that the total fracture energy in a direct tension bar depends both on the complex state of stress in the composite material and on the local distribution of material properties.

In a specimen with an arbitrarely chosen geometry material properties (modulus of elasticity, tensile strength, strain softening) may be generated numerically in such a way that they vary statistically in space. If the distribution functions are chosen in a realistic way the size effect on fracture energy can be predicted. In a more complex approach the structure of concrete can be simulated as is done by numerical concrete [Sadouki 1987, Roelfstra & al. 1985, Wittmann & Roelfstra 1987]. The size effect is determined by the interaction of the complex state of stress in a given specimen (taking the composite structure of the material into consideration if necessary) and the locally varying material properties.

7. CONCLUSIONS

- It is possible to determine fracture energy and tensile strain softening of concrete with a displacement controlled test on CT-specimens.

- The experimentally observed behaviour can be reasonably simulated numerically by means of the fictitious crack model after parameters have been determined by a computer module called SOFTFIT.
- The experimental findings show a marked size effect. This is in agreement with findings of other authors.
- A numerical model has been developed to distinguish between real fracture energy used for crack formation and damage energy dissipated in highly stressed zones.
- The numerical model explains the influence of the state of stress and the local variability of a material on fracture energy and this explains the size effect.
- The model can be further elaborated to take the heterogeneous structure of concrete into consideration.

REFERENCES

Hillerborg, A. 1978. A Model for Fracture Analysis. Report TVBM-3005, Division of Building Materials, Lund Institute of Technology, Lund, Sweden.

Hillerborg, A. 1983. Analysis of One Single Crack. In : Fracture Mechanics of Concrete, edited by F.H. Wittmann, Elsevier Science Publishers, Amsterdam, p. 223-249.

Bazant, Z.P. and B.H. Oh. 1983. Crack Band Theory for Fracture of Concrete. Materials and Structures 16:155-177.

RILEM Draft Recommendation (50-FMC). 1985. Determination of the Fracture Energy of Mortar and Concrete by means of Three-Point Bending Tests on Notched Beams. Material and Structures 18:285-290.

Sadouki, H. 1987. Simulation et analyse numérique du comportement mécanique de structures composites. Ph.D. Thesis, Swiss Federal Institute of Technology, Lausanne.

Roelfstra, P.E. & F.H. Wittmann. 1986. Numerical Method to link Strain Softening with Failure of Concrete. In : Fracture Toughness and Fracture Energy of Concrete, edited by F.H. Wittmann, Elsevier Science Publishers, Amsterdam, p. 163-175.

Rots, J.G. 1986. Strain-Softening Analysis of Concrete Fracture Specimens. (See Roelfstra, P.E. & F.H. Wittmann) p. 137-148.

Carpenteri, A., A. Di Tommaso & M. Fanelli. 1986. Influence of Material Parameters and Geometry on Cohesive Crack Propagation. (See Roelfstra, P.E. & F.H. Wittmann) p. 117-135.

Roelfstra, P.E. 1986. Manual of Program SOFTFIT, Theory and Application. Interna. Report, Laboratory for Building Materials, Swiss Federal Institute of Technology, Lausanne.

Wittmann, F.H., Yiun-Yuan Huang, P.E. Roelfstra, H. Mihashi, N. Nomura & Xin-Hua Zhang. 1987. Influence of Age of Loading, Water-Cement Ratio and Rate of Loading on Fracture Energy of Concrete. (Accepted for publication in Materials and Structures).

Wittmann, F.H., E. Brühwiler, E. Rokugo & H. Mihashi. 1987. Constitutive Laws based on the Concept of Fracture Energy to predict the Behaviour of Concrete under Seismic Conditions. Proceedings Int. Conf. on Fracture and Fracture Mechanics, Shanghai.

Brühwiler, E., K. Rokugo & F.H. Wittmann. 1987. Influence of Rate of Loading on Fracture Energy and Strain Softening of Concrete. Contribution to this volume.

Rokugo, K., E. Brühwiler, H. Mihashi & F.H. Wittmann. 1987. Methods to determine Fracture Energy and Strain Softening of Concrete. Contribution to this volume.
Wittmann, F.H. & P.E. Roelfstra. 1987. Numerical Concrete Applied to Predict Constitutive Relations of Composite Materials. (See this volume elsewhere).
Nallathambi, P. & B.L. Karihaloo. 1986. Prediction of Load-Deflection Behaviour of Plain Concrete from Fracture Energy. Cement and Concrete Research 16, 3:373-382.
Roelfstra, P.E. 1987. Numerical Analysis and Simulation of Crack Formation in Composite Materials such as Concrete. In : Fracture of Non-Metallic Materials, edited by K.P. Herrmann and L.H. Larsson, ECSC, EEC, EAEC, Brussels and Luxembourg, p. 359-384.
Roelfstra, P.E., H. Sadouki & F.H. Wittmann. 1985. Le bèton numérique. Matériaux et Constructions, Vol. 18, No 107, p. 309-317.
Cedolin, L. 1987. Fracture Mechanics Parameters and Fracture Process Zone of Concrete. (See Roelfstra 1987), p. 341-358.
Cedolin, L., S. Dei Poli & I. Iori. 1983. Experimental Determination of the Fracture Process Zone in Concrete. Cement and Concrete Research, Vol. 13, p. 557-567.
Cedolin, L. S. Dei Poli & I. Iori. 1987. Tensile behaviour of concrete. To be published, J. of Eng. Mech. Dev., ASCE.