ABSTRACT

MOHEB ALIZADEH GASHTI, HADI. Efficient Design of Sustainable Supply Chain Networks: Development and Extensions. (Under the direction of Dr. Robert B. Handfied.)

In recent years, consumers and legislation have pressured companies to design and plan their supply chain networks toward diminishing negative environmental and social impacts, and consequently becoming more sustainable. On the other hand, due to the shortage of valuable resources utilized through a logistics network, it is of great importance to design a supply chain network in which available scarce resources are used as efficiently as possible by located facilities. To address efficiency issue in a sustainable supply chain network, we develop in this dissertation three programming models to design a sustainable supply chain network, in which not only all sustainability pillars, i.e. economic, environmental and social responsibility, are considered, but also efficiency of facilities is dealt with.

Particularly, we first develop an integrated multi-objective multi-period mixed integer nonlinear programming model to design a sustainable closed loop supply chain network, where the efficiency concept is formulated using a bi-objective output-oriented data envelopment analysis (DEA) model. Since the developed mixed integer nonlinear programming model is non-convex because of the existence of bilinear terms, we present a solution procedure whose first step is to linearize bilinear terms by piecewise McCormick envelopes. In order to cope with the elevated dimension of the relaxed mixed integer linear programming model, we then enhance the performance of the proposed solution procedure by adding a set of valid inequalities. Finally, we derive an aggregated single objective mixed integer linear model by exploiting a fuzzy multi-objective programming approach. The computational experiment exhibits how to balance sustainability and efficiency in the supply chain network design problem. In particular, it presents that integration of efficiency results in improving economic aspect of sustainability, while there exist trade-offs between efficiency and both environmental and social responsibility aspects.

Afterwards, we extend the above-mentioned approach by taking uncertainty into consideration, where demand is assumed to be stochastic in a supplier selection and order allocation problem. In this regard, we propose a multi-objective mixed integer nonlinear programming model for efficient and sustainable supplier selection and order allocation with stochastic demand. Our approach considers sustainability dimensions and seeks to design the most efficient supply network given constraints of the supply market. Enterprise efficiency is assessed using a bi-objective DEA model whose inputs include raw materials, current expenses and labor force capacity. The resulting model is again non-convex because of the presence of bilinear terms in DEA-related constraints, so we introduce a multi-stage solution procedure that first uses piecewise McCormick envelopes to linearize the bilinear terms. Next, we introduce a set of valid

inequalities in order to improve solution time of the problem whose dimension significantly increases after being linearized. Afterwards, we exploit chance constrained programming to deal with stochastic demand. Finally, a single aggregated objective function is derived using a fuzzy multi-objective programming approach. A manufacturing case study demonstrates the validity of the proposed approach, and its effectiveness in designing a supply network that addresses the triple bottom line of people, profit, and planet that comprises many sustainability initiatives.

Ultimately, we develop a novel non-integrated approach for sustainable and efficient supplier selection and order allocation problem. In particular, our research seeks to develop an inclusive multi-objective mixed integer linear programming model, which accounts for multiple periods, multiple products and multimodal transportation, to evaluate suppliers and allocate order quantities. In addition, the model takes sustainability criteria, as well as both shortage and discount conditions into consideration. A hybrid three-step solution methodology is presented, by first converting the original multi-objective problem to a single objective model using the ε-constraint method. In the second step, the single objective programming model is solved using the Benders decomposition algorithm that in turn is accelerated by a variety of algorithmic enhancements. Finally, among all the Pareto optimal solutions of the original multi-objective programming problem, the preferable solution is intelligently selected based on the DEA super efficiency score of all purchasing firms as a decision support tool. The applicability of the proposed approach is illustrated using an empirical case study from the automotive sector.

To the best of our knowledge, all above-mentioned three approaches with respect to problem formulations and solution procedures are novel and significantly contribute to the current literature as there does not exist any prior study to investigate both sustainability and efficiency when planing a supply chain network. We elucidate the contributions of each developed model and its solution procedure in the following chapters.

Efficient Design of Sustainable Supply Chain Networks: Development and Extensions

by
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DEDICATION

To my beloved parents and sisters
Yadollah, *Sakineh*, *Fatemeh* and *Mahnaz*.

BIOGRAPHY

Hadi Moheb Alizadeh Gashti is a Ph.D. student in Graduate Program in Operations Research at North Carolina State University. He was born in Tehran, Iran. He has received his Bachelor and Masters degree in Industrial Engineering from Azad University (Tehran South Branch) and The Kharazmi University in September 2006 and February 2010, respectively. To further his education, he came to the United States in August 2012 and received a Master of Engineering degree in Industrial and Systems Engineering from North Carolina State University in May 2014 and continued into the PhD program in Operations research at North Carolina State University in August 2014.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

Introduction

Supply chain network design (SCND) as a strategic decision plays the key role in the performance of a supply chain. It includes the determination of location, number and capacity of network facilities as well as the material flow through the network. The configuration of the logistics network cannot be changed in short term as its change is very costly and time-consuming. SCND is also a crucially important decision due to the subsequent tactical and operational decisions.

While for many years, minimizing total cost or maximizing profit was the main objective of supply chains, they are nowadays responsible for the environmental impacts of their products and operations, and the health and safety of their employees and the whole society. In this way, activists, non-governmental organizations and media are demanding the firms to take the responsibility of not only their own actions but also other partners in the supply chains. Moreover, governments have enacted legislations in favor of the environment such as regulations for greenhouse gases reduction in European Union, Australia and Canada. The concern about environmental and social impacts of business activities has led to development of a new paradigm called sustainable development. Sustainable development is defined as "a development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (WCED 1987). While diverse comprehensions of sustainability exist, one central concept helping operationalize sustainability is the triple bottom line approach, where a minimum performance is to be achieved in the economic, environmental and social dimensions (Elkington 1998). Dyllick and Hockerts (2002) framed the three dimensions of sustainability as the business case (economic), the natural case (environmental), and the societal case (social). Despite its simple definition, sustainable development is a complex concept whose principles implementation encounters significant difficulties at both macro and micro-levels. To deploy sustainability into business environment, it is not sufficient to only control the level of sustainability within the boundary of a corporations ownership, rather the level of sustainability

should be assured at the whole supply chain network (Cruz 2009). Accordingly, sustainable supply chain management (SSCM) is defined as the consideration of environmental and social impacts of supply chain operations as well as its economic performance in the management of information, material and capital flow (Seuring and Muller 2008).

To respond the need of sustainability, a number of research works have ever been presented in the context of SCND problem. Nevertheless, the literature on sustainable SCND that models all three aspects of sustainability quantitatively is very scarce (Seuring and Muller 2008, Brandenburg et al. 2014). In this limited pool of quantitative studies, a sustainable supply chain network is designed such that economic, environmental and social responsibility aspects of sustainability are optimized by, usually but not limited to, using linear or mixed integer linear programming. In other words, they focus on the responsiveness feature of a sustainable supply chain, while there is another critical characteristic in supply chain management called *efficiency* that needs to be explored (Chopra and Meindl 2012). The key issue in devising supply chain strategy for a company is to design a supply chain that balances efficiency and responsiveness given its portfolio of products, customer segments, and supply sources (Chopra and Meindl 2012). Considering efficiency in planning a supply chain network is an important issue in the recent years because of encountering limited amount of resources including human, budget, equipment, space, fuel, etc. (Mitropoulos et al.2013). Scarcity of resources is becoming one of the new areas of concern for the economic, industrial and political systems (Matopoulos et al. 2015). Numerous industry and government reports (PwC 2011, European Commission 2011, SCU 2012) have been released recently where growing concerns over the short-term availability of resources and the potential implications for firms are expressed. This highlights the necessity of using available rare resources as efficient as possible through upstream and downstream supply chains of various industries. According to DEFRA (2012), it is estimated that 2 percent of the UK business profits per year may be lost through inefficient use of resources, whereas the UK businesses could save around 23 billion per year by making simple changes to use resources more efficiently and help protect the environment. Consequently, resource efficiency should be regarded as a part of the overall sustainability research agenda. However, surprisingly even in recent literature reviews on SSCM (Seuring and Muller 2008, Hassini et al. 2012, Zimmer et al. 2016), very few references are made to the specific resources used, to the methods and tools applied to assess resources usage and to the overall supply chain configuration and logistics implications of the use of specific resources.

In order to fill the present gap in the current literature of SSCM, we propose in this dissertation a novel approach to design the generic efficient and sustainable supply chain networks in which resources efficiency is defined by data envelopment analysis (DEA). In particular, the explicit research questions addressed in this dissertation are: *i) how should the resources efficiency be modeled using DEA to measure efficiency of facilities in utilizing rare resources through the*

sustainable supply chain network, where all sustainability aspects including economic, environmental and social responsibility are taken into account? and ii) how does the resources efficiency defined by DEA impact the performance of the sustainable supply chain network? In this regard, we develop in the first phase of this dissertation a multi-objective mixed integer nonlinear programming (MINLP) model in multiple periods in which DEA efficiency of located facilities is maximized, alongside optimizing all triple bottom lines of sustainability. The proposed model enables the supply chain executives to balance sustainability and efficiency in the supply chain network and derive the natural trade-off between a sustainability aspect and DEA efficiency of facilities. Due to taking the efficiency into account, the designed sustainable supply chain poses to have facilities located on both upstream and downstream chains that utilize their available scarce resources as efficiently as possible. However, since the developed multi-objective MINLP model is non-convex, we propose a three-step solution procedure in which nonlinear terms are relaxed using piecewise McCormick envelopes (step 1), valid inequalities are then added to the obtained relaxed mixed integer linear programming (MILP) model to accelerate the solution procedure with respect to its solution time (step 2) and finally an aggregated single objective function is derived using fuzzy multi-objective programming (step 3).

In the second phase of this dissertation, we extend the above-mentioned approach in the following two directions:

1) we develop an integrated chance-constrained stochastic model for supplier selection and order allocation problem. In particular, we propose a multi-objective mixed integer nonlinear programming model for efficient and sustainable supplier selection and order allocation with stochastic demand. Our approach considers sustainability dimensions including economic, environmental and social responsibility, but also seeks to design the most efficient supply network given constraints of the supply market. Enterprise efficiency is assessed using a bi-objective data envelopment analysis whose inputs include raw materials, current expenses and labor force capacity. The resulting model is non-convex because of the presence of bilinear terms in DEA-related constraints, so we introduce a multi-stage solution procedure that first uses piecewise McCormick envelopes to linearize the bilinear terms. Next, we introduce a set of valid inequalities in order to improve solution time of the problem whose dimension significantly increases after being linearized. Afterwards, we exploit chance constrained programming to deal with stochastic demand. Finally, a single aggregated objective function is derived using a fuzzy multi-objective programming approach. A manufacturing case study demonstrates the validity of the proposed approach, and its effectiveness in designing a supply network that addresses the triple bottom line of people, profit, and planet that comprises many sustainability initiatives in an efficient manner.

2) contrary to the previous developments, we develop a non-integrated approach to examine supplier selection and order allocation problem. Particularly, we propose an inclusive

multi-objective mixed integer linear programming model, which accounts for multiple periods, multiple products and multimodal transportation, to evaluate suppliers and allocate order quantities. Furthermore, the developed model that includes all sustainability aspects also takes both shortage and discount conditions into account. A hybrid three-step solution methodology is then presented, using which the original multi-objective problem is firstly converted to a single objective model by -constraint method. In the second step, the current single objective programming model is solved using the Benders decomposition algorithm that in turn is accelerated by a variety of algorithmic enhancements. Finally, among all the Pareto optimal solutions of the original multi-objective programming problem, the preferable solution is intelligently selected based on the DEA super efficiency score of all purchasing firms as a decision support tool. The applicability of the proposed approach is illustrated by an empirical study in automotive industry.

The contributions of each of the aforementioned model developments are elaborated in the corresponding chapter, where they are presented.

The rest of this dissertation is organized as follows: in chapter 2, we design the efficient and sustainable closed loop supply chain network and develop its solution procedure. We also review the related literature to the addressed topic and present a computational experiment. In chapter 3, we present the first extension to the problem, where we examine the an integrated efficient and sustainable supplier selection and order allocation problem with stochastic demand parameter and its solution procedure. Likewise the former chapter, we review the related literature and present an empirical study. We develop the second extension in chapter 4, where we study the sustainable supplier selection and order allocation problem whose final solution is chosen based on efficiency of purchasing firms. The developed approach is a non-integrated modeling of the addressed problem whose solution procedure alongside an empirical implementation are also presented in this chapter. Chapter 5 presents the concluding remarks and future works.

Chapter 2

Efficient and Sustainable Closed Loop Supply Chain Network Design

In this chapter, we develop an integrated multi-objective multi-period mixed integer nonlinear programming (MINLP) model to incorporate DEA efficiency in planning a generic sustainable supply chain network, in which all sustainability aspects are considered. To accomplish this target, we first formulate a bi-objective output-oriented DEA model to derive the efficiency score of all DMUs (located facilities in this context) simultaneously. In the current DEA model, physical input, current expense and labor force constitute the input factors, whereas the total amount of outgoing materials/products defines the output factor of each facility in each period. As the original MINLP model is non-convex, we linearize the bilinear terms in DEA efficiency constraints by piecewise McCormick envelopes approach in the first step of the developed solution procedure. In order to cope with the elevated dimension of the obtained relaxed MILP model, we then add a set of valid inequalities in the second step of solution procedure. Finally, we obtain an aggregated single objective MILP model using a weighted sum fuzzy aggregation function. The contributions of the proposed approach in this chapter over all studies in the relevant literature are as follows:

1) A novel and comprehensive multi-objective MINLP model is proposed to plan a sustainable and efficient supply chain network, which includes all sustainability aspects.

2) The efficiency of network facilities is taken into account and modeled by a bi-objective output-oriented DEA model. In DEA model, physical input, current expense and labor force are regarded as the input factors, whereas physical outgoing commodity is defined as the output factor for each facility.

3) A trade-off between facilities efficiency and each sustainability aspect can be derived. Sustainability and efficiency issues can be balanced through the supply chain network. 4) An innovative solution procedure is developed that includes applying piecewise McCormick envelopes

to eliminate non-convexity, adding valid inequalities to improve solution time and implementing fuzzy multi-objective programming to derive a single objective function.

To the best of our knowledge, there does not exist such a holistic integrated approach to efficient and sustainable SCND problem in the current literature.

2.1 Literature Review

In this section, we present a selective overview of the relevant literature on SSCM, in which all sustainability aspects are quantitatively examined. For a detailed review on other papers studying sustainability partially, the interested reader is referred to Hassini et al. (2012), Tang and Zhou (2012), Brandenburg et al. (2014). We also provide a brief review of the existing studies in which DEA is somehow incorporated into a relevant subject to supply chain management.

2.1.1 Quantitative models in SSCM

Holistic approaches of SSCM that reflect all three sustainability dimensions are relatively rare in the academic literature (Seuring and Muller 2008). Given that SSCM can positively influence a firms profitability, performance, and competitive advantage, SSCM research tends to focus primarily on environmental issues, while social facets are widely neglected in empirical and in analytical SSCM modeling research (Seuring and Mller 2008, Tang and Zhou 2012).

Bouzembrak et al. (2013) discussed the design of a multi-modal supply chain network in order to satisfy the demand of the treated sediments and to respect environmental, social and economic requirements by proposing a multi-modal and single period MILP model for SCND problem of waterway sediments of French waterways. Dehghanian and Mansour (2009) designed a sustainable recovery network, in which economic, environmental (derived by lifecycle analysis) and social (calculated by analytical hierarchy process) impacts are balanced. The authors treated the multiple objectives by apply a multi-objective genetic algorithm to find the Pareto-optimal solutions. Tuzkaya et al. (2011) developed a two-stage procedure, where in the first stage they centralized return centers by an integrated analytical network process and fuzzy-TOPSIS methodology and then, in the second stage, solved an MILP model for reverse logistics network design problem by genetic algorithms. The authors also implemented their approach on a Turkish white goods industry. Devika et al. (2014) developed a multi-objective MILP model for closed-loop SCND problem and applied three hybrid metaheuristic methods based on adapted imperialist competitive algorithms and variable neighborhood search to conquer NP-hardness difficulty. Using a lexicographic mixed integer goal programming approach, Harraz and Galal (2011) presented the design of a sustainable recovery network for end-of-life vehicles in Egypt. To explore bio-based supply chains that use locally available biomass, Perez-Fortes et al. (2012)

formulated a multi-objective MILP model that supports decision-making about location and capacity of technologies, connectivity between the supply entities, biomass storage periods, matter transportation and biomass utilization. Mota et al. (2015) presented a generic multi-objective mathematical programming model for the design and planning of sustainable supply chains, where the authors used ReCiPe as assessment methodology for environmental impacts and considered the impact of social and political concerns on company's performance as social indicator. Measuring environmental and social impacts of sustainability by the life cycle greenhouse gas emissions and the number of accrued local jobs, You et al. (2012) addressed the optimal design and planning of cellulosic ethanol sustainable supply chains by developing a multi-objective MILP model, which is solved by an -constraint method. Santibanez-Aguilar et al. (2014) presented a multi-objective MILP model to design and plan sustainable bio-refinery supply chains and exploited the -constraint method to treat multiple objectives. The authors modeled environmental and social impacts by the Eco-indicator99 according to the life-cycle assessment technique and the number of jobs generated, respectively. Covering the cradle-to-gate life cycle of bioelectricity, Yue et al. (2014) proposed a multi-objective mixed-integer linear fractional programming problem following the functional-unit-based life cycle optimization approach for the sustainable design and operation of bioelectricity supply chain networks. Pishvaee et al. (2014) proposed a multi-objective possibilistic programming model to design a sustainable medical supply chain network under uncertainty and devised an accelerated Benders decomposition algorithm utilizing three acceleration mechanisms to cope with computational complexity of solving the proposed model. Zhalechian et al. (2017) developed a multi-period, multi-product multi-objective, mixed-integer, nonlinear mathematical programming model for a closed loop supply chain network problem under uncertainty.

2.1.2 DEA applications to similar fields

DEA has been applied to evaluate the relative efficiency in various contexts (Liu et al. 2013). Among the fields with the closest proximity to the present paper are location problem, resources assignment problem and efficiency evaluation of existing supply chains. For instance, Guo (2009) proposed fuzzy DEA models for evaluating the efficiencies of objects with fuzzy input and output data and analyzed a case study involving a restaurant location problem in detail. Mohajeri and Amin (2010) dealt with the problem of finding the optimum site for a railway station using the methods of analytical hierarchy process and DEA and identified a four-level hierarchy model for the railway station site-selection problem. In selection problem of the appropriate site for solid waste facilities, Khadivi and Fatemi-Ghomi (2012) proposed a location selection procedure to construct an undesirable facility applying analytical network process and DEA approaches, in which managerial preferences and subjective data are taken into consideration, along with

quantitative factors. Klimberg and Ratick (2008) applied the approach proposed by Thomas et al. (2002) to the assignment problem. In other words, the authors identified the optimal location of facilities by applying a location problem and assigned the demand requirements of each node to the pre-located facilities by combining the capacitated and un-capacitated facility location problems models with DEA such that the average efficiency of each facility becomes maximum. Moheb-Alizadeh et al. (2011) developed an approach to deal with both location and allocation problems simultaneously by integrating DEA into locationallocation models in a fuzzy environment and presented the natural trade-off among multiple objective functions. To effective allocation of scarce resources in a health-care system, Mitropoulos et al. (2013) proposed a methodology that takes into account health service provider efficiencies based on multiple measures. The efficiencies are then employed to determine health providers locations and service allocations, which include new services distribution as well as existing services redistribution. Neto et al. (2008) presented the expected computational difficulties of using multi-objective programming approach in the design of sustainable networks and evaluated the efficiency of existing logistic networks utilizing a technique based on the commonalities between DEA and multi-objective programming. Lau (2013) investigated the feasibility of using DEA to measure efficiency and rationalize an existing distribution network as an alternative approach to the conventional method of optimizing delivery routes and schedules through linear programming, which can be complex and data demanding. Xu et al. (2009) studied the supply chain performance evaluation of a furniture manufacture industry under uncertainty situations by analyzing rough data envelopment analysis models, where cost, time and human resource are regarded as input factors, while flexibility, financial and service are considered as output factors. Azadi et al. (2014) proposed two DEA approaches to set achievable targets for two-stage network structures in green supply chain management.

2.2 Multi-criteria data envelopment analysis (MCDEA)

Since initially proposed by Charnes et al. (1978), DEA has been extensively exploited to assess the efficiency of many activities (Seiford, 1994). DEA evaluates the relative efficiency of a set of decision making units (DMUs) by using the ratio of the weighted sum of outputs to the weighted sum of inputs. Specifically, DEA determines a set of weights such that the efficiency of under evaluation DMU is maximized rather than the other DMUs. This model is a non-parametric technique that can be used to identify the source and amount of inefficiency of each input rather than each output for each DMU. The efficiency score varies in the interval [0,1] and a DMU with the efficiency score of 1 is called efficient.

The initial DEA model suffers from two shortcomings including low discriminating power and unrealistic weights distribution. In order to overcome either of or both these problems,

various approaches have been proposed in the literature such as cone ratio model (Charnes et al. 1990), assurance region (Thompson et al. 1986), super efficiency (Andersen and Petersen 1993), cross-evaluation (Sexton 1986) and multi-objective approach (Li and Reeves 1999). The latter approach has several advantages over the former ones to be used in this paper, as follows:

- Once integrated into the proposed SCND model, it results in fewer and more tractable nonlinear terms.
- It is a one-stage approach, i.e. it eliminates both aforementioned drawbacks by solving only one multi-objective problem that produces unique solution for the weight of inputs and outputs.
- It does not need any priori information from decision maker, which may cause infeasibility of the problem.

However, Ghasemi et al. (2014) recently modified Li and Reeves's model, which provides better weight dispersion and discrimination power, as follows:

$$Min \qquad ES_1 = \eta \tag{2.1}$$

$$Min \qquad ES_2 = \sum_{j=1}^{n} d_j \tag{2.2}$$

s.t.

$$\sum_{i=1}^{m} \lambda_i x_{ij_0} = 1; \qquad \forall j \tag{2.3}$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \lambda_i x_{ij} + d_j = 1; \qquad \forall j \tag{2.4}$$

$$\eta - d_j \geq 0; \qquad \forall j \tag{2.5}$$

$$\mu_r, \lambda_i \geq \epsilon; \qquad \forall i, r \tag{2.6}$$

$$d_j \geq 0; \qquad \forall j \tag{2.7}$$

The above-mentioned model evaluates the efficiency of DMU_0 where s and m represent the number of outputs and inputs, respectively. Moreover, y_{rj} is the amount of output r from DMU j, while x_{ij} is the amount of input i to DMU j. Furthermore, μ_r and λ_i are the weights given to output r and input i, respectively. Objective functions (2.1) and (2.2) are to minimize the maximum quantity and summation of all deviation (inefficiency) variables , respectively. η serves as the maximum inefficiency score of all DMUs.

As stated before, the efficiency score of a DMU ranges from zero to one. A DMU with an efficiency score less than one is called inefficient because it uses too much input and/or it does not produce enough output. There are two ways to improve the performance of such a DMU including i) increasing its outputs while retaining its inputs fixed, and ii) decreasing its inputs while keeping its outputs unchanged. Consequently, DEA models can be classified into two categories: output-oriented and input-oriented. The output (input)-oriented model refers to the capacity of a DMU to achieve the maximum (minimum) volume of output (input) with the current level of inputs (outputs). The bi-objective DEA model (2.1)-(2.7) has input orientation. We propose the output-oriented instance of it as follows:

$$Min \qquad ES_1 = \eta$$

$$Min \qquad ES_2 = \sum_{j=1}^{n}(1 + d_j)$$

$$\text{s.t.}$$

$$\sum_{i=1}^{m} \mu_r y_{rj_0} = 1; \qquad \forall j$$

$$-\sum_{r=1}^{s} \mu_r y_{rj} + \sum_{i=1}^{m} \lambda_i x_{ij} - d_j = 0; \qquad \forall j$$

$$\eta - d_j \geq 0; \qquad \forall j$$

$$\mu_r, \lambda_i \geq \epsilon; \qquad \forall i, r$$

$$d_j \geq 0; \qquad \forall j$$

We intend to proceed with the output-oriented bi-objective DEA model as developed above because, based on the defined inputs and outputs of facilities in this paper, we are willing to focus on the outputs of facilities more profoundly rather than their inputs.

Assuming n DMUs to be evaluated, one needs to solve this model n times in order to determine the efficiency score of all DMUs. In order to derive the efficiency score of all DMUs simultaneously, we propose the following simultaneous output oriented bi-objective DEA model, which is adapted from the simultaneous DEA model by Klimberg and Ratick (2008):

$$Min \qquad ES_1 = \eta \tag{2.8}$$

$$Min \qquad ES_2 = \sum_{j=1}^{n} d_j \tag{2.9}$$

$$\text{s.t.}$$

$$\sum_{r=1}^{s} \mu_{rj} y_{rj} = 1; \qquad \forall j \tag{2.10}$$

$$\sum_{i=1}^{m} \lambda_{ij} x_{ij} - d_j = 1; \qquad \forall j \tag{2.11}$$

$$\sum_{r=1}^{s} \mu_{rj} y_{r\bar{j}} - \sum_{i=1}^{m} \lambda_{ij} x_{i\bar{j}} + d_j = 1; \qquad \forall j, \bar{j}, j \neq \bar{j} \tag{2.12}$$

$$\eta - d_j \geq 0; \qquad \forall j \tag{2.13}$$

$$\mu_r, \lambda_i \geq \epsilon; \qquad \forall i, r \tag{2.14}$$

$$d_j \geq 0; \qquad \forall j \tag{2.15}$$

where, the efficiency score of DMU j is derived as $\frac{1}{1+d_j}$.

2.3 Problem Description

In this paper, we examine a generic closed loop supply chain with a multi-echelon network in multiple periods that handles both forward and reverse flows in an integrated scheme. The underlined closed loop network has different components, which are configured as illustrated in Figure 2.1. The closed loop supply chain network in various industries can be modeled as the network presented in this figure; including appliance industry (Jeihoonian et al. 2016, Alumur et al., 2012), electronic industry (Ninlawan et al. 2010), vehicle industry (ster et al. 2007, Demirel et al. 2016) and textile industry (Fahimnia et al. 2013). In this figure, the solid lines demonstrate the forward flows, whereas the dashed lines represent the reverse flows. Through the forward flows, multiple products are produced by manufacturing centers and sent to distribution centers and then to customers to satisfy their demand requirements. However, the defective products produced in manufacturing processes are sent to recycling centers in order to obtain their useful materials. On the other hand, at the end of their useful lives, end-of-life products are returned to collection centers via reverse flows where they are sorted, inspected and classified into two distinct parts: recoverable and recyclable products. The recoverable products are transported to recovery (also known as remanufacturing) centers to bring them back into an as new condition by carrying out the necessary disassembly, overhaul and replacement operations. The recyclable products, however, are shipped to recycling centers to extract their useful materials. Examples of recycled materials are metals, glass, paper and plastic. Such recycled materials are sent to materials customers to satisfy their demands and to manufacturing centers to produce new products. The scraps that are unrecyclable parts are transported from recycling centers to disposal centers to be treated by landfilling and/or incinerating. While product structures are

not conserved in recycling operations, recovery operations preserve product identity.

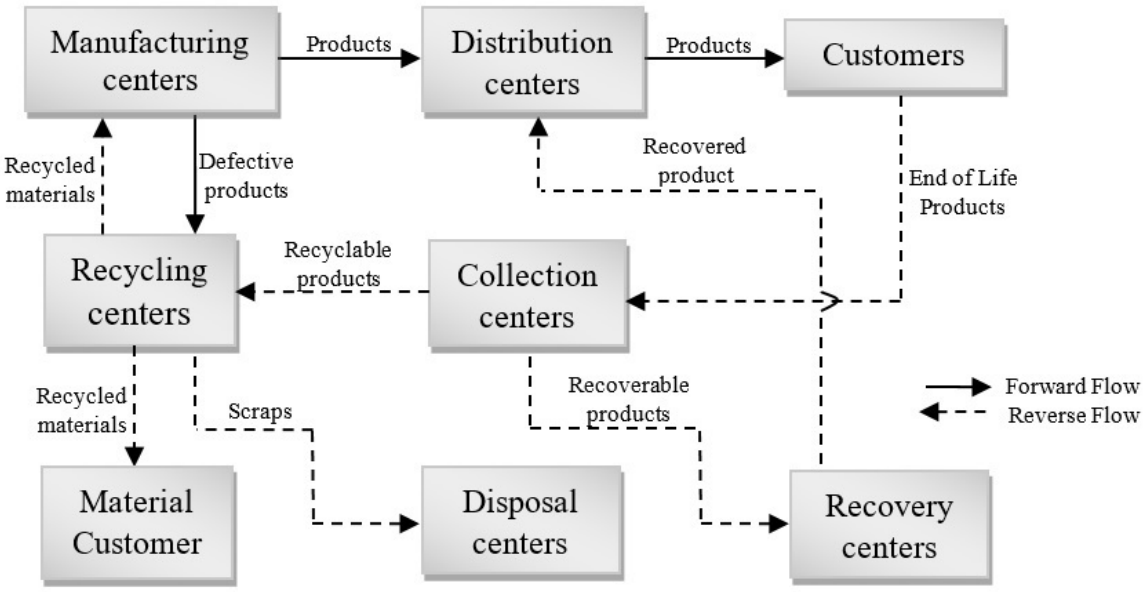


Figure 2.1: The closed loop supply chain network examined in this paper.

For planning the abovementioned supply chain network, we propose an integrated multi-objective MINLP model. The developed model aims to determine the strategic level decisions including the number, location and used capacity of various facilities in the network as well as the tactical level decisions involving aggregate quantities and mass flows through the network so as to minimize total CO2 emission and to maximize total profit, social responsibility and DEA efficiency of located facilities.

The list of assumptions we postulate in this paper to formulate the proposed model is as follows:

- Customers' demands for all products and recycled materials are deterministically known at each period of the planning horizon. .
- Number and location of candidate sites for different facilities are known.
- Number and location of customers and material customers are known.
- Processing and holding capacities of all facilities are known.
- Inventory levels of materials and products are known at the beginning of the planning horizon.

- Defective and scraped products in manufacturing and recycling centers are not held over periods, i.e. they are shipped to recycling centers and disposal centers, respectively, during the periods they are generated.

2.4 Model Development

In this section, we elucidate the integrated multi-objective MINLP model to explore the problem described in the preceding section. The associated indices, parameters and decision variables used to build the proposed model are presented in Appendix of this chapter.

2.4.1 Objective functions

Profit objective function: Total profit gained through the closed loop network includes six main components: selling revenue, transportation cost, production/processing cost, holding cost, fixed opening cost and purchasing cost in all periods. In other words, total profit equals selling revenue minus the summation of transportation, production/processing, holding, fixed and purchasing costs.

Relation (2.16) characterizes total selling revenue of products and recycled materials to customers and materials customers, respectively.

$$SeR = \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{c=1}^{C}\sum_{t=1}^{T} spr_{jct}^{pro} DTC_{djct} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T} spr_{imt}^{mat} YTC_{yimt} \tag{2.16}$$

On the other hand, total transportation cost between any two connected facilities in the closed loop network is derived as follows:

$$\begin{aligned} TrC = &\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{t=1}^{T} tco_{kjdt}^{man-dis} MTD_{kjdt} + \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{y=1}^{Y}\sum_{t=1}^{T} tco_{kjyt}^{man-rcy} MTY_{kjyt} + \\ &\sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{c=1}^{C}\sum_{t=1}^{T} tco_{djct}^{dis-cus} DTC_{djct} + \sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} tco_{cjlt}^{cus-co} CTC_{cjlt} + \\ &\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{y=1}^{Y}\sum_{t=1}^{T} tco_{ljyt}^{col-rcy} CTY_{ljyt} + \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{t=1}^{T} tco_{ljvt}^{col-rcv} CTV_{ljvt} + \\ &\sum_{v=1}^{V}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{t=1}^{T} tco_{vjdt}^{rcv-dis} VTDvjdt + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{t=1}^{T} tco_{yipt}^{rcy-dip} YTPvipt + \\ &\sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T} tco_{yimt}^{rcy-mcu} YTCvimt + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} tco_{yikt}^{rcy-man} YTMyikt \end{aligned} \tag{2.17}$$

Moreover, total processing cost, total holding cost of remained items at the end of periods and total fixed cost of opening facilities are derived as relations (2.18)-(2.20), respectively. Relation (2.21) presents other cost components including total purchasing cost of end-of-life products from customers (term 1), penalty cost of uncollected end-of-life products (term 2) and shortage cost of unsatisfied demand at customers (terms 3 and 4).

$$\begin{aligned} PrC = &\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} pco_{kjt}^{man} PLP_{kjdt}^{man} + \sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} pco_{ljt}^{col} CTC_{cjlt} + \\ &\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{t=1}^{T} pco_{vjt}^{rcv} CTV_{ljvt} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} pco_{yit}^{rcy} PLP_{yit}^{rcy} + \\ &\sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{t=1}^{T} pco_{pit}^{dip} YTP_{yipt} \end{aligned} \tag{2.18}$$

$$\begin{aligned} HoC = &\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{t=1}^{T} \overline{hco}_{kit}^{man} \overline{LOI}_{kit}^{man} + \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} hco_{kjt}^{man} LOI_{kjt}^{man} + \\ &\sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{t=1}^{T} hco_{djt}^{dis} LOI_{djt}^{dis} + \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{t=1}^{T} hco_{ljt}^{col-y} LOI_{ljt}^{col-y} + \\ &\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{t=1}^{T} hco_{ljt}^{col-v} LOI_{ljt}^{col-v} + \sum_{v=1}^{V}\sum_{j=1}^{J}\sum_{t=1}^{T} hco_{vjt}^{rcv} LOI_{vjt}^{rcv} + \\ &\sum_{y=1}^{Y}\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{hco}_{yjt}^{rcy} \overline{LOI}_{yjt}^{rcy} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} hco_{yit}^{rcy} LOI_{yit}^{rcy} \end{aligned} \tag{2.19}$$

$$\begin{aligned} FiC = &\sum_{k=1}^{K} X_{k}^{man} fco^{man} + \sum_{d=1}^{D} X_{d}^{dis} fco^{dis} + \sum_{l=1}^{L} X_{l}^{col} fco^{col} + \sum_{v=1}^{V} X_{v}^{rev} fco^{rev} + \\ &\sum_{y=1}^{Y} X_{y}^{rey} fco^{rey} + \sum_{d=1}^{D} X_{d}^{dip} fco^{dip} + \end{aligned} \tag{2.20}$$

$$\begin{aligned} OtC = &\sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} ppo_{jt} CTC_{cjlt} + \sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{t=1}^{T} cup_{cjt} UEP_{cjt} + \\ &\sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{t=1}^{T} pcs_{cjt}^{cus} STG_{cjt}^{cus} + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T} pcs_{imt}^{mat} STG_{imt}^{mat} \end{aligned} \tag{2.21}$$

Hence, we derive the objective function maximizing total profit as follows:

$$Max \qquad TP = SeR - TrC - PrC - HoC - FiC - OtC \tag{2.22}$$

Emission objective function: This objective function contains three different components including total transportation emission, total production/processing emission and total holding emission, which are derived, respectively, as follows:

$$\begin{aligned}TrE =& \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{t=1}^{T} tem_{kjdt}^{man-dis} MTD_{kjdt} + \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{y=1}^{Y}\sum_{t=1}^{T} tem_{kjyt}^{man-rcy} MTY_{kjyt} + \\ & \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{c=1}^{C}\sum_{t=1}^{T} tem_{djct}^{dis-cus} DTC_{djct} + \sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} tem_{cjlt}^{cus-co} CTC_{cjlt} + \\ & \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{y=1}^{Y}\sum_{t=1}^{T} tem_{ljyt}^{col-rcy} CTY_{ljyt} + \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{t=1}^{T} tem_{ljvt}^{col-rcv} CTV_{ljvt} + \\ & \sum_{v=1}^{V}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{t=1}^{T} tem_{vjdt}^{rcv-dis} VTD_{vjdt} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{t=1}^{T} tem_{yipt}^{rcy-dip} YTP_{vipt} + \\ & \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T} tem_{yimt}^{rcy-mcu} YTC_{vimt} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} tem_{yikt}^{rcy-man} YTM_{yikt}\end{aligned} \tag{2.23}$$

$$\begin{aligned}PrE =& \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} pem_{kjt}^{man} PLP_{kjdt}^{man} + \sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} pem_{ljt}^{col} CTC_{cjlt} + \\ & \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{t=1}^{T} pem_{vjt}^{rcv} CTV_{ljvt} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} pem_{yit}^{rcy} PLP_{yit}^{rcy} + \\ & \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{t=1}^{T} pem_{pit}^{dip} YTP_{yipt}\end{aligned} \tag{2.24}$$

$$\begin{aligned}HoE =& \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{t=1}^{T} \overline{hem}_{kit}^{man} \overline{LOI}_{kit}^{man} + \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} hem_{kjt}^{man} LOI_{kjt}^{man} + \\ & \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{t=1}^{T} hem_{djt}^{dis} LOI_{djt}^{dis} + \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{t=1}^{T} hem_{ljt}^{col-y} LOI_{ljt}^{col-y} + \\ & \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{t=1}^{T} hem_{ljt}^{col-v} LOI_{ljt}^{col-v} + \sum_{v=1}^{V}\sum_{j=1}^{J}\sum_{t=1}^{T} hem_{vjt}^{rcv} LOI_{vjt}^{rcv} + \\ & \sum_{y=1}^{Y}\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{hem}_{yjt}^{rcy} \overline{LOI}_{yjt}^{rcy} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} hem_{yit}^{rcy} LOI_{yit}^{rcy}\end{aligned} \tag{2.25}$$

Accordingly, we define the emission objective function as:

$$Min \qquad TE = TrE + PrE + HoE \tag{2.26}$$

Social responsibility objective function: The current objective function contains two components including job opportunities created (JoC) and lost days caused from works damages (LoD), which are formulated as relations (2.27) and (2.28), respectively. In formulating JoC and LoD, we adapt the procedure proposed by Pishvaee et al. (2012) and Devika et al. (2014), respectively.

$$\begin{aligned} JoC = & \sum_{k=1}^{K}\sum_{t=1}^{T} lab_{kt}^{man} X_k^{man} + \sum_{d=1}^{D}\sum_{t=1}^{T} lab_{dt}^{dis} X_d^{dis} + \sum_{l=1}^{L}\sum_{t=1}^{T} lab_{lt}^{col} X_l^{col} + \\ & \sum_{v=1}^{V}\sum_{t=1}^{T} lab_{vt}^{rcv} X_v^{rcv} + \sum_{y=1}^{Y}\sum_{t=1}^{T} lab_{yt}^{rcy} X_y^{rcy} + \\ & \sum_{p=1}^{P}\sum_{t=1}^{T} lab_{pt}^{dip} X_p^{dip} \end{aligned} \tag{2.27}$$

$$\begin{aligned} LoD = & \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} los_k^{man} PLP_{kjt}^{man} / pca_{kjt}^{man} + \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{t=1}^{T} (los_d^{dis} / hca_{djt}^{dis})[\sum_{k=1}^{K} MTD_{kjdt} + \\ & \sum_{v=1}^{V} VTD_{vjdt}] + \sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} los_l^{col} CTC_{cjlt} / pca_{ljt}^{col} + \\ & \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{t=1}^{T} los_v^{rcv} CTV_{ljvt} / pca_{vjt}^{rcv} + \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} los_y^{rcy} PLP_{vit}^{rcy} / pca_{vit}^{rcy} + \\ & \sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{t=1}^{T} los_p^{dip} YTP_{yipt}^{rcy} / pca_{pit}^{dip} \end{aligned} \tag{2.28}$$

Consequently, we derive the social responsibility objective function as follows, where ζ and $\bar{\zeta}$ and are normalizing weight factors given to JoC and LoD components, respectively.

$$Max \qquad SR = \zeta.JoC + \bar{\zeta}.LoD \tag{2.29}$$

DEA efficiency objective functions: Objective functions (2.30) and (2.31) characterize the DEA efficiency objective functions in this study:

$$Min \qquad ES_1 = \eta \tag{2.30}$$

$$Min \qquad ES_2 = \sum_{k=1}^{K}(X_k^{man} + d_k^{man}) + \sum_{d=1}^{D}(X_d^{dis} + d_d^{dis}) + \sum_{l=1}^{L}(X_l^{col} + d_l^{col}) + \sum_{v=1}^{V}(X_v^{rcv} + d_v^{rcv}) + \sum_{y=1}^{Y}(X_y^{rcy} + d_y^{rcy}) \tag{2.31}$$

2.4.2 Constraints

We define eight types of constraints including inventory level constraints, inoperative facilities constraints, minimum operational level constraints, capacity constraints, constraints for number of opened facilities, production constraints, demand constraints and DEA efficiency constraints.

Inventory level constraints: This type of constraints determines the inventory level of each material/product in each facility at the end of each period. The general rule in defining such constraints is that: the inventory level of each material/product at the end of current period is equal to that at the end of previous period plus total received/produced amount of the same material/product minus total outgoing amount of that material/product during the current period. According to this definition, constraints (2.32)-(2.39) derive the inventory level of each material/product at the end of each period for different facilities, where $\overline{LOI}_{ki0}^{man} = \overline{iil}_{ki}^{man}$, $LOI_{kj0}^{man} = iil_{kj}^{man}$, $LOI_{dj0}^{dis} = iil_{dj}^{dis}$, $LOI_{lj0}^{col-y} = iil_{lj}^{col-y}$, $LOI_{lj0}^{col-v} = iil_{lj}^{col-v}$, $LOI_{vj0}^{rcv} = iil_{vj}^{rcv}$, $\overline{LOI}_{yj0}^{rcy} = \overline{iil}_{yj}^{rcy}$, and $LOI_{yi0}^{rcy} = iil_{yi}^{rcy}$.

$$\overline{LOI}_{kit}^{man} = (1 - \sum_{j=1}^{J}\alpha_{ij})(\overline{LOI}_{ki,t-1}^{man} + \sum_{y=1}^{Y} YTM_{yikt}); \qquad \forall k, i, t \tag{2.32}$$

$$LOI_{kjt}^{man} = LOI_{kj,t-1}^{man} + (1 - \beta_j)PLP_{kjt}^{man} - \sum_{d=1}^{D} MTD_{kjdt}; \qquad \forall k, j, t \tag{2.33}$$

$$LOI_{djt}^{dis} = LOI_{dj,t-1}^{dis} + \sum_{k=1}^{K} MTD_{kjdt} + \sum_{v=1}^{V} VTD_{vjdt} - \sum_{c=1}^{C} DTC_{djct}; \qquad \forall d, j, t \tag{2.34}$$

$$LOI_{ljt}^{col-y} = LOI_{lj,t-1}^{col-y} + (1 - \psi_j)\sum_{c=1}^{C} CTC_{cjlt} - \sum_{y=1}^{Y} CTY_{ljyt}; \qquad \forall l, j, t \tag{2.35}$$

$$LOI_{ljt}^{col-v} = LOI_{lj,t-1}^{col-v} + \psi_j \sum_{c=1}^{C} CTC_{cjlt} - \sum_{v=1}^{V} CTV_{ljvt}; \qquad \forall l, j, t \tag{2.36}$$

$$LOI_{vjt}^{rcv} = LOI_{vj,t-1}^{rcv} + \sum_{l=1}^{L} CTV_{ljvt} - \sum_{d=1}^{D} VTD_{vjdt}; \qquad \forall v, j, t \tag{2.37}$$

$$\overline{LOI}^{rcy}_{yjt} = (1 - \sum_{i=1}^{I} \delta_{ji})(\overline{LOI}^{rcy}_{yj,t-1} + \sum_{k=1}^{K} MTY_{kjyt} + \sum_{l=1}^{L} CTY_{ljyt}); \qquad \forall y, j, t \tag{2.38}$$

$$LOI^{rcy}_{yit} = LOI^{rcy}_{yi,t-1} + (1 - \tau_i)PLP^{rcy}_{yit} - \sum_{k=1}^{K} YTM_{yikt} - \sum_{m=1}^{M} YTC_{yimt}; \qquad \forall y, i, t \tag{2.39}$$

Inoperative facilities constraints: Once a facility is not opened in one of its respective candidate sites, then it does not rationally have any incoming (outgoing) material/product from (to) other facilities in the network. We define inoperative facilities constraints to guarantee the current conditions for all facilities as follows:

$$\sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} YTM_{yikt} + \sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{t=1}^{T} MTD_{kjdt} + \sum_{j=1}^{J}\sum_{y=1}^{Y}\sum_{t=1}^{T} MTY_{kjyt} \leq X^{man}_{k} M; \qquad \forall k \tag{2.40}$$

$$\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} MTD_{kjdt} + \sum_{v=1}^{V}\sum_{j=1}^{J}\sum_{t=1}^{T} VTD_{vjdt} + \sum_{j=1}^{J}\sum_{c=1}^{C}\sum_{t=1}^{T} DTC_{djct} \leq X^{dis}_{d} M; \qquad \forall d \tag{2.41}$$

$$\sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{t=1}^{T} CTC_{cjlt} + \sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{t=1}^{T} CTV_{ljvt} + \sum_{j=1}^{J}\sum_{y=1}^{Y}\sum_{t=1}^{T} CTY_{ljyt} \leq X^{col}_{l} M; \qquad \forall l \tag{2.42}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{t=1}^{T} CTV_{ljvt} + \sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{t=1}^{T} VTD_{vjdt} \leq X^{rcv}_{v} M; \qquad \forall v \tag{2.43}$$

$$\begin{aligned} \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{t=1}^{T} MTY_{kjyt} + \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{t=1}^{T} CTY_{ljyt} + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T} YTC_{yimt} + \\ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} YTM_{yikt} + \sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{t=1}^{T} YTP_{yipt} \leq X^{rcy}_{y} M; \qquad \forall y \end{aligned} \tag{2.44}$$

$$\sum_{y=1}^{Y}\sum_{i=1}^{I}\sum_{t=1}^{T} YTP_{yipt} + \leq X^{dip}_{p} M; \qquad \forall v \tag{2.45}$$

Minimum operational level constraints: This type of constraints prevents any opened facility to be idle in practice. In order for an opened facility to be operative in the current period, it needs to either hold materials/products transferred from the previous period or receives incoming materials/products in the current period or both. In this regard, constraints (2.46)-(2.51) require the operationality of opened facilities in their respective minimum levels.

$$LOI^{man}_{kj,t-1} + PLP^{man}_{kjt} \geq X^{man}_{k} mic^{man}_{kjt}; \qquad \forall k, j, t \tag{2.46}$$

$$LOI^{dis}_{dj,t-1} + \sum_{k=1}^{K} MTD_{kjdt} + \sum_{v=1}^{V} VTD_{vjdt} \geq X^{dis}_{d} mic^{dis}_{djt}; \qquad \forall d, j, t \tag{2.47}$$

$$LOI^{col-y}_{lj,t-1} + LOI^{col-v}_{lj,t-1} + \sum_{c=1}^{C} CTC_{cjlt} \geq X^{col}_{l} mic^{col}_{ljt}; \qquad \forall l, j, t \tag{2.48}$$

$$LOI^{rcv}_{vj,t-1} + \sum_{l=1}^{L} CTV_{ljvt} \geq X^{rcv}_{v} mic^{rcv}_{vjt}; \qquad \forall v, j, t \tag{2.49}$$

$$LOI^{rcy}_{yi,t-1} + PLP^{rcy}_{yit} \geq X^{rcy}_{y} mic^{rcy}_{yit}; \qquad \forall y, i, t \tag{2.50}$$

$$\sum_{y=1}^{Y} YTP_{yipt} \geq X^{dip}_{p} mic^{dip}_{pit}; \qquad \forall p, i, t \tag{2.51}$$

Capacity constraints: A facility that processes materials/products needs to have its corresponding production/processing capacity satisfied if it is opened in one of its respective candidate sites. Otherwise, it does not have any production/process. In this regard, constraints (2.52) and (2.53) denote production capacity constraints for manufacturing and recycling centers, while constraints (2.54)-(2.56) satisfy the processing constraints for collection, recovery and disposal centers, respectively.

$$PLP^{man}_{kjt} \leq X^{man}_{k} pca^{man}_{kjt}; \qquad \forall k, j, t \tag{2.52}$$

$$PLP^{rcy}_{yit} \leq X^{rcy}_{y} pca^{rcy}_{yit}; \qquad \forall y, i, t \tag{2.53}$$

$$\sum_{c=1}^{C} CTC_{cjlt} \leq X^{col}_{l} pca^{col}_{ljt}; \qquad \forall l, j, t \tag{2.54}$$

$$\sum_{l=1}^{L} CTV_{ljvt} \leq X^{rcv}_{v} pca^{rcv}_{vjt}; \qquad \forall v, j, t \tag{2.55}$$

$$\sum_{y=1}^{Y} YTP_{yipt} \leq X^{dip}_{p} pca^{dip}_{pit}; \qquad \forall p, i, t \tag{2.56}$$

In addition, holding capacity needs to be satisfied for all facilities in the network. If a facility is to be operative, then the amount of material/product held at the end of each period should satisfy holding capacity as follows:

$$\overline{LOI}^{man}_{kit} \leq X^{man}_{k} \overline{hca}^{man}_{kit}; \qquad \forall k, i, t \tag{2.57}$$

$$LOI^{man}_{kjt} \leq X^{man}_{k} hca^{man}_{kjt}; \qquad \forall k, j, t \tag{2.58}$$

$$LOI_{djt}^{dis} \leq X_d^{dis} hca_{djt}^{dis}; \qquad \forall d, j, t \tag{2.59}$$

$$LOI_{ljt}^{col-y} \leq X_l^{col} hca_{ljt}^{col-y}; \qquad \forall l, j, t \tag{2.60}$$

$$LOI_{ljt}^{col-v} \leq X_l^{col} hca_{ljt}^{col-v}; \qquad \forall l, j, t \tag{2.61}$$

$$LOI_{vjt}^{rcv} \leq X_v^{rcv} hca_{vjt}^{rcv}; \qquad \forall v, j, t \tag{2.62}$$

$$\overline{LOI}_{yjt}^{rcy} \leq X_y^{rcy} \overline{hca}_{yjt}^{rcy}; \qquad \forall y, j, t \tag{2.63}$$

$$LOI_{yit}^{rcv} \leq X_y^{rcy} hca_{yit}^{rcy}; \qquad \forall y, i, t \tag{2.64}$$

Constraints for number of opened facilities: Using this type of constraints, we set bounds on the number of opened facilities of each type as follows:

$$1 \leq \sum_{k=1}^{K} X_k^{man} \leq mxn^{man} \tag{2.65}$$

$$1 \leq \sum_{d=1}^{D} X_d^{dis} \leq mxn^{did} \tag{2.66}$$

$$1 \leq \sum_{l=1}^{L} X_l^{col} \leq mxn^{col} \tag{2.67}$$

$$1 \leq \sum_{v=1}^{V} X_v^{rcv} \leq mxn^{rcv} \tag{2.68}$$

$$1 \leq \sum_{y=1}^{Y} X_y^{rcy} \leq mxn^{rcy} \tag{2.69}$$

$$1 \leq \sum_{p=1}^{P} X_p^{dip} \leq mxn^{dip} \tag{2.70}$$

Production constraints: At each period, constraint (2.71) ((2.72)) denotes the production level of each product (recycled material) in each manufacturing (recycling) center as a linear function of its incoming materials (defective and recyclable products). Constraint (2.73) derives the amount of each defective product produced in each manufacturing center at each period based on the amount of corresponding production level. Similarly, constraint (2.74) characterizes the amount of each scraped material in each recycling center at each period based on the respective production level. In addition, constraint (2.75) ensures the total amount of each

end-of-life product in each customer at each period is collected and sent to collection centers.

$$PLP_{kjt}^{man} = \sum_{i=1}^{I} \alpha_{ij}(\overline{LOI}_{ki,t-1}^{man} + \sum_{y=1}^{Y} YTM_{yikt}); \qquad \forall k, j, t \tag{2.71}$$

$$PLP_{yit}^{rcy} = \sum_{j=1}^{J} \delta_{ji}(\overline{LOI}_{yj,t-1}^{rcy} + \sum_{l=1}^{L} CTY_{ljyt} + \sum_{k=1}^{K} MTY_{kjyt}); \qquad \forall y, i, t \tag{2.72}$$

$$\sum_{y=1}^{Y} MTY_{kjyt} = \beta_j PLP_{kjt}^{man}; \qquad \forall k, j, t \tag{2.73}$$

$$\sum_{p=1}^{P} YTP_{yipt} = \tau_i PLP_{yit}^{rcy}; \qquad \forall y, i, t \tag{2.74}$$

$$\sum_{l=1}^{L} CTC_{cjlt} + UEP_{cjt} = \phi_j \sum_{d=1}^{D} DTC_{djct}; \qquad \forall c, j, t \tag{2.75}$$

Demand constraints: Constraint (2.76) satisfies demand requirement for each product in each customer at each period. The same requirement for each recycled material in each materials customer at each period is implied by constraint (2.77). We assume the amount of shortage at the beginning of planning horizon for all products and recycled materials at any customer and materials customer is equal to zero, i.e. $STG_{cj0}^{cus} = 0; \forall c, j$ and $STG_{im0}^{mat} = 0; \forall i, m$.

$$\sum_{d=1}^{D} DTC_{djct} + STG_{cjt}^{cus} = dem_{cjt}^{pro} + STG_{cj,t-1}^{cus}; \qquad \forall c, j, t \tag{2.76}$$

$$\sum_{y=1}^{Y} YTC_{yimt} + STG_{imt}^{mat} = dem_{imt}^{mat} + STG_{im,t-1}^{mat}; \qquad \forall i, m, t \tag{2.77}$$

DEA efficiency constraints: In order to derive DEA constraints for various facilities in the presented closed loop supply chain network, it is first required determining their respective inputs and outputs. In this study, as indicated earlier, we define three types of inputs for each facility at each period as follows:

- Physical input: this type of input is defined as the amount of physical materials/products that is available in each facility at each period. Therefore, it includes the inventory level of a material/product at the beginning of each period plus the amount of incoming materials/products during each period.
- Current expense: this input characterizes the total cost of keeping each facility going at

each period. Examples of current expense of a facility include rent and electricity bills.

- Labor force: the last input is equal to the total number of labor forces that a facility needs during each period.

On the other hand, we define one output for each facility, which is the total amount of outgoing materials/products to all other connected facilities in the network.

Tables 2.1 and 2.2 represent the total amount of outputs and inputs for each type of facilities over all periods, respectively. That the inputs and outputs are derived over all periods in this study is based on the aggregate model proposed by Kao and Liu (2014). Furthermore, the defective products and scraped materials that are produced in manufacturing and recycling centers respectively are undesirable outputs in evaluating the respective DEA efficiency scores. Such undesirable outputs are converted in this study using a monotone decreasing linear transformation proposed by Seiford and Zhu (2002).

Table 2.1: Characterization of outputs in various facilities in closed loop network.

Facility	No. of outputs	Output
Manufacturing center	$2J$	$\sum_{d=1}^{D}\sum_{t=1}^{T} MTD_{kjdt}$ and $\sum_{y=1}^{Y}\sum_{t=1}^{T} MTY_{kjyt}$
Distribution center	J	$\sum_{c=1}^{C}\sum_{t=1}^{T} DTC_{djct}$
Collection center	$2J$	$\sum_{v=1}^{V}\sum_{t=1}^{T} CTV_{ljvt}$ and $\sum_{y=1}^{Y}\sum_{t=1}^{T} CTY_{ljyt}$
Recovery center	J	$\sum_{d=1}^{D}\sum_{t=1}^{T} VTD_{vjdt}$
Recycling center	$2I$	$\sum_{k=1}^{K}\sum_{t=1}^{T} YTM_{yikt} + \sum_{m=1}^{M}\sum_{t=1}^{T} YTC_{yimt}$ and $\sum_{p=1}^{P}\sum_{t=1}^{T} YTP_{yipt}$

According to Table 2.1, constraints (2.78) to (2.82) denote the weighted sum of outputs from each open facility is equal to one. The current constraints are equivalent to constraint

Table 2.2: Characterization of inputs in various facilities in closed loop network.

Facility	No. of inputs	Physical input	Current expense	Labor force
Manufacturing center	$2+I$	$\sum_{t=1}^{T} \overline{LOI}_{ki,t-1,man} + \sum_{y=1}^{Y}\sum_{t=1}^{T} YTM_{yikt}$	$\sum_{t=1}^{T} cex_{kt}^{man}$	$\sum_{t=1}^{T} lab_{kt}^{man}$
Distribution center	$2+J$	$\sum_{t=1}^{T} LOI_{dj,t-1}^{dis} + \sum_{k=1}^{K}\sum_{t=1}^{T} MTD_{kjdt} + \sum_{v=1}^{V}\sum_{t=1}^{T} VTD_{vjdt}$	$\sum_{t=1}^{T} cex_{dt}^{dis}$	$\sum_{t=1}^{T} lab_{dt}^{dis}$
Collection center	$2J+2$	$\sum_{t=1}^{T} LOI_{lj,t-1}^{col-y} + (1-\psi_j)\sum_{c=1}^{C}\sum_{t=1}^{T} CTC_{cjlt}$ and $\sum_{t=1}^{T} LOI_{lj,t-1}^{col-v} + \psi_j \sum_{c=1}^{C}\sum_{t=1}^{T} CTC_{cjlt}$	$\sum_{t=1}^{T} cex_{lt}^{col}$	$\sum_{t=1}^{T} lab_{lt}^{col}$
Recovery center	J	$\sum_{t=1}^{T} LOI_{vj,t-1}^{rcv} + \sum_{l=1}^{L}\sum_{t=1}^{T} CTV_{ljvt}$	$\sum_{t=1}^{T} cex_{vt}^{rcv}$	$\sum_{t=1}^{T} lab_{vt}^{rcv}$
Recycling center	J	$\sum_{t=1}^{T} \overline{LOI}_{yj,t-1}^{rcy} + \sum_{k=1}^{K}\sum_{t=1}^{T} MTY_{kjyt} + \sum_{l=1}^{L}\sum_{t=1}^{T} CTY_{ljyt}$	$\sum_{t=1}^{T} cex_{yt}^{rcy}$	$\sum_{t=1}^{T} lab_{yt}^{rcy}$

(2.10) in DEA model.

$$\sum_{j=1}^{J} \mu_{kj}^{man} \sum_{d=1}^{D}\sum_{t=1}^{T} MTD_{kjdt} + \sum_{j=1}^{J} \bar{\mu}_{kj}^{man}[w_j - \sum_{y=1}^{Y}\sum_{t=1}^{T} MTY_{kjyt}] = X_k^{man}; \qquad \forall k \tag{2.78}$$

$$\sum_{j=1}^{J} \mu_{dj}^{dis} \sum_{c=1}^{C}\sum_{t=1}^{T} DTC_{djct} = X_d^{dis}; \qquad \forall d \tag{2.79}$$

$$\sum_{j=1}^{J} \mu_{lj}^{col-v} \sum_{v=1}^{V}\sum_{t=1}^{T} CTV_{ljvt} + \sum_{j=1}^{J} \mu_{lj}^{col-y} \sum_{y=1}^{Y}\sum_{t=1}^{T} CTY_{ljyt} = X_l^{col}; \qquad \forall l \tag{2.80}$$

$$\sum_{j=1}^{J} \mu_{vj}^{rcv} \sum_{d=1}^{D} \sum_{t=1}^{T} VTD_{vjdt} = X_v^{rcv}; \qquad \forall v \tag{2.81}$$

$$\sum_{i=1}^{I} \mu_{yi}^{rcy} [\sum_{k=1}^{K} \sum_{t=1}^{T} YTM_{yikt} + \sum_{m=1}^{M} \sum_{t=1}^{T} YTC_{yimt}] + \sum_{i=1}^{I} \bar{\mu}_{yi}^{rcy} [\bar{w}_i - \sum_{p=1}^{P} \sum_{t=1}^{T} YTP_{yipt}] = X_y^{rcy}; \forall y \tag{2.82}$$

Moreover, based on Table 2.2, constraints (2.83) to (2.87) that are equivalent to constraint (2.11) in DEA model imply, after subtracting the inefficiency variables, the weighted sum of inputs to any open facility in the network is equal to one. In addition, constraints (2.88) to (2.92) that are equivalent to constraint (2.12) in DEA model show the efficiency score of any facility is less than or equal to one. That η is the maximum of inefficiencies of open facilities is denoted by constraints (2.93) to (2.97).

$$\sum_{i=1}^{I} \lambda_{ki}^{man} [\sum_{t=1}^{T} \overline{LOI}_{ki,t-1}^{man} + \sum_{y=1}^{Y} \sum_{t=1}^{T} YTM_{yikt}] + \bar{\lambda}_k^{man} \sum_{t=1}^{T} cux_{kt}^{man} + \bar{\bar{\lambda}}_k^{man} \sum_{t=1}^{T} lab_{kt}^{man} - d_k^{man} = X_k^{man}; \qquad \forall k \tag{2.83}$$

$$\sum_{j=1}^{J} \lambda_{dj}^{dis} [\sum_{t=1}^{T} LOI_{dj,t-1}^{dis} + \sum_{k=1}^{K} \sum_{t=1}^{T} MTD_{kjdt} + \sum_{v=1}^{V} \sum_{t=1}^{T} VTD_{vjdt}] + \bar{\lambda}_d^{dis} \sum_{t=1}^{T} cux_{dt}^{dis} + \bar{\bar{\lambda}}_d^{dis} \sum_{t=1}^{T} lab_{dt}^{dis} - d_d^{dis} = X_d^{dis}; \qquad \forall d \tag{2.84}$$

$$\sum_{j=1}^{J} \lambda_{lj}^{col-y} [\sum_{t=1}^{T} LOI_{lj,t-1}^{col-y} + (1-\psi_j) \sum_{c=1}^{C} \sum_{t=1}^{T} CTC_{cjlt}] + \sum_{j=1}^{J} \lambda_{lj}^{col-v} [\sum_{t=1}^{T} LOI_{lj,t-1}^{col-v} + \psi_j \sum_{c=1}^{C} \sum_{t=1}^{T} CTC_{cjlt}] + \bar{\lambda}_l^{col} \sum_{t=1}^{T} cux_{lt}^{col} + \bar{\bar{\lambda}}_l^{col} \sum_{t=1}^{T} lab_{lt}^{col} - d_l^{col} = X_l^{col}; \qquad \forall l \tag{2.85}$$

$$\sum_{j=1}^{J} \lambda_{vj}^{rcv} [\sum_{t=1}^{T} LOI_{vj,t-1}^{rcv} + \sum_{l=1}^{L} \sum_{t=1}^{T} CTV_{ljvt}] + \bar{\lambda}_v^{rcv} \sum_{t=1}^{T} cux_{vt}^{rcv} + \bar{\bar{\lambda}}_v^{rcv} \sum_{t=1}^{T} lab_{vt}^{rcv} - d_v^{rcv} = X_v^{rcv}; \qquad \forall v \tag{2.86}$$

$$\sum_{j=1}^{J} \lambda_{yj}^{rcy} [\sum_{t=1}^{T} \overline{LOI}_{yj,t-1}^{rcy} + \sum_{k=1}^{K} \sum_{t=1}^{T} MTY_{kjyt} + \sum_{l=1}^{L} \sum_{t=1}^{T} CTY_{ljyt}] + \bar{\lambda}_y^{rcy} \sum_{t=1}^{T} cux_{yt}^{rcy} + \bar{\bar{\lambda}}_y^{rcy} \sum_{t=1}^{T} lab_{yt}^{rcy} - d_y^{rcy} = X_y^{rcy}; \qquad \forall y \tag{2.87}$$

$$\sum_{j=1}^{J}\mu_{kj}^{man}\sum_{d=1}^{D}\sum_{t=1}^{T}MTD_{\bar{k}jdt}+\sum_{j=1}^{J}\bar{\mu}_{kj}^{man}[w_j-\sum_{y=1}^{Y}\sum_{t=1}^{T}MTY_{\bar{k}jyt}]-\sum_{i=1}^{I}\lambda_{ki}^{man}[\sum_{t=1}^{T}\overline{LOI}_{\bar{k}i,t-1}^{man}+$$
$$\sum_{y=1}^{Y}\sum_{t=1}^{T}YTM_{yi\bar{k}t}]-\bar{\lambda}_{k}^{man}\sum_{t=1}^{T}cux_{\bar{k}t}^{man}-\bar{\bar{\lambda}}_{k}^{man}\sum_{t=1}^{T}lab_{\bar{k}t}^{man}\leq 0;\qquad \forall k,\bar{k};k\neq\bar{k} \tag{2.88}$$

$$\sum_{j=1}^{J}\mu_{dj}^{dis}\sum_{c=1}^{C}\sum_{t=1}^{T}DTC_{\bar{d}jct}-\sum_{j=1}^{J}\lambda_{dj}^{dis}[\sum_{t=1}^{T}LOI_{\bar{d}j,t-1}^{dis}+\sum_{k=1}^{K}\sum_{t=1}^{T}MTD_{kj\bar{d}t}+\sum_{v=1}^{V}\sum_{t=1}^{T}VTD_{vj\bar{d}t}]-$$
$$\bar{\lambda}_{d}^{dis}\sum_{t=1}^{T}cux_{\bar{d}t}^{dis}-\bar{\bar{\lambda}}_{d}^{dis}\sum_{t=1}^{T}lab_{\bar{d}t}^{dis}\leq 0;\qquad \forall d,\bar{d};d\neq\bar{d} \tag{2.89}$$

$$\sum_{j=1}^{J}\mu_{lj}^{col-v}\sum_{v=1}^{V}\sum_{t=1}^{T}CTV_{\bar{l}jvt}+\sum_{j=1}^{J}\mu_{lj}^{col-y}\sum_{y=1}^{Y}\sum_{t=1}^{T}CTY_{\bar{l}jyt}-\sum_{j=1}^{J}\lambda_{lj}^{col-y}[\sum_{t=1}^{T}LOI_{\bar{l}j,t-1}^{col-y}+$$
$$(1-\psi_j)\sum_{c=1}^{C}\sum_{t=1}^{T}CTC_{cj\bar{l}t}]-\sum_{j=1}^{J}\lambda_{lj}^{col-v}[\sum_{t=1}^{T}LOI_{\bar{l}j,t-1}^{col-v}+\psi_j\sum_{c=1}^{C}\sum_{t=1}^{T}CTC_{cj\bar{l}t}]- \tag{2.90}$$
$$\bar{\lambda}_{l}^{col}\sum_{t=1}^{T}cux_{\bar{l}t}^{col}-\bar{\bar{\lambda}}_{l}^{col}\sum_{t=1}^{T}lab_{\bar{l}t}^{col}\leq 0;\qquad \forall l,\bar{l};l\neq\bar{l}$$

$$\sum_{j=1}^{J}\mu_{vj}^{rcv}\sum_{d=1}^{D}\sum_{t=1}^{T}VTD_{\bar{v}jdt}-\sum_{j=1}^{J}\lambda_{vj}^{rcv}[\sum_{t=1}^{T}LOI_{\bar{v}j,t-1}^{rcv}+\sum_{l=1}^{L}\sum_{t=1}^{T}CTV_{lj\bar{v}t}]-\bar{\lambda}_{v}^{rcv}\sum_{t=1}^{T}cux_{\bar{v}t}^{rcv}-$$
$$\bar{\bar{\lambda}}_{v}^{rcv}\sum_{t=1}^{T}lab_{\bar{v}t}^{rcv}\leq 0;\qquad \forall v,\bar{v};v\neq\bar{v} \tag{2.91}$$

$$\sum_{i=1}^{I}\mu_{yi}^{rcy}[\sum_{k=1}^{K}\sum_{t=1}^{T}YTM_{\bar{y}ikt}+\sum_{m=1}^{M}\sum_{t=1}^{T}YTC_{\bar{y}imt}]+\sum_{i=1}^{I}\bar{\mu}_{yi}^{rcy}[\bar{w}_i-\sum_{p=1}^{P}\sum_{t=1}^{T}YTP_{\bar{y}ipt}]-$$
$$\sum_{j=1}^{J}\lambda_{yj}^{rcy}[\sum_{t=1}^{T}\overline{LOI}_{\bar{y}j,t-1}^{rcy}+\sum_{k=1}^{K}\sum_{t=1}^{T}MTY_{kj\bar{y}t}+\sum_{l=1}^{L}\sum_{t=1}^{T}CTY_{lj\bar{y}t}]- \tag{2.92}$$
$$\bar{\lambda}_{y}^{rcy}\sum_{t=1}^{T}cux_{\bar{y}t}^{rcy}-\bar{\bar{\lambda}}_{y}^{rcy}\sum_{t=1}^{T}lab_{\bar{y}t}^{rcy}\leq 0;\qquad \forall y,\bar{y};y\neq\bar{y}$$

$$\eta-d_k^{man}\geq 0;\qquad \forall k \tag{2.93}$$

$$\eta-d_d^{dis}\geq 0;\qquad \forall d \tag{2.94}$$

$$\eta-d_l^{col}\geq 0;\qquad \forall l \tag{2.95}$$

$$\eta-d_v^{rcv}\geq 0;\qquad \forall v \tag{2.96}$$

$$\eta - d_y^{rcy} \geq 0; \qquad \forall y \tag{2.97}$$

No inefficiency score is defined for an unopened facility. The current condition is satisfied by constraints (2.98)-(2.102).

$$d_k^{man} \leq X_k^{man} M0; \qquad \forall k \tag{2.98}$$

$$d_d^{dis} \leq X_d^{dis} M0; \qquad \forall d \tag{2.99}$$

$$d_l^{col} \leq X_l^{col} M0; \qquad \forall l \tag{2.100}$$

$$d_v^{rcv} \leq X_v^{rcv} M0; \qquad \forall v \tag{2.101}$$

$$d_y^{rcy} \leq X_y^{rcy} M0; \qquad \forall y \tag{2.102}$$

Decision variables constraints: Constraint (2.103) requires all mass flow, inventory and production levels decision variables to be continuous. That DEA weights decision variables are greater than or equal to positive small value is denoted by constraints (2.104)-(2.108). Moreover, constraint (2.109) defines location decision variables as binary.

$$\begin{aligned} &MTD_{kjdt}, MTY_{kjyt}, DTC_{djct}, CTC_{cjlt}, CTV_{ljvt}, CTY_{ljyt}, VTD_{vjdt}, YTP_{yipt}, YTM_{yikt}, \\ &YTC_{yimt}, \overline{LOI}_{kit}^{man}, LOI_{kjt}^{man}, LOI_{djt}^{dis}, LOI_{ljt}^{col-y}, LOI_{ljt}^{col-v}, LOI_{vjt}^{rcv}, \overline{LOI}_{yjt}^{rcy}, LOI_{yit}^{rcy}, \\ &PLP_{kjt}^{man}, PLP_{yit}^{rcy}, d_k^{man}, d_d^{dis}, d_l^{col}, d_v^{rcv}, d_y^{rcy} \geq 0; \qquad \forall i,j,k,d,c,l,v,y,p,m,t \end{aligned} \tag{2.103}$$

$$\lambda_{ki}^{man}, \bar{\lambda}_k^{man}, \bar{\bar{\lambda}}_k^{man}, \mu_{kj}^{man}, \bar{\mu}_{kj}^{man} \geq \varepsilon X_k^{man}; \qquad \forall i,j,k \tag{2.104}$$

$$\lambda_{dj}^{dis}, \bar{\lambda}_d^{dis}, \bar{\bar{\lambda}}_d^{dis}, \mu_{dj}^{dis} \geq \varepsilon X_d^{dis}; \qquad \forall j,d \tag{2.105}$$

$$\lambda_{lj}^{col-y}, \lambda_{lj}^{col-v}, \bar{\lambda}_l^{col}, \bar{\bar{\lambda}}_l^{col}, \mu_{lj}^{col-v}, \mu_{lj}^{col-y} \geq \varepsilon X_l^{col}; \qquad \forall j,l \tag{2.106}$$

$$\lambda_{vj}^{rcv}, \bar{\lambda}_v^{rcv}, \bar{\bar{\lambda}}_v^{rcv}, \mu_{vj}^{rcv} \geq \varepsilon X_v^{rcv}; \qquad \forall j,v \tag{2.107}$$

$$\lambda_{yj}^{rcy}, \bar{\lambda}_y^{rcy}, \bar{\bar{\lambda}}_y^{rcy}, \mu_{yi}^{rcy}, \bar{\mu}_{yi}^{rcy} \geq \varepsilon X_y^{rcy}; \qquad \forall i,j,y \tag{2.108}$$

$$X_k^{man}, X_d^{dis}, X_l^{col}, X_v^{rcv}, X_y^{rcy}, X_p^{dip} \in \{0,1\}; \qquad \forall k,d,l,v,y,p \tag{2.109}$$

2.5 Solution procedure

The programming model developed in the preceding section is a type of MINLP problem in which nonlinearity is due to the presence of bilinear terms in the DEA efficiency constraints (2.78)-(2.92). In the current constraints, a bilinear term is formed by multiplying either a mass flow decision variable or inventory level decision variable by its respective DEA weight. This makes the proposed programming problem nonconvex, for which a global optimum solution is not guaranteed to attain using the ordinary optimization methods. In order to deal with

the non-convexity issue, we relax the bilinear terms by means of the piecewise McCormick envelopes approach. Since relaxing the bilinear terms by the latter approach requires adding some extra mixed integer linear constraints to the original MINLP problem, we introduce a set of valid inequalities to improve the performance of the proposed solution procedure in terms of its solution time. Another issue to encounter is the presence of the multiple objective functions in the proposed programming problem. To tackle the current concern, we use a fuzzy programming approach in which a weighted sum fuzzy aggregation function is applied to derive a single objective function.

Therefore, the steps of solution procedure are summarized as follows:

- In the first step, the bilinear terms in the DEA efficiency constraints are relaxed and subsequently a multi-objective mixed integer *linear* programming (MILP) model is derived.
- Then, a set of valid inequalities are added to the current MILP model to improve the solution time.
- Finally, a fuzzy programming approach is used to attain a single objective MILP model.

2.5.1 Bilinear relaxation by Piecewise McCormick envelopes

Various relaxation strategies have been developed in the literature to deal with bilinear terms, including piecewise McCormick envelopes, also known as convex hull, (McCormick 1976, Bergamini et al. 2005), outer approximation (Floudas 1995), Eigen vector projections (Cambini and Sodini 2005), semi-definiteness condition (Anstreicher 2009), piecewise linear under estimators (Karuppiah and Grossmann 2006) and etc., among which piecewise McCormick envelopes methodology is a very frequently applied approach with satisfactory results (Castro and Teles 2013).

One can apply McCormick (1976) envelopes to obtain different types of relaxations to a bilinear term. In the standard approach, a bilinear term involving variables X_i and X_j is replaced with a new variable $W_{ij} = X_i X_j$ to derive a linear programming relaxation. However, uniformly partitioning the domain of one of the variables (X_j) of the bilinear term into N disjoint regions results in a tighter MILP relaxation, in which new binary variables are added to the formulation to select the optimal partition for X_j. This causes a linear increase in problem size with the number of partitions (Castro 2015).

Let $x_i^L \leq X_i \leq x_i^U$ and $x_j^L \leq X_j \leq x_j^U$ denote the bounds on the variables involved in the bilinear term. As stated before, we uniformly partition X_j into N regions, where x_{jn}^L and x_{jn}^U represent the upper and lower bounds of X_j in partition $n \in \{1, ..., N\}$, respectively. If the value of X_j lies within such partition, then binary variable $Y_{jn} = 1$ and the McCormick envelopes hold. In this case, we relax the bilinear term $W_{ij} = X_i X_j$ by adding the following mixed integer linear constraints (Castro 2015):

$$W_{ij} \geq \sum_{n=1}^{N} (\hat{X}_{ijn} x_{jn}^{L} + x_i^{L} \hat{X}_{jn} - x_i^{L} x_{jn}^{L} Q_{jn}) \tag{2.110}$$

$$W_{ij} \geq \sum_{n=1}^{N} (\hat{X}_{ijn} x_{jn}^{U} + x_i^{U} \hat{X}_{jn} - x_i^{U} x_{jn}^{U} Q_{jn}) \tag{2.111}$$

$$W_{ij} \leq \sum_{n=1}^{N} (\hat{X}_{ijn} x_{jn}^{L} + x_i^{U} \hat{X}_{jn} - x_i^{U} x_{jn}^{L} Q_{jn}) \tag{2.112}$$

$$W_{ij} \leq \sum_{n=1}^{N} (\hat{X}_{ijn} x_{jn}^{U} + x_i^{L} \hat{X}_{jn} - x_i^{L} x_{jn}^{U} Q_{jn}) \tag{2.113}$$

$$X_i = \sum_{n=1}^{N} \hat{X}_{ijn} \tag{2.114}$$

$$X_j = \sum_{n=1}^{N} \hat{X}_{jn} \tag{2.115}$$

$$\sum_{n=1}^{N} Q_{jn} = 1 \tag{2.116}$$

$$x_i^{L} Q_{jn} \leq \hat{X}_{ijn} \leq x_i^{U} Q_{jn} \tag{2.117}$$

$$x_{jn}^{L} Q_{jn} \leq \hat{X}_{jn} \leq x_j^{U} Q_{jn} \tag{2.118}$$

$$Q_{jn} \in \{0, 1\} \tag{2.119}$$

where, we define:

$$x_{jn}^{L} = x_j^{L} + \frac{(x_j^{U} - x_j^{L}) n(n-1)}{N} \tag{2.120}$$

$$x_{jn}^{U} = x_j^{L} + \frac{(x_j^{U} - x_j^{L}) nn}{N} \tag{2.121}$$

Relaxation of bilinear terms using piecewise McCormick envelopes provides a lower (upper) bound for the original MINLP problem in which the objective function is maximized (minimized). However, the following global optimization algorithm adapted from Kolodziej et al. (2013) assists with obtaining global optimum solution of the original MINLP problem:

PCMGO Algorithm

Step 1: Set $N = 1$.

Step 2: Obtain an upper bound z^{U} for the MINLP problem using a local solver.

Step 3: Solve the relaxed MILP problem to obtain a lower bound z^{L} for the MINLP prob-

lem.

Step 4: If $(z^U - z^L)/z^L \leq \varepsilon$ or the relaxed MILP problem is integer infeasible; stop. Otherwise, set $N = \Delta + N$, where Δ is a positive step size to increase the accuracy of partitioned variable x_j, and return to step 3.

Relaxing the bilinear terms of the proposed programming problem requires determining the variables on which partitioning is performed. The quality of the relaxation is influenced by the choice of variable selected for partitioning. In this study, we select mass flows and inventory levels decision variables to be partitioned mainly because we are able to derive their upper and lower bounds either by intuition or using mathematics of the proposed programming problem. While the lower bound of all mass flows and inventory levels decision variables is equal to zero, Table 2.3 presents their upper bounds:

For instance, YTM_{yikt} in period 1 cannot be intuitively greater than the inventory level of material i in recycling center y at the beginning of period 1, i.e. iil_{yi}^{rcy}, plus the production level of this material in recycling center y, which itself has an upper bound of pca_{yi1}^{rcy}. Therefore, we derive an upper bound for YTM_{yikt} in period 1 as $iil_{yi}^{rcy} + pca_{yi1}^{rcy}$. However, there might be other upper bounds for partitioned decision variables that could have been obtained by more complicated arithmetic. The upper bounds given in Table 2.3 are ones that are easily accessible either by intuition or from constraints of developed programming problem. Using these upper bounds, we relax the bilinear terms in the DEA efficiency constraints (2.78)-(2.92). For instance, $\lambda_{ki}^{man} YTM_{yikt}$ in constraint (2.83) is relaxed by including the following mixed integer linear constraints:

$$W_{yikt}^{YTM} \geq \sum_{n=1}^{N} \hat{\lambda}_{kin}^{man} YTM_{yiktn}^{L}; \qquad \forall y, i, k, t$$

$$W_{yikt}^{YTM} \geq \sum_{n=1}^{N} (\hat{\lambda}_{kin}^{man} YTM_{yiktn}^{U} + \lambda_{ki}^{man-U} Y\hat{T}M_{yiktn} - \lambda_{ki}^{man-U} YTM_{yiktn}^{U} Y_{yiktn}^{man}); \qquad \forall y, i, k, t$$

$$W_{yikt}^{YTM} \leq \sum_{n=1}^{N} (\hat{\lambda}_{kin}^{man} YTM_{yiktn}^{L} + \lambda_{ki}^{man-U} Y\hat{T}M_{yiktn} - \lambda_{ki}^{man-U} YTM_{yiktn}^{L} Y_{yiktn}^{man}); \qquad \forall y, i, k, t$$

$$W_{yikt}^{YTM} \leq \sum_{n=1}^{N} \hat{\lambda}_{kin}^{man} YTM_{yiktn}^{U}; \qquad \forall y, i, k, t$$

$$\lambda_{ki}^{man} = \sum_{n=1}^{N} \hat{\lambda}_{kin}^{man}; \qquad \forall k, i$$

Table 2.3: Upper bounds of mass flow decision variables.

Decision variable	Upper bound
YTM_{yikt}	$iil_{yi}^{rcy} + pca_{yi1}^{rcy}; t = 1, \quad hca_{yi,t-1}^{rcy} + pca_{yit}^{rcy}; t \geq 2$
MTD_{kjdt}	$iil_{kj}^{man} + pca_{kj1}^{man}; t = 1, \quad hca_{kj,t-1}^{man} + pca_{kjt}^{man}; t \geq 2$
DTC_{djct}	$iil_{dj}^{dis} + \underset{k}{max}(iil_{kj}^{man} + pca_{kj1}^{man}) + \underset{v}{max}(iil_{vj}^{rcv} + pca_{vj1}^{rcv}); t = 1, \quad hca_{dj,t-1}^{dis} + \underset{k}{max}(hca_{kj,t-1}^{man} + pca_{kjt}^{man}) + \underset{v}{max}(iil_{vj,t-1}^{rcv} + pca_{vjt}^{rcv}); t \geq 2$
CTC_{cjlt}	pca_{ljt}^{col}
CTV_{ljvt}	pca_{vjt}^{rcv}
CTY_{ljyt}	$iil_{lj}^{col-y} + pca_{lj1}^{col}; t = 1, \quad hca_{lj,t-1}^{col-y} + pca_{ljt}^{col}; t \geq 2$
VTD_{vjdt}	$iil_{vj}^{rcv} + pca_{vj1}^{rcv}; t = 1, \quad hca_{vj,t-1}^{rcv} + pca_{vjt}^{rcv}; t \geq 2$
YTP_{yipt}	$iil_{yi}^{rcy} + pca_{yi1}^{rcy}; t = 1, \quad hca_{yi,t-1}^{rcy} + pca_{yit}^{rcy}; t \geq 2$
YTC_{yict}	$iil_{yi}^{rcy} + pca_{yi1}^{rcy}; t = 1, \quad hca_{yi,t-1}^{rcy} + pca_{yit}^{rcy}; t \geq 2$
$\overline{LOI}_{kit}^{man}$	$\overline{hca}_{kit}^{man}$
LOI_{kjt}^{man}	hca_{kjt}^{man}
LOI_{djt}^{dis}	hca_{djt}^{dis}
LOI_{ljt}^{col-y}	hca_{ljt}^{col-y}
LOI_{ljt}^{col-v}	hca_{ljt}^{col-v}
LOI_{vjt}^{rcv}	hca_{vjt}^{rcv}
$\overline{LOI}_{yjt}^{rcy}$	$\overline{hca}_{yjt}^{rcy}$
LOI_{yit}^{rcy}	hca_{yit}^{rcy}

$$YTM_{yikt} = \sum_{n=1}^{N} Y\hat{T}M_{yiktn}; \qquad \forall y, i, k, t$$

$$YTM_{yiktn}^{L} Y_{yiktn}^{man} \leq Y\hat{T}M_{yiktn} \leq YTM_{yiktn}^{U} Y_{yiktn}^{man}; \qquad \forall y, i, k, t, n$$

$$0 \leq \hat{\lambda}^{man}_{kin} \leq \lambda^{man-U}_{ki} Y^{man}_{yiktn}; \qquad \forall y, i, k, t, n$$

$$\sum_{n=1}^{N} Y^{man}_{yiktn} = 1; \qquad \forall y, i, k, t$$

$$Y^{man}_{yiktn} \in \{0, 1\}$$

where, λ^{man-U}_{ki} denotes an upper bound for λ^{man}_{ki}. In this case, the bilinear term $\lambda^{man}_{ki} YTM_{yikt}$ in constraint (2.83) is replaced by W^{YTM}_{yikt}. It should also be noted that we derive the upper and lower bounds of the partitioned decision variable, i.e. YTM_{yikt}, in each disjoint partition n as follows:

$$YTM^{U}_{yiktn} = \frac{nn(iil^{rcy}_{yi} + pca^{rcy}_{yit})}{N}; \qquad \forall y, i, k, t = 1, n \in \{1...N\}$$

$$YTM^{U}_{yiktn} = \frac{nn(hca^{rcy}_{yi,t-1} + pca^{rcy}_{yit})}{N}; \qquad \forall y, i, k, t \geq 2, n \in \{1...N\}$$

$$YTM^{L}_{yiktn} = \frac{n(n-1)(iil^{rcy}_{yi} + pca^{rcy}_{yit})}{N}; \qquad \forall y, i, k, t = 1, n \in \{1...N\}$$

$$YTM^{L}_{yiktn} = \frac{n(n-1)(hca^{rcy}_{yi,t-1} + pca^{rcy}_{yit})}{N}; \qquad \forall y, i, k, t \geq 2, n \in \{1...N\}$$

2.5.2 Improving the solution procedure using valid inequalities

Relaxing the bilinear terms by piecewise McCormick envelopes requires adding extra mixed integer linear constraints to the original programming problem. Consequently, the solution time of the problem might increase to some degree. In order to accelerate the global optimization algorithm presented in the former section, we introduce a set of valid inequalities to add to the relaxed MILP model.

Definition 2.1: Consider constraint (2.65) for the number of opened manufacturing centers. Let $\bar{N} = \{1, ..., K\}$. A set $\dot{K} \subseteq \bar{N}$ is a *cover* if $mxn^{man} < |\dot{K}|$. For the remaining constraints on the number of opened facilities, the corresponding covers are derived in the same way.

Proposition 2.1: If $\dot{K} \subseteq \bar{N}$ is a cover, the cover inequality $\sum_{k \in \dot{K}} X^{man}_{k} \leq |\dot{K}| - 1$ is valid for constraint (2.65).

Proof: The proof is straightforward based on Wolsey (1998); page: 147.

Proposition 2.2: If $\dot{K} \subseteq \bar{N}$ is a cover, then the valid inequality

$$\sum_{k \in \dot{K}} X^{man}_{k} + \sum_{k \notin \dot{K}} \theta^{man}_{k} X^{man}_{k} \leq |\dot{K}| - 1 \tag{2.122}$$

is non-redundant (facet-defining) and thus as strong as possible, where

$$\theta_k^{man} = |\dot{K}| - mxn^{man}; \forall k \notin \dot{K}$$

.

Proof: Following the procedure to obtain a lifted cover inequality by Wolsey (1998); page: 149, the proof is straightforward.

The same procedure is employed to attain the corresponding strong valid inequalities for constraints (2.66) to (2.70), as follows:

$$\sum_{d \in \dot{D}} X_d^{dis} + \sum_{d \notin \dot{D}} \theta_d^{dis} X_d^{dis} \leq |\dot{D}| - 1 \tag{2.123}$$

$$\sum_{l \in \dot{L}} X_l^{col} + \sum_{l \notin \dot{L}} \theta_l^{col} X_l^{col} \leq |\dot{L}| - 1 \tag{2.124}$$

$$\sum_{v \in \dot{V}} X_v^{rcv} + \sum_{v \notin \dot{V}} \theta_v^{rcv} X_v^{rcv} \leq |\dot{V}| - 1 \tag{2.125}$$

$$\sum_{y \in \dot{Y}} X_y^{rcy} + \sum_{y \notin \dot{Y}} \theta_y^{rcy} X_y^{rcy} \leq |\dot{Y}| - 1 \tag{2.126}$$

$$\sum_{p \in \dot{P}} X_p^{dip} + \sum_{p \notin \dot{P}} \theta_p^{dip} X_p^{dip} \leq |\dot{P}| - 1 \tag{2.127}$$

where the sets $\dot{D}$, $\dot{L}$, $\dot{V}$, $\dot{Y}$ and $\dot{P}$ are covers and $\theta_d^{dis} = |\dot{D}| - mxn^{dis}; \forall d \notin \dot{D}$, $\theta_l^{col} = |\dot{L}| - mxn^{col}; \forall l \notin \dot{L}$, $\theta_v^{rcv} = |\dot{V}| - mxn^{rcv}; \forall v \notin \dot{V}$, $\theta_y^{rcy} = |\dot{Y}| - mxn^{rcy}; \forall y \notin \dot{Y}$ and $\theta_p^{dip} = |\dot{P}| - mxn^{dip}; \forall p \notin \dot{P}$.

In addition, constraints (2.128) and (2.129) imply that the selected collection and recycling centers provide sufficient capacity to process end-of-life products and recycled materials, respectively.

$$\sum_{c=1}^{C} dem_{cjt}^{pro} \phi_j \leq \sum_{l=1}^{L} pca_{ljt}^{col} X_j^{col}; \qquad \forall j, t \tag{2.128}$$

$$\sum_{m=1}^{M} dem_{imt}^{mat} \leq \sum_{y=1}^{Y} pca_{yit}^{rcy} X_y^{rcy}; \qquad \forall i, t \tag{2.129}$$

2.5.3 Fuzzy multi-objective programming

Fuzzy mathematical programming is an effective approach to derive a Pareto-optimal solution for a multi-objective optimization problem, in which each objective function is modeled as a fuzzy set. The membership function of the latter fuzzy set characterizes the degree of satisfaction

of its corresponding objective function. Zimmermann (1978) firstly introduced the maxmin operator to aggregate the fuzzy objective functions for making a compromise decision. However, Zimmermann's approach has a couple of shortcomings; *i*) a Pareto-optimal solution is not guaranteed to obtain, and *ii*) it does not allow for compensation among objective functions. In order to overcome these pitfalls, we exploit a posteriori method based on a weighted sum fuzzy aggregation function proposed by Pishvaee et al. (2014), which ensures the achievement of Pareto-optimal solutions.

In order to implement the approach developed by Pishvaee et al. (2014), we proceed as follows:

Step 1: Obtain the global optimum solution of each objective function over the constraints of the relaxed MILP problem by the means of global optimization algorithm in section 2.5.1. Let $\mathbb{X} = \{X^*_{TP}, X^*_{TE}, X^*_{SR}, X^*_{ES_1}, X^*_{ES_2}\}$, where X^*_{TP}, X^*_{TE}, X^*_{SR}, $X^*_{ES_1}$ and $X^*_{ES_1}$ are the obtained global optimum solutions corresponding to *TP*, *TE*, *SR*, ES_1 and ES_2 objective functions.

Step 2: Find the values of the objective functions *TP*, *TE*, *SR*, ES_1 and ES_2 at each point of $\mathbb{X}$.

Step 3: Derive the minimum value of objective functions as $TP^{min} = min\{TP(x), x \in \mathbb{X}\}$, $TE^{min} = min\{TE(x), x \in \mathbb{X}\}$, $SR^{min} = min\{SR(x), x \in \mathbb{X}\}$, $ES_1^{min} = min\{SE_1(x), x \in \mathbb{X}\}$ and $ES_2^{min} = min\{SE_2(x), x \in \mathbb{X}\}$, and the maximum value of them as follows: $TP^{max} = max\{TP(x), x \in \mathbb{X}\}$, $TE^{max} = max\{TE(x), x \in \mathbb{X}\}$, $SR^{max} = max\{SR(x), x \in \mathbb{X}\}$, $ES_1^{max} = max\{SE_1(x), x \in \mathbb{X}\}$ and $ES_2^{max} = max\{SE_2(x), x \in \mathbb{X}\}$.

Step 4: Construct the membership function for each objective function as follows:

$$\vartheta^{TP}(x) = \begin{cases} 1; & TP > TP^{max} \\ \dfrac{TP - TP^{min}}{TP^{max} - TP^{min}}; & TP^{min} \leq TP \leq TP^{max} \\ 0; & TP < TP^{min} \end{cases} \tag{2.130}$$

$$\vartheta^{TE}(x) = \begin{cases} 1; & TE < TE^{min} \\ \dfrac{TE^{max} - TE}{TE^{max} - TE^{min}}; & TE^{min} \leq TE \leq TE^{max} \\ 0; & TE > TE^{max} \end{cases} \tag{2.131}$$

$$\vartheta^{SR}(x) = \begin{cases} 1; & SR > SR^{max} \\ \dfrac{SR - SR^{min}}{SR^{max} - SR^{min}}; & SR^{min} \leq SR \leq SR^{max} \\ 0; & SR < SR^{min} \end{cases} \tag{2.132}$$

$$\vartheta^{ES_1}(x) = \begin{cases} 1; & ES_1 < ES_1^{min} \\ \dfrac{ES_1^{max} - ES_1}{ES_1^{max} - ES_1^{min}}; & ES_1^{min} \leq ES_1 \leq ES_1^{max} \\ 0; & ES_1 > ES_1^{max} \end{cases} \tag{2.133}$$

$$\vartheta^{ES_2}(x) = \begin{cases} 1; & ES_2 < ES_2^{min} \\ \dfrac{ES_2^{max} - ES_2}{ES_2^{max} - ES_2^{min}}; & ES_2^{min} \leq ES_2 \leq ES_2^{max} \\ 0; & ES_2 > ES_2^{max} \end{cases} \tag{2.134}$$

In the attained relaxed MILP model, total profit and social responsibility objective functions are to be maximized. In this case, the decision maker's satisfaction increases as the solution moves toward upper bound of these objective functions. Therefore, the decision maker is completely satisfied if these two objective functions reach to their upper bounds. On the other hand, all remained objective functions are to be minimized. Under this condition, the level of satisfaction of the decision maker increases if the obtained solution approaches the lower bound of each objective function. In this case, the decision maker is fully satisfied if the objective functions reach to their respective lower bounds.

Step 5: Aggregate the above-mentioned membership functions to formulate the following single objective programming model:

$$\begin{aligned} Max \quad & \omega^{TP}\vartheta^{TP}(x) + \omega^{TE}\vartheta^{TE}(x) + \omega^{SR}\vartheta^{SR}(x) + \omega^{ES_1}\vartheta^{ES_1}(x) + \omega^{ES_2}\vartheta^{ES_2}(x) \\ s.t. \quad & x \in \mathbb{A} \end{aligned} \tag{2.135}$$

where $\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}$ and ω^{ES_2} are the weights (relative importance) of the corresponding objective functions such that $\omega^{TP}+\omega^{TE}+\omega^{SR}+\omega^{ES_1}+\omega^{ES_2} = 1$. Furthermore, $\mathbb{A}$ characterizes the set of all constraints in the obtained relaxed MILP model. We can now solve the single objective MILP model (2.135) by an ordinary optimization approach such as branch-and-bound algorithm.

2.6 Computational experiment

In this section, we explain how to apply the proposed model and its solution procedure on a test problem whose dimension is given in Table 2.4. According to the list of indexes, the capital letters given in this table refer to the magnitude of the presented problem.

We illustrate the influence of incorporating DEA efficiency on the sustainable closed loop supply chain depicted in Figure 2.1. Particularly, we examine how strategic and tactical decisions through the network are effected by incorporating DEA efficiency. Although we do not intend to solve a real problem of a particular company in the test problem, all parameters are defined

Table 2.4: Dimension of the studied test problem.

I	J	K	D	C	L	V	Y	P	M	T
4	2	5	4	5	5	3	2	4	6	2

in a realistic order of magnitude for a small company.

2.6.1 Data acquisition

Without loss of generality, we generate the precise quantity of each parameter by Monte Carlo sampling method, which is implemented on a particular uniform distribution function with predetermined interval. Such uniform distribution functions are defined in such a way that the proposed model becomes feasible and the obtained results sound reasonable. In this regard, if $U \sim (a, b)$ denotes a uniform random variable in the interval (a, b), then the demands of products and recycled materials for customers and materials customers are defined as $U \sim (46, 69)$ tons/year and $U \sim (3, 7)$ tons/year, respectively. Furthermore, defining production/processing and holding capacities of all facilities as $U \sim (75, 85)$ tons/year and $U \sim (36, 44)$ tons/year, respectively, guarantees the feasibility of the proposed model. Table 2.5 presents the uniform distribution functions that are used to generate the quantity of the remaining parameters. It should be noted here that all other initial inventory levels are set to zero. In addition, we set $mxn^{man} = 5$, $mxn^{dis} = 4$, $mxn^{col} = 5$, $mxn^{rcv} = 3$, $mxn^{rcy} = 5$ and $mxn^{dip} = 5$, $w_j = 5000; \forall j$, $\bar{w}_j = 5000; \forall j$, $\xi = 0.3$, $\bar{\xi} = 0.7$ and all fixed opening costs to 4,000. Due to space limitation, we do not present the generated values by Monte Carlo sampling method in the paper. They are accessible for any interested readers at supplementary materials.

2.6.2 Results

As the first step in the proposed solution procedure, all bilinear terms in DEA constraints (2.78)-(2.92) need to be relaxed by Piecewise McCormick envelopes. We presented how to implement such a relaxation on the bilinear term $\lambda_{ki}^{man} YTM_{yikt}$ before. In addition, according to the definition of a cover inequality, we can derive a strong valid inequality for constraint (2.70) as inequality (2.127). In this regard, we define a set of cover inequalities associated with constraint (2.70) as $X_1^{dip} + X_2^{dip} + X_3^{dip} \leq 2$, $X_1^{dip} + X_2^{dip} + X_4^{dip} \leq 2$, $X_1^{dip} + X_3^{dip} + X_4^{dip} \leq 2$ and $X_2^{dip} + X_3^{dip} + X_4^{dip} \leq 2$. Hence, strong valid inequality for constraint (2.70) are as follows:

$$X_1^{dip} + X_2^{dip} + X_3^{dip} + 2X_4^{dip} \leq 2$$

Table 2.5: Characterization of various parameters in small size test problem.

parameter	value	parameter	value
All transportation costs	$U \sim (10, 12)$	lab_{lt}^{col}	$U \sim (18, 28)$
All processing/ production costs	$U \sim (5.2, 5.5)$	lab_{vt}^{rcv}	$U \sim (30, 42)$
All holding costs	$U \sim (3.1, 3.4)$	lab_{yt}^{rcy}	$U \sim (45, 125)$
All transportation emission	$U \sim (0.075, 0.08)$	lab_{pt}^{dip}	$U \sim (18, 28)$
All processing/ production emissions	$U \sim (0.005, 0.0053)$	spr_{jct}^{pro}	$U \sim (195, 207)$
All holding emissions	$U \sim (0.0028, 0.003)$	spr_{imt}^{mat}	$U \sim (31, 43)$
All lost days	$U \sim (0.2, 0.6)$	ppo_{jt}	$U \sim (110, 128)$
iil_{kj}^{man}	$U \sim (45, 75)$	α_{ij}	$U \sim (0.48, 0.55)$
$\overline{iil}_{ki}^{man}$	$U \sim (22, 38)$	δ_{ji}	$U \sim (0.18, 0.25)$
$\overline{iil}_{yj}^{rcy}$	$U \sim (18, 30)$	β_j	$U \sim (0.04, 0.06)$
cux_{kt}^{man}	$U \sim (2, 2.75) \times 10^5$	ϕ_j	$U \sim (0.35, 0.38)$
cux_{dt}^{dis}	$U \sim (2, 3) \times 10^5$	τ_i	$U \sim (0.35, 0.42)$
cux_{lt}^{col}	$U \sim (5, 12.5) \times 10^4$	ψ_j	$U \sim (0.45, 0.55)$
cux_{vt}^{rcv}	$U \sim (1.6, 2.7) \times 10^5$	cup_{cjt}	$U \sim (10.3, 14.9)$
cux_{yt}^{rcy}	$U \sim (4, 5.1) \times 10^5$	pcs_{cjt}^{cus}	$U \sim (45.6, 58.1)$
lab_{kt}^{man}	$U \sim (80, 180)$	pcs_{imt}^{mat}	$U \sim (18.3, 25.7)$
lab_{dt}^{dis}	$U \sim (30, 50)$		

$$X_1^{dip} + X_2^{dip} + 2X_3^{dip} + X_4^{dip} \leq 2$$

$$X_1^{dip} + 2X_2^{dip} + X_3^{dip} + X_4^{dip} \leq 2$$

$$2X_1^{dip} + X_2^{dip} + X_3^{dip} + X_4^{dip} \leq 2$$

Using all defined data and the derived strong valid inequalities, we optimize each of five objective functions in the relaxed MILP model by following the global optimization algorithm presented in section 6.1. In this regard, we first program the original MINLP model in GAMS

24.1.2 and use BARON solver on a computer with CPU of 2.5 GHz and 6.0 GB RAM to derive a *local* optimum solution of each objective function. For TP and SR objective functions, such a local optimum solution provides a lower bound, while it is an upper bound for TE, ES_1 and ES_2 objective functions. Furthermore, we define $\Delta = 1$ as the step size to increase the accuracy of partitioned decision variables and $\varepsilon = 10^{-3}$ as the maximum acceptable optimality gap. In this case, Table 2.6 presents the global optimum solution of each objective function alongside its corresponding number of partitions used for algorithm convergence. It also presents the local optimum solution of each objective function. We utilize GAMS 24.1.2 and use CPLEX solver to solve these five relaxed MILP models.

In order to derive the single objective function based on the step 3 of fuzzy multi-objective programming, Table 2.6 also presents both minimum and maximum values of each objective function.

Table 2.6: Solution report of each objective function in the computational problem.

Obj.	MINLP model		Relaxed MINLP model			Optimality	Minimum	Maximum
Fun.	Bound type (value)	CPU time (sec)	Optimum value	CPU time (sec)	N	gap	value	value
TP	Lower (83,682.01)	33.80	83,753.2	24.50	5	0.00085	25,870.67	83,753.2
TE	Upper (201.16)	29.37	201.01	23.94	5	0.00073	201.01	261.12
SR	Lower (727.96)	24.74	728.17	16.13	5	0.00029	660.12	728.17
ES_1	Upper (0.2164)	29.31	0.2164	17.78	1	0	0.21635	1
ES_2	Upper (19.31)	46.52	19.31	39.94	1	0	19.31	30.11

Table 2.7 gives the dimensions of both original MINLP and relaxed MILP models. It appears that the dimension of original MINLP model is significantly smaller than that of relaxed MILP model. However, according to Table 2.6, the substantially large dimension of relaxed MILP model does not have a bad effect on solution time of each objective function. That is because the relaxed MILP model can be effectively solved using the available heuristics and that, as presented later, the valid inequalities are effective in reducing the solution time of the relaxed MILP model.

Using the minimum and maximum values of each objective function, we can construct the respective membership functions, which themselves are used to formulate the following single

Table 2.7: Dimensions of original MINLP and relaxed MILP models.

Model	N	# of integer variables	# of total variables	# of constraints
Original MINLP	-	23	1182	855
Relaxed MILP	1	23	3487	9825
	5	4103	11743	21153

objective (aggregated) programming model:

$$\begin{aligned} Max \quad & \omega^{TP}\left(\frac{TP - 25,870.65}{57,882.49}\right) + \omega^{TE}\left(\frac{261.12 - TE}{60.12}\right) + \omega^{SR}\left(\frac{SR - 660.12}{68.06}\right) + \omega^{ES_1}\left(\frac{1 - ES_1}{0.7837}\right) + \\ & \omega^{ES_2}\left(\frac{30.11 - ES_2}{10.81}\right) \\ s.t. \quad & x \in \mathbb{A} \end{aligned}$$

We again utilize GAMS 24.1.2 with CPLEX solver to solve the current single objective relaxed MILP model. In order to examine the effect of incorporation of DEA efficiency, we consider various values for ω^{TP}, ω^{TE}, ω^{SR}, ω^{ES_1} and ω^{ES_2}. Particularly, we evaluate three instances in which we set the importance weight of objective functions as follows:

1. $\omega_1 = (\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = ((1-2\varepsilon)/3, (1-2\varepsilon)/3, (1-2\varepsilon)/3, \varepsilon, \varepsilon)$. In this instance, we assign extremely small positive importance weights to DEA objective functions to ignore their effects on the obtained optimum solution. We notice that one should not assign zero to the importance weight of DEA objective functions in this instance because of assuring non-dominated solutions. However, since ε is a very small quantity that is close to zero, this instance appropriately represents a situation in which DEA efficiency score is not taken into account when planning the closed loop supply chain network..

2. $\omega_2 = (\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = (0.2, 0.2, 0.2, 0.2, 0.2)$. In the current instance, we consider all objective functions that are equally weighted.

3. $\omega_3 = (\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = (\varepsilon, \varepsilon, \varepsilon, (1-3\varepsilon)/2, (1-3\varepsilon)/2)$. In this instance, we set the importance weight of sustainability objective functions to a diminutive quantity. Hence, this instance properly models a situation in which we disregard the sustainability objective functions while assuring non-dominated solutions.

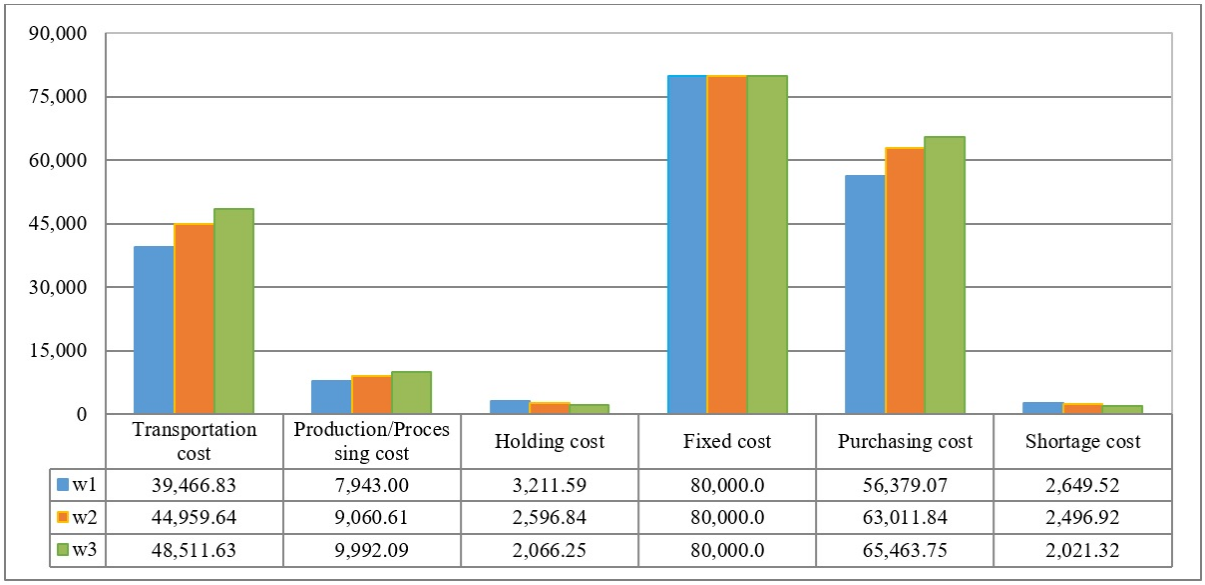
Table 2.8 presents the located facilities under each instance with their respective DEA efficiencies. It also gives the average efficiency score of each facility type in each instance. The bottom row of Table 2.8 shows the average DEA efficiency score of all facilities opened through

Table 2.8: The located facilities in each defined instance with their respective DEA efficiencies.

Facility	ω_1		ω_2		ω_3	
	Facility	Efficiency	Facility	Efficiency	Facility	Efficiency
Manufacturing centers	1	0.6893	1	0.8122	1	1
	2	0.5708	2	0.7095	2	1
	3	0.5708	3	0.6827	3	1
	4	0.7200	4	0.7412	4	1
	5	0.6155	5	0.8019	5	1
Avg. efficiencies		**0.63333**		**0.7495**		**1**
Distribution centers	1	0.9228	1	0.9272	1	0.9272
	2	1	2	1	2	1
	3	0.8191	3	0.8221	3	0.8221
	4	0.8833	4	0.9883	4	0.9883
Avg. efficiencies		**0.9063**		**0.9344**		**0.9344**
Collection centers	1	1	1	1	1	1
	2	1	2	1	2	1
	3	0.8997	3	0.8997	3	1
	4	1	4	1	4	1
	5	0.6802	5	0.7202	5	1
Avg. efficiencies		**0.9160**		**0.9240**		**1**
Recovery centers	1	1	1	1	1	1
	2	1	2	1	2	1
	3	1	3	1	3	1
Avg. efficiencies		**1**		**1**		**1**
Avg. efficiency of all facilities		0.7840		0.9020		0.9869

the network. It appears that as ω^{ES_1} and ω^{ES_2} increase in instances ω_2 and ω_3, the average efficiency score of each type of located facilities raises as well. Since ω_1 reflects a situation in which we do not take DEA efficiency into account when planning the closed loop supply chain network, we observe some facilities operating very inefficiently in this instance. Manufacturing centers with average DEA efficiency score of 0.6333 are the most inefficient facilities of the network in ω_1 instance. However, the located facilities reach their most efficient conditions in instance ω_3, where we maximize their respective DEA efficiency scores while disregarding total obtained profit, total generated CO2 emission and social responsibility. According to the bottom row of Table 2.8, it appears that the DEA efficiency score of the network, i.e. all located facilities, grows up by %25.9.

Figure 2.2 demonstrates cost components of total profit objective function (2.22). It turns out that total transportation cost increases when the importance weights ω^{ES_1} and ω^{ES_2} grow up in instances ω_2 and ω_3. It follows because the bi-objective DEA model (2.8)-(2.15) has output orientation. As stated earlier, to improve efficiency in an output-oriented DEA model, we increase the output levels of a DMU while retaining its input levels unchanged. Hence, we



	Transportation cost	Production/Processing cost	Holding cost	Fixed cost	Purchasing cost	Shortage cost
w1	39,466.83	7,943.00	3,211.59	80,000.0	56,379.07	2,649.52
w2	44,959.64	9,060.61	2,596.84	80,000.0	63,011.84	2,496.92
w3	48,511.63	9,992.09	2,066.25	80,000.0	65,463.75	2,021.32

Figure 2.2: The components of total profit objective function.

conclude the level of outgoing materials/products in located facilities increases as their DEA efficiency scores raise from instance ω_1 to instance ω_3 . Since the outgoing mass flows of a located facility constitute the incoming mass flows of some other facilities in the network, it turns out that overall level of throughputs passing through the network increases. Consequently, total transportation cost increases as the DEA efficiency score of opened facilities improves.

It appears in Figure 2.2 that the production/processing cost has an increasing trend when the efficiency of located facilities raises from instance ω_1 to instance ω_3. It follows because as the output level of a particular facility grows up in order to increase its respective efficiency, then there will be more raw materials/products to process in all other facilities that are connected to this specific facility in the network. This implies that the production/processing cost of such connected facilities goes up.

Improving the efficiency of a facility by rising its output level, i.e. outgoing materials/products, requires holding less inventory by that particular facility. In other words, the less inventory in a facility, the more outgoing materials/products sent out and consequently the greater efficiency score the facility has. Therefore, the inventory cost in the network reduces once the DEA efficiency of facilities improves from instance ω_1 to instance ω_3, as depicted in Figure 2.2.

Figures 2.2 illustrates that total fixed cost is unchanged when increasing DEA efficiency score of located facilities. Since the capacity of located facilities is sufficient to cover the amount of increased mass flows in instances ω_2 and ω_3, no new facility is opened through the network while increasing the DEA efficiency of pre-located facilities. However, generally, once the incoming

mass flows raise for a facility, it might reach either its production/processing or its holding capacity. In this case, the plethora amount of incoming mass flows should be transferred to another facility of the same type, which might result in opening a new facility. It implies that, under any circumstance, total fixed opening cost of the network is not decreasing while improving DEA efficiency of opened facilities.

The next cost component is the purchasing cost, which as demonstrated in Figure 2.2 has an increasing trend with improving the efficiency score of opened facilities. We incur the purchasing cost when buying the end-of-life products from customers in collection centers. According to constraint (2.75), the quantity of end-of-life products purchased from customers is a linear function of the quantity of products transported from the distribution centers to customers. Since the latter quantity grows up to increase the DEA efficiency score of distribution centers, we conclude the quantity of end-of-life products returned from customers increases as well. Accordingly, total purchasing cost of returned end-of-life products raises when increasing DEA efficiency score of distribution centers in instances ω_2 and ω_3.

The shortage cost has a decreasing trend as presented in Figure 2.2. It follows because increasing the quantity of products and recycled materials shipped from distribution and recycling centers, respectively, to improve their corresponding efficiencies in instances ω_2 and ω_3 results in having further products and recycled materials in customers and materials customers. Shipping more products and recycled materials implies customers and materials customers will have less shortage in their demands. This observation also follows from constraints (2.76) and (2.77), where the level of demand requirements remains unchanged in instances ω_2 and ω_3. In this case, increasing the level of DTC_{djct} and YTC_{yimt} leads to reducing STG^{cus}_{cjt} and STG^{mat}_{imt}.

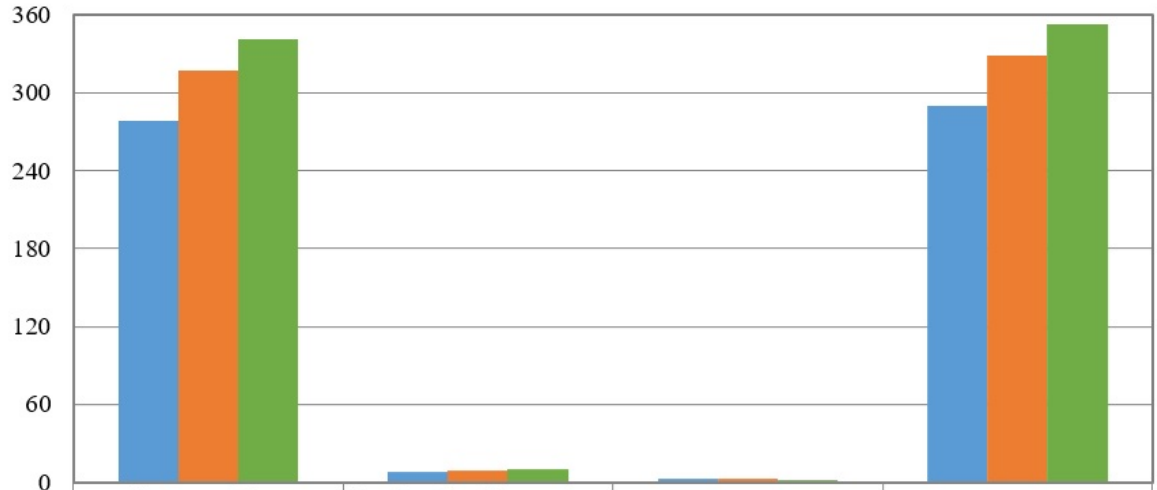
Table 2.9: Selling revenue and total profit in defined instances.

Instance	Selling revenue	Total profit
ω_1	244,340.24	54,690.23
ω_2	263,925.54	61,799.69
ω_3	283,933.32	75,878.28

Table 2.9 presents selling revenue and total profit in all three instances defined. In the closed loop network, we earn selling revenue by selling products and recycled materials to customers

and materials customers, respectively. As stated earlier, the quantity of products (recycled materials) shipped from distribution (recycling) centers to customers (materials customers) increases in order to improve the efficiency score of distribution (recycling) centers in instances ω_2 and ω_3. Consequently, the selling revenue obtained by selling products (recycled materials) to customers (materials customers) also grows up.

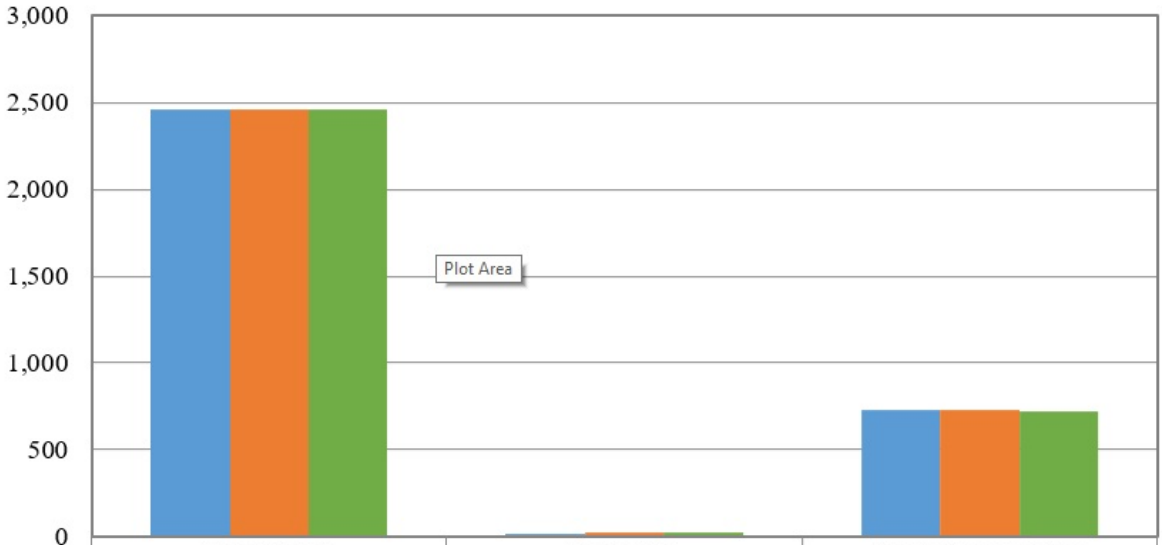
Table 2.9 also shows that the total profit generated in the network increases once the DEA efficiency score of opened facilities raises from instance ω_1 to instance ω_3. In other words, by improving the DEA efficiency of located facilities, we are able to make them more profitable. As a rationally expected implication, we observe that the more efficiency, the more profit we earn through the network.



	Transportation emission	Producstion/Processing emission	Holding emission	Total emission
w1	279.11	7.64	2.86	289.6
w2	317.49	8.70	2.32	328.5
w3	341.22	9.60	1.86	352.7

Figure 2.3: The components of total emission CO2 emission objective function.

Figure 2.3 illustrates the components of total CO2 emission objective function (2.26). Transportation emission has an increasing trend when improving DEA efficiency score of located facilities. It follows because, as explained for transportation cost, increasing DEA efficiency scores in instances ω_2 and ω_3 implies raising in total mass flows transported through the network, which itself results in growing up the transportation emission as demonstrated in Figure 2.3. In addition, as explained earlier for production/processing cost, the amount of raw materials/products



	Created jobs	Lost days	Social resposibility
w1	2,460.00	17.34	725.86
w2	2,460.00	19.25	724.52
w3	2,460.00	21.01	723.29

Figure 2.4: The components of total social responsibility objective function.

received by facilities in the network goes up in instances ω_2 and ω_3. Thus, they have further raw materials/products to process in these instances rather than instance ω_1, which results in more processing emission. Moreover, as concluded for holding cost, less inventory should be kept by facilities in order to improve their respective efficiencies. This leads to decreasing holding emission as presented in Figure 2.3. Finally, total CO2 emission of the network also raises when improving DEA efficiency score of facilities. Therefore, there exists a trade-off between total CO2 emission and DEA efficiency, which implies the more DEA efficiency, the more CO2 emission generated through the network.

Figure 2.4 demonstrates two components of social responsibility objective function (2.29) alongside its total value. As stated before, the located facilities in the network provide sufficient production/processing and holding capacities. Hence, no new facilities are needed to open in instances ω_2 and ω_3, which itself results in the unchanged number of jobs created in the latter instances.

As explained for production/processing cost, further raw materials/products are processed by facilities in instances ω_2 and ω_3 rather than instance ω_1. Accordingly, more damages might happen for labors under the former instances. In other words, the more production/processing level in a facility, the more damages occur for its labors.

Total social responsibility diminishes if DEA efficiency of located facilities increases, as illustrated in Figure 2.4. Hence, it turns out that there exists a trade-off between DEA efficiency and the social responsibility.

2.6.3 Performance of valid inequalities

The purpose of adding valid inequalities to the relaxed MILP model is to improve the solution procedure in terms of its solution time. Therefore, it is worth evaluating the performance of the added valid inequalities in achieving this target. In this study, we derive the strong valid inequality (2.127) as $X_1^{dip} + X_2^{dip} + X_3^{dip} + 2X_4^{dip} \leq 2$, $X_1^{dip} + X_2^{dip} + 2X_3^{dip} + X_4^{dip} \leq 2$, $X_1^{dip} + 2X_2^{dip} + X_3^{dip} + X_4^{dip} \leq 2$ and $2X_1^{dip} + X_2^{dip} + X_3^{dip} + X_4^{dip} \leq 2$ (cut I). Furthermore, we enforce the valid inequalities (2.128) and (2.129) (cut II) into the relaxed MILP model. In order to compare the performance of cuts in reducing the solution time of the relaxed MILP model, we implement a one-way analysis of variance (ANOVA) with Fisher's grouping method, in which we define four factors as "No Cut", "Cut I", "Cut II"and "Cuts I & II". For "No Cut"factor, we solve the relaxed MILP model when no cut, neither cut I nor cut II, is put into the model. However, for "Cut I"("Cut II") factor, we enforce only cut I (cut II) into the relaxed MILP model. For "Cuts I & II"factor, we apply both cut I and cut II into the model. Table 2.10 presents solution time mean and standard deviation once each of five objective functions and the aggregated single objective function in which each objective function is equally weighted is optimized for 30 times (sample size) in the relaxed MILP model.

Table 2.10: ANOVA result for evaluating various cuts performance.

Obg. fun.	No Cut		No Cut	Cut I			Cut II			Cuts I & II			p-value
	Mean	StDev	Group	Mean	StDev	Group	Mean	StDev	Group	Mean	StDev	Group	
TP	27.572	2.683	A	25.688	4.506	B	22.867	2.383	A	23.889	1.140	C	0.00
TE	30.989	9.129	A	25.688	4.506	B	22.709	2.356	C	21.338	1.627	C	0.00
SR	22.531	8.646	A	21.027	4.691	A	15.631	3.187	B	15.284	2.719	B	0.00
ES_1	30.599	11.816	A	22.076	6.452	B	21.091	8.473	B	15.434	4.391	C	0.00
ES_2	41.650	21.220	A	41.23	12.87	A	41.630	14.03	A	41.880	11.80	A	0.99
Aggregated	120.87	24.64	A	92.23	19.63	B	88.55	6.25	B	50.08	6.82	C	0.00

Figure 2.5 also demonstrates the box plots for the obtained distributions of solution times of each objective function. The reason why we consider all objective functions in this evaluation is that we need to optimize each objective function separately in the fuzzy multi-objective programming step of the solution procedure. Table 2.10 also gives the grouping information and the p-value of the test.

If we maximize the objective function TP individually, the obtained p-value of zero indicates

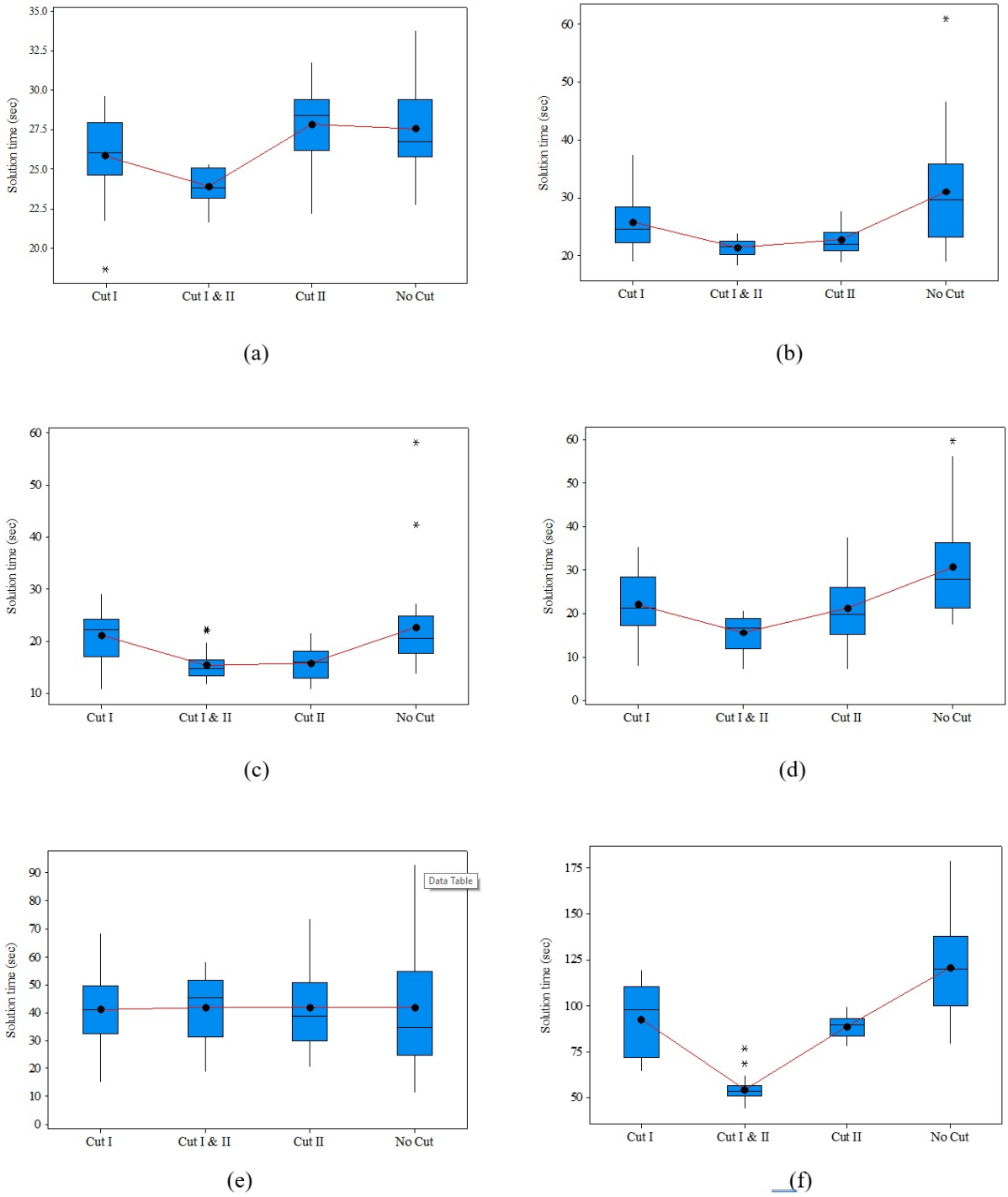


Figure 2.5: Box plot representation of solution time means with various cuts for (a) TP, (b) TE, (c) SR, (d) ES_1, (e) ES_2 and (f) aggregated objective functions.

there exists sufficient evidence that not all solution time means are equal with the confidence level 0.95. To explore the differences among the solution time means, we examine the grouping information obtained by Fisher's method. Since "No Cut" and "Cut II" factors share the grouping letter A, it turns out that cut II does not significantly reduce the solution time if individually applied to the relaxed MILP model with TP objective function. However, due to different grouping letters associated with "Cut I" and "Cuts I & II" factors, we conclude that cut I and cuts I & II are able to significantly diminish the solution time of the relaxed MILP model. In other words, cut I and cuts I & II reduce the solution time by 6.3% (=100(27.572-25.846)/27.572) and 13.4% (=100(27.572-23.889)/27.572), respectively.

The derived p-value when minimizing TE objective function solely indicates there exists sufficient evidence that not all solution time means are equal with the confidence level 0.95. Particularly, cut I with the distinct grouping letter B reduces the solution time by 17.1% (=100(30.989-25.688)/30.989) in the relaxed MILP model rather than a situation in which no cut is applied to the model. Moreover, exploring the grouping information reveals that there does not exist statistically significant difference between the effects of "Cut II" and "Cuts I & II" factors in reducing the solution time of relaxed MILP model. Cut II and cuts I & II decrease the solution time mean by 26.7% (=100(30.989-22.709)/30.989) and 31.1% (=100(30.989-21.338)/ 30.989), respectively.

According to p-value obtained for maximization of SR objective function, it appears that there exists sufficient evidence that not all solution time means are equal with the confidence level 0.95. However, since "Cut I" and "No Cut" factors share the same grouping letter, we conclude cut I is not able to individually diminish the solution time of the relaxed MILP problem with SR objective function. In addition, because of the same grouping letter shared, there is no statistically significant difference between effects of cut II and cuts I & II on the solution time mean. Meanwhile, cut II and cuts I & II reduce the solution time mean by 30.6% and 32.2%, respectively.

Based on the p-value calculated for minimizing ES_1 objective function with different cut factors, it turns out that there exists sufficient evidence that not all solution time means are equal with the confidence level 0.95. That "Cut I" and "Cut II" factors have the same grouping information letter implies cut I and cut II have the statistically same effect on the solution time of the relaxed MILP problem. Particularly, they are able to decrease the solution time mean by 27.9% and 31.1%, respectively, rather than no cut is enforced. Furthermore, "Cuts I & II" factor has also a distinct grouping letter, which indicates cuts I & II are able to significantly diminish the solution time in comparison with a condition in which no cut is applied. Particularly, the reduction percentage in the solution time by cuts I & II is equal to 49.6%.

None of the defined cuts is effective in reducing the solution time of the relaxed MILP model with the objective function ES_1 because the derived p-value is extremely larger than the critical

degree of the test, i.e. 0.05. In other words, the solution time means obtained under all four factors are statistically the same.

Once we maximize the aggregated single objective function with equal weights in the relaxed MILP model, the calculated p-value displays there exists sufficient evidence that not all solution time means are equal with the confidence level 0.95. Meanwhile, the solution time means associated with cut I and cut II are statistically the same because they share similar grouping information letter. Cut I (cut II) reduces the solution time mean by 23.7% (26.7%) in the relaxed MILP model rather than the situation in while no cut is applied into the model. Moreover, because of distinct grouping information letter for "Cuts I & II" factor, cuts I & II are significantly effective in decreasing the solution time mean of the model. Particularly, the current cuts reduce the solution time mean by 55.3%.

According to the above evaluation, it turns out that cuts I & II are immensely effective in reducing the solution time mean of relaxed MILP model with each of five objective functions.

2.7 Appendix

Indexes

i: Index of recycled materials; $i = 1, ..., I$

v: Index of candidate sites for recovery centers; $v = 1, ..., V$

j: Index of products; $j = 1, ..., J$

y: Index of candidate sites for recycling centers; $y = 1, ..., Y$

k: Index of candidate sites for manufacturers; $k = 1, ..., K$

p: Index of candidate sites for disposal centers; $p = 1, ..., P$

d: Index of candidate sites for distribution centers; $d = 1, ..., D$

m: Index of materials' customers; $m = 1, ..., M$

c: Index of customers; $c = 1, ..., C$

t: Index of period; $t = 1, ..., T$

l: Index of candidate sites for collection centers; $l = 1, ..., L$

Parameters

$tco_{kjdt}^{man-dis}$: unit transportation cost of product j between manufacturer k and distribution center d in period t.

$tco_{kjyt}^{man-rcy}$: unit transportation cost of defective product j between manufacturer k and recycling center d in period t.

$tco_{djct}^{dis-cus}$: unit transportation cost of product j between distribution center d and customer c in period t.

$tco_{cjlt}^{cus-rcy}$: unit transportation cost of EOL product j between customer c and collection center

l in period t.

$tco_{ljyt}^{col-rcy}$: unit transportation cost of recyclable product j between collection center l and recycling center y in period t.

$tco_{ljvt}^{col-rcv}$: unit transportation cost of recoverable product j between collection center l and recovery center v in period t.

$tco_{vjdt}^{rcv-dis}$: unit transportation cost of recovered product j between recovery center v and distribution center d in period t.

$tco_{yikt}^{rey-man}$: unit transportation cost for recycled material i between recycling center y and manufacturing center k in period t.

$tco_{yimt}^{rey-cus}$: unit transportation cost of recycled material i between recycling center y and material customer m in period t.

$tco_{yipt}^{rey-dip}$: unit transportation cost of recycled material i between recycling center y and disposal center p in period t.

pca_{kjt}^{man}: production capacity of product j in manufacturer k in period t.

pca_{yit}^{rcy}: production capacity of recycled material i in recycling center y in period t.

$\overline{hca}_{kit}^{man}$: holding capacity of recycled material i in manufacturer k in period t.

hca_{kjt}^{man}: holding capacity of product j in manufacturer k in period t.

hca_{djt}^{dis}: holding capacity of product j in distribution center d in period t.

hca_{ljt}^{col-y}: holding capacity of recyclable EOL product j in collection center l in period t.

hca_{ljt}^{col-v}: holding capacity of recoverable EOL product j in collection center l in period t.

hca_{vjt}^{rcv}: holding capacity of recoverable product j in recovery center v in period t.

$\overline{hca}_{yjt}^{rcy}$: holding capacity of recyclable product j in recycling center v in period t.

hca_{yit}^{rcy}: holding capacity of recycled material i in recycling center v in period t.

hca_{pit}^{dip}: holding capacity of recycled material i in disposal center p in period t.

pco_{kjt}^{man}: unit production cost of product j in manufacturer k in period t.

pco_{ljt}^{col}: unit processing cost of EOL product j in collection center l in period t.

pco_{vjt}^{rcv}: unit processing cost of recoverable product j in recovery center v in period t.

pca_{ljt}^{col}: processing capacity of EOL product j in collection center l in period t.

pca_{vjt}^{rcv}: processing capacity of recoverable product j in recovery center v in period t.

pca_{pit}^{dip}: processing capacity of material i in disposal center p in period t.

hco_{kjt}^{man}: unit holding cost of product j in manufacturing center k in period t.

hco_{djt}^{dis}: unit holding cost of product j in distribution center d in period t.

hco_{ljt}^{col}: unit holding cost of EOL product j in collection center l in period t.

hco_{vjt}^{rcv}: unit holding cost of EOL product j in recovery center v in period t.

$\overline{hco}_{yjt}^{rcy}$: holding capacity of recyclable product j in recycling center y in period t.

hco_{yit}^{rcy}: unit holding cost of recycled material i in recycling center y in period t.

hco_{pit}^{dip}: unit holding cost of recycled material i in disposal center p in period t.

mic_{kjt}^{man}: minimum operation level of product j in manufacturer k in period t.

mic_{djt}^{dis}: minimum operation level of product j in distribution center d in period t.

mic_{ljt}^{col}: minimum operation level of EOL product j in collection center l in period t.

mic_{vjt}^{rcv}: minimum operation level of recoverable product j in recovery center v in period t.

mic_{yit}^{rcy}: minimum operation level of recycled material i in recycling center y in period t.

mic_{pit}^{dip}: minimum operation level of recycled material i in disposal center p in period t.

$tem_{kjdt}^{man-dis}$: CO2 transportation emission of product j between manufacturer k and distribution center d in period t.

$tem_{djct}^{dis-cus}$: CO2 transportation emission of product j between distribution center d and customer c in period t.

$tem_{cjlt}^{cus-col}$: CO2 transportation emission of EOL product j between customer c and collection center l in period t.

$tem_{ljyt}^{col-rcy}$: CO2 transportation emission of EOL product j between collection center l and recycling center y in period t.

$tem_{kjvt}^{col-rcv}$: CO2 transportation emission of EOL product j between collection center l and recovery center v in period t.

pco_{yit}^{rcy}: unit processing cost of recycled material i in recycling center y in period t.

pco_{pit}^{dip}: unit processing cost of recycled material i in disposal center p in period t.

$\overline{hco}_{kit}^{man}$: unit holding cost of recycled material i in manufacturing center k in period t.

$tem_{yimt}^{rcy-mcu}$: CO2 transportation emission of recycled material i between recycling center y and material customer m in period t.

$tem_{yipt}^{rcy-dip}$: CO2 transportation emission of recycled material i between recycling center y and disposal center p in period t.

pem_{kjt}^{man}: CO2 production emission of product j in manufacturer k in period t.

pem_{ljt}^{col}: CO2 processing emission of end-of-life product j in collection center l in period t.

pem_{vjt}^{rcv}: CO2 processing emission of end-of-life product j in recovery center v in period t.

pem_{yit}^{rcy}: CO2 processing emission of recycled material i in recycling center y in period t.

pem_{pit}^{dip}: CO2 processing emission of recycled material i in disposal center p in period t.

$\overline{hem}_{kit}^{man}$: CO2 holding emission of material i in manufacturer **k** in period t.

hem_{kjt}^{man}: CO2 holding emission of product j in manufacturer k in period t.

hem_{djt}^{dis}: CO2 holding cost of product j in distribution center d in period t.

hem_{ljt}^{col}: CO2 holding emission of end-of-life product j in collection center l in period t.

hem_{vjt}^{rcv}: CO2 holding emission of end-of-life product j in recovery center v in period t.

$\overline{hem}_{yjt}^{rcy}$: CO2 holding emission of recyclable product j in recycling center y in period t.

hem_{yit}^{rcy}: CO2 holding emission of recycled material i in recycling center y in period t.

hem_{pit}^{dip}: CO2 holding emission of end-of-life product j in disposal center p in period t.

$\overline{iil}_{ki}^{man}$: inventory level of recycled material i in manufacturing center k at the beginning of

planning horizon.

iil_{kj}^{man}: inventory level of product j in manufacturing center k at the beginning of planning horizon.

iil_{dj}^{dis}: inventory level of product j in distribution center k at the beginning of planning horizon.

$tem_{vjdt}^{rcv-dis}$: CO2 transportation emission of EOL product j between recovery center v and distribution center d in period t.

$tem_{kjyt}^{man-rcy}$: CO2 transportation emission of defective product j between manufacturer k and recycling center y in period t.

$tem_{yikt}^{rcy-man}$: CO2 transportation emission of recycled material i between recycling center y and manufacturer k in period t.

$\overline{iil}_{yj}^{rcy}$: inventory level of product j in recycling center y at the beginning of planning horizon.

iil_{yi}^{rcy}: inventory level of recycled material i in recycling center k at the beginning of planning horizon.

spr_{imt}^{mat}: unit selling price of recycled material i to material customer m in period t.

spr_{jct}^{pro}: unit selling price of product j to customer c in period t.

cux_{kt}^{man}: current expense for manufacturing center k in period t.

cux_{dt}^{dis}: current expense for distribution center d in period t.

cux_{lt}^{col}: current expense for collection center l in period t.

cux_{vt}^{rcv}: current expense for recovery center v in period t.

cux_{yt}^{rcy}: current expense for recycling center y in period t.

cux_{pt}^{dip}: current expense for disposal center p in period $\mathbf{t}$.

mxn^{man}: maximum number of manufacturers to open.

mxn^{dis}: maximum number of distribution center to open.

mxn^{col}: maximum number of collection centers to open.

mxn^{rcv}: maximum number of recovery centers to open.

mxn^{rcy}: maximum number of recycling centers to open.

mxn^{dip}: maximum number of disposal centers to open.

ppo_{jt}: unit purchasing price of EOL product j in period t.

dem_{cjt}^{pro}: required demand for product j in customer c in period t.

dem_{imt}^{mat}: required demand for material j in material customer c in period t.

ϕ_j: return rate of EOL product j by customers.

β_j: defective rate of product j in a manufacturing center.

ψ_j: recovery rate of EOL product j in a recovery center.

τ_j: scrap rate of recycled material i in a recycling center.

α_{ij}: consumption coefficient of material i in producing product j in a manufacturing center.

cup_{cjt}: penalty cost of uncollected EOL product j at customer c in period t.

pcs_{cjt}^{cus}: shortage cost of unsatisfied demand for product j at customer c in period t.

iil_{lj}^{col-y}: inventory level of recyclable EOL product j in collection center l at the beginning of planning horizon.

iil_{lj}^{col-v}: inventory level of recoverable EOL product j in collection center l at the beginning of planning horizon.

iil_{vj}^{rcv}: inventory level of product j in recovery center k at the beginning of planning horizon.

δ_{ji}: consumption coefficient of product j in producing recycled material i in a recycling center.

lab_{kt}^{man}: labor force needed for manufacturing center k in period t.

lab_{dt}^{dis}: labor force needed for distribution center d in period t.

lab_{lt}^{col}: labor force needed for collection center l in period t.

lab_{vt}^{rcv}: labor force needed for recovery center v in period t.

lab_{yt}^{rcy}: labor force needed for recycling center y in period t.

lab_{pt}^{dip}: labor force needed for disposal center p in period t.

fco^{man}: fixed cost of opening a manufacturer.

fco^{dis}: fixed cost of opening a distribution center.

fco^{col}: fixed cost of opening a collection center.

fco^{rcv}: fixed cost of opening a recovery center.

fco^{rcy}: fixed cost of opening a recycling center.

fco^{dip}: fixed cost of opening a disposal center.

los_{k}^{man}: lost days caused from works damages during production in manufacturing center k.

los_{d}^{dis}: lost days caused from works damages during processing in distribution center d.

los_{l}^{col}: lost days caused from works damages during processing in collection center l.

los_{v}^{rcv}: lost days caused from works damages during processing in recovery center v.

los_{y}^{rcy}: lost days caused from works damages during production in recycling center y.

los_{p}^{dip}: lost days caused from works damages during processing in disposal center p.

pcs_{imt}^{mat}: shortage cost of unsatisfied demand for recycled material i at material customer m in period t.

M: A big positive value.

Decision variables

MTD_{kjdt}: quantity of product j shipped from manufacturing center k to distribution center d in period t.

MTY_{kjyt}: quantity of defective product j shipped from manufacturer k to recycling center y in period t.

DTC_{djct}: quantity of product j shipped from distribution center d to customer c in period t.

CTC_{cjlt}: quantity of EOL product j shipped from customer c to collection center l in period t.

CTV_{ljvt}: quantity of recoverable product j shipped from collection center l to recovery center v in period t.

CTY_{ljyt}: quantity of recyclable product j shipped from collection center l to recycling center y in period t.

VTD_{vjdt}: quantity of recovered product j shipped from recovery center v to distribution center d in period t.

YTP_{yipt}: quantity of scrapped material i shipped from recycling center y to disposal center p in period t.

YTM_{yikt}: quantity of recycled material i shipped from recycling center y to manufacturing center k in period t.

YTC_{yimt}: quantity of recycled material i shipped from recycling center y to material customer m in period t.

$\overline{LOI}^{man}_{kjt}$: inventory level of material i in manufacturing center k at the end of period t.

LOI^{man}_{kjt}: inventory level of product j in manufacturing center k at the end of period t.

$\overline{LOI}^{rcy}_{yjt}$: inventory level of recyclable product j in recycling center y at the end of period t.

LOI^{rcy}_{yit}: inventory level of recycled material i in recycling center y at the end of period t.

LOI^{dis}_{djt}: inventory level of product j in distribution center d in period t.

LOI^{col-y}_{ljt}: inventory level of recyclable product j in collection center l at the end of period t.

LOI^{col-v}_{ljt}: inventory level of recoverable product j in collection center l at the end of period t.

LOI^{rcv}_{vjt}: inventory level of recovered product j in recovery center v at the end of period t.

PLP^{man}_{kjt}: production level of product j in manufacturer k in period t.

PLP^{rcy}_{yit}: recycling level of recycled material e in recycling center y in period t.

μ^{man}_{kj}: DEA weight of outgoing product j in manufacturing center k.

$\overline{\mu}^{man}_{kj}$: DEA weight of defective product j in manufacturing center k.

μ^{dis}_{dj}: DEA weight of outgoing product j in distribution center d.

μ^{col-v}_{lj}: DEA weight of outgoing recoverable product j in collection center l.

μ^{col-y}_{lj}: DEA weight of outgoing recyclable product j in collection center l.

μ^{rcv}_{vj}: DEA weight of outgoing product j in recovery center v.

μ^{rcy}_{yi}: DEA weight of recycled material i in recycling center y.

$\overline{\mu}^{rcy}_{yi}$: DEA weight of scraped material i in recycling center y.

λ^{man}_{ki}: DEA weight of material i in manufacturing center k.

$\bar{\lambda}^{man}_{k}$: DEA weight of current expense in manufacturing center k.

$\bar{\bar{\lambda}}^{man}_{k}$: DEA weight of labor force in manufacturing center k.

λ^{dis}_{dj}: DEA weight of incoming product j in distribution center d.

$\bar{\lambda}^{dis}_{d}$: DEA weight of current expense in distribution center d.

$\bar{\bar{\lambda}}^{dis}_{d}$: DEA weight of labor force in distribution center d.

λ^{col-y}_{lj}: DEA weight of incoming recyclable product j in collection center l.

λ^{col-v}_{lj}: DEA weight of incoming recoverable product j in collection center l.

$\bar{\lambda}^{col}_{l}$: DEA weight of current expense in collection center l.

$\bar{\bar{\lambda}}_l^{col}$: DEA weight of labor force in collection center l.
λ_{vj}^{rcv}: DEA weight of incoming product j in recovery center v.
$\bar{\lambda}_v^{rcv}$: DEA weight of current expense in recovery center v.
$\bar{\bar{\lambda}}_v^{rcv}$: DEA weight of labor force in recovery center v.
λ_{yj}^{rcy}: DEA weight of incoming product j in recycling center y.
$\bar{\lambda}_y^{rcy}$: DEA weight of current expense in recycling center y.
$\bar{\bar{\lambda}}_y^{rcy}$: DEA weight of labor force in recycling center y.
X_k^{man}: 1 if a manufacturer is located in candidate site k; otherwise, 0.
X_d^{dis}: 1 if a distribution center is located in candidate site k; otherwise, 0.
X_l^{col}: 1 if a collection center is located in candidate site k; otherwise, 0.
X_v^{rcv}: 1 if a recovery center is located in candidate site k; otherwise, 0.
X_y^{rcy}: 1 if a recycling center is located in candidate site k; otherwise, 0.
X_p^{dip}: 1 if a disposal center is open in candidate site k; otherwise, 0.
d_k^{man}: inefficiency of manufacturer center k.
d_d^{dis}: inefficiency of distribution center d.
d_l^{col}: inefficiency of collection center l.
d_v^{rcv}: inefficiency of recovery center v.
d_y^{rcy}: inefficiency of recycling center y.

Chapter 3

Efficient and sustainable supplier selection and order allocation in stochastic environment

Effective allocation of scarce resources across supply chain environments is an emerging issue, as enterprises face shortfalls in raw materials, human labor, budgetary resources, equipment, energy, and capacity. In this chapter, we consider these related objectives in designing efficient and sustainable supply networks using a multi-objective mixed integer nonlinear programming (MINLP) model for efficient and sustainable supplier selection and order allocation with stochastic demand. Our approach considers sustainability dimensions including economic, environmental and social responsibility, but also seeks to design the most efficient supply network given constraints of the supply market. Enterprise efficiency is assessed using a bi-objective data envelopment analysis (DEA) whose inputs include raw materials, current expenses and labor force capacity. The resulting model is non-convex because of the presence of bilinear terms in DEA-related constraints, so we introduce a multi-stage solution procedure that first uses piecewise McCormick envelopes (PCM) to linearize the bilinear terms. Next, we introduce a set of valid inequalities in order to improve solution time of the problem whose dimension significantly increases after being linearized. Afterwards, we exploit chance constrained programming to deal with stochastic demand. Finally, a single aggregated objective function is derived using a fuzzy multi-objective programming approach. A manufacturing case study demonstrates the validity of the proposed approach, and its effectiveness in designing a supply network that addresses the triple bottom line of people, profit, and planet that comprises many sustainability initiatives in an efficient manner.

3.1 Introduction

Supplier selection is an important component of effective supply chain management, involving the identification of quality suppliers and allocation of materials and product requirements to each supplier. Since supplier selection and order allocation result in reduced cost of goods sold and competitive value, researchers suggest that it is one of the most salient elements of supply management (Willis et al. 1993; Dobler et al. 1990).

Reducing the total cost of ownership has become one of the vital aligned objectives for partners in a supply network. More recently, however, ensuring the societal and environmental impacts of operations and safeguarding the health and welfare of employees have become as important as profitability. Public concern on the environmental and social impacts of business activities has led to a business paradigm known as sustainable development. The integration of ecological, economic, and societal concerns in business decisions that promotes sustainable development has become an important strategic activity across many global industries (Benn et al., 2014). Sustainable supplier selection and order allocation are critical decisions in industrial supply chains that contribute to improved business performance and competitive advantage (Govindan et al. 2013; Grimm et al. 2014). For this reason, decision models that support sustainable supplier selection (SSS) have become an important component of sustainability-focused supply chains (Amindoust et al. 2012). Sustainable supplier selection is therefore critical to establishing sustainable initiatives in supply chains (Grimm et al. 2016).

Several prior research streams have focused on supplier selection problem. However, these researches are comparatively sparse when limited to models that consider the combined requirements of efficient, safe, and sustainable supply (Zimmer et al. 2016). This limited pool of quantitative studies primarily focuses on the responsiveness of sustainable supply networks, yet often ignores another important characteristic of supply management known as efficiency (Chopra and Meindl 2012). Chopra and Meindl (2012) suggest that an optimum network strategy should design supply network that balance efficiency and responsiveness given a portfolio of products, customer segments, and supply sources (Chopra and Meindl 2012). Considering efficiency in the design process of a supply network is fundamental given the pressures faced by organizations in terms of limited resources including labor, budget funds, equipment, space, and energy (Mitropoulos et al. 2013). Resource scarcity concerns span public and private sector systems due to economic, industrial and political pressure (Matopoulos et al. 2015). Numerous industry and government reports (PwC 2011; European Commission 2011; SCU 2012) emphasize growing concern over the short-term availability of resources and the potential implications of scarcity on economic and social outcomes. These reports highlight the necessity of employing available rare resources as efficiently as possible through effective management of supply networks across industrial sector. According to DEFRA (2012), it is estimated that 2% of UK

business profits per year are lost due to inefficient allocation of resources, and estimated savings of 23 billion per year could result by better allocation of resources efficiently that also protect the environment. This research suggests that resource efficiency is closely linked to a sustainability research agenda, yet recent literature reviews on sustainable supplier selection (Zimmer et al. 2012) make no reference to the specific resources used, the methods and tools applied to assess resources usage, or the overall supply network configuration and logistics implications associated with the use of specific resources.

In addressing this gap in research on sustainable supplier selection, we propose in this chapter a novel approach that addresses the concept of efficiency as defined through data envelopment analysis (DEA). Our approach applies DEA method to the problem of sustainable supplier selection and order allocation (SSSOA), and considers all three pillars of sustainability including economic, environmental and social responsibility implicit in this decision. In particular, we cope with the following research question: how does resource efficiency modeled by DEA impact the design of a supply network, where economic, environmental and social responsibility aspects of sustainability are taken into account? In this regard, we develop a multi-objective MINLP model in multiple periods with discount and shortage conditions in which DEA efficiency of purchasing firms are maximized, alongside optimizing all triple bottom lines of sustainability, where the demands of purchasing firms are assumed to be stochastic. The proposed model enables the supply chain executives to derive the natural trade-off between a sustainability aspect and DEA efficiency of their firms. Due to taking the efficiency into account, the purchasing firms poses to utilize their available rare resources as efficiently as possible. However, since the developed stochastic MINLP model is non-convex, we propose a four-step solution procedure in which nonlinear terms are relaxed using PCM approach (step 1), valid inequalities are added to the obtained relaxed programming model (step 2), chance-constrained programming is utilized to derive the deterministic equivalent of the original stochastic model (step 3) and then an aggregated single objective function is derived using fuzzy multi-objective programming (step 4). To the best of the authors' knowledge, there does not exist such a holistic integrated approach to efficient design of sustainable supply network in the current literature.

3.2 Literature review

The papers reviewed in this section are classified into three categories: i) the papers examining sustainable supplier selection and/or order allocation, and ii) the papers that apply DEA model to evaluate suppliers and/or to allocate orders, and iii) the papers that use other methodologies for supplier selection and/or order allocation.

3.2.1 Sustainable supplier selection and/or order allocation

In this section, we review the papers studying sustainable supplier selection and/or order allocation problem, in which all sustainability aspects are included.

Bai and Sarkis (2010) utilized grey system and rough set theory to sustainable supplier selection problem by explicitly considering sustainability attributes. To handle the subjectivity of decision makers assessments on the sustainable supplier selection criteria and sub-criteria, Amindoust et al. (2012) applied fuzzy logic and proposed a ranking method on the basis of fuzzy inference system (FIS) for supplier selection problem. Identifying 22 sustainable supplier selection criteria and three dimensions of criteria (economic, environmental, and social), Luthra et al. (2016) used an integrated Analytical Hierarchy Process (AHP), (VIKOR), a multi-criteria optimization and compromise solution approach to evaluate sustainable supplier selection. Orji and Wei (2015) presented a modeling approach of integrating information on supplier behavior in fuzzy environment with system dynamics simulation modeling technique. Taking a triple-bottom-line (profit, people and planet) approach and considering business operations as well as environmental impacts and social responsibilities of the suppliers, Sarkis and Dhavale (2015) developed an approach based on a Bayesian framework and Monte Carlo Markov Chain (MCMC) simulation to rank and select suppliers using specific selection objectives. Buyukozkan and Cifci (2011) developed an approach based on fuzzy analytic network process within multi-person decision-making schema under incomplete preference relations to evaluate sustainable suppliers. Trapp and Sarkis (2016) developed an optimization model that simultaneously addressed supplier selection, supplier development, and sustainability considerations. Kumar et al. (2016) used integrated fuzzy AHP and fuzzy multi-objective linear programming approach for order allocation among suppliers. Using both quantitative and qualitative criteria taking into account the economic, Gupta et al. (2016) developed an optimization model that integrated fuzzy multi-objective integer linear programming and analytic hierarchy process techniques. Aktin and Gergin (2016) introduced a questionnaire for measuring the sustainability scores of a company's potential suppliers, which are then used as an input to mixed-integer linear programming models distributing demand to the most sustainable suppliers. Azadnia et al. (2015) proposed an integrated approach of rule-based weighted fuzzy method, fuzzy analytical hierarchy process and multi-objective mathematical programming for sustainable supplier selection and order allocation combined with multi-period multi-product lot-sizing problem. The papers reviewed in this subsection have examined all sustainability aspects, i.e. economic, environmental and social, in their proposed frameworks. However, there are papers in the literature in which not all sustainability pillars are studied. Govindan et al. (2015) provided an overview of such papers. An immediate contribution of the present chapter over this type of studies, beside taking efficiency into account, is that all sustainability aspects are studied in this chapter.

3.2.2 DEA application to supplier selection and/or order allocation

In this context, Weber (1996) applied DEA model to identify the best suppliers based on the criteria purchasing cost, on time delivery and percent of rejected items. Webers study was developed by Liu et al. (2000). They exploited DEA model proposed by Banker et al. (1986) to evaluate the total efficiency of suppliers and applied their proposed model to decrease the number of suppliers. Weber et al. (2000) proposed an approach to determine suitable vendors. They firstly obtained the efficient solutions of a multi-objective supplier selection programming by changing the weights of objective functions and the number of suppliers. Then exploiting the values of objective functions corresponding to obtained efficient solutions as inputs of DEA model, they specified the number of suppliers, which resulted in maximizing the average of DEA efficiency score. Talluri et al. (2006) used chance constrained data envelopment analysis (CCDEA) to take into account intrinsic variation in characteristics of suppliers performance and subsequently to evaluate suppliers risk. When suppliers do not use the same inputs and outputs, Saen (2007) proposed an approach using AHP and interpolation to estimate missed data and then, applied CCDEA model to obtain relative efficiency of each supplier. Celebi and Bayraktar (2008) developed a hybrid approach including neural network and DEA with missed data. Wu (2009) proposed a hybrid approach using DEA, decision tree and neural network, where DEA model is applied to classify suppliers into efficient and inefficient groups, whereas Ha and Krishnan (2008) applied a another hybrid approach including AHP, DEA and neural networks to evaluate suppliers. Saen (2008) proposed an algorithm for ranking suppliers in the presence of volume discount offers based on super-efficiency analysis. Proposing a new model of IDEA considering weight restriction and imprecise data simultaneously, Saen (2010) applied it to evaluate suppliers. Jafari-Songhori et al. (2011) presented a framework based on DEA and multi-objective mixed integer programming model to firstly select suppliers and then separately allocate orders. Kuo and Lin (2012) considered supplier selection problem using analysis network process (ANP) and DEA, where ANP is used to consider the interdependency between criteria. Falagario et al. (2012) addressed the decision problem of supplier selection by applying DEA cross-efficiency. Dotoli and Falagario (2012) addressed supplier selection and order allocation problem using DEA, TOSIS and linear programming. Karsak and Dursun (2014) implemented an imprecise DEA for supplier selection, which employs the weights of supplier assessment criteria computed by fuzzy weighted average (FWA) utilizing the data from the house of quality (HOQ) and the supplier ratings with respect to supplier assessment criteria. Zhou et al. (2016) developed a multi-objective DEA model in a setting of type-2 fuzzy modeling to evaluate and select the most appropriate sustainable suppliers.

In all above-reviewed papers, sustainability issue is not considered in evaluating suppliers and assigning orders. Moreover, they do not develop an integrated approach, in which both

efficiency and the main objective function(s) of the supplier selection problem are optimized simultaneously. Accordingly, the solution provided by them suffers from the sub-optimality issue. In addition, in these papers, the main resources utilized by a purchasing firm are regarded as inputs and the main performance measures are viewed as outputs of DEA model. However, in the present chapter, we define the physical raw materials, current expenses and labor forces as the inputs of the purchasing firms with a single constant output. The defined inputs seem far comprehensible for decision maker from an econometric perspective when evaluating the DEA efficiency scores.

3.2.3 Other methodologies

There exist numerous studies in the literature in which other solution procedures for supplier selection and/or order allocation problem are used. For instance, Scott et al. (2015) proposed an integrated method for dealing with supplier selection and order allocation in stochastic, multi-stakeholder and multi-criteria environments using a combined Analytic Hierarchy Process-Quality Function Deployment (AHP-QFD) and chance constrained optimization algorithm approach. Mogale et al. (2017a) developed a MINLP model after studying the Indian wheat supply chain scenario to investigate the multi-period multi-modal bulk wheat transportation and storage problem in a two-stage supply chain network of Public Distribution System (PDS). Mogale et al. (2017b) studied the transportation of bulk food grain by capacitated vehicles from surplus states to deficit states through silo storage by formulating a MINLP model that sought to minimize the overall cost. Fallahpour et al. (2017) first developed the most important criteria and their corresponding sub-criteria for sustainable supplier selection through a questionnaire-based survey and then proposed a hybrid model to identify the most sustainable supplier with respect to the determined attributes. To provide a decision-making tool to solve a multi-period green supplier selection and order allocation problem, Hamdan and Cheaitou (2017) integrated three components: fuzzy TOPSIS to assign two preference weights to each potential supplier, AHP to assign a global importance weight to each of the two sets of criteria and then multiplication of the preference weight obtained from fuzzy TOPSIS by the global importance weight of the set of traditional criteria. Maiyar and Thakkar (2017) proposed a combined tactical and operational two-stage food grain transportation model with linear formulation in the first stage and a MINLP in the second stage taking the case of India. Ghadimi et al. (2017) proposed a Multi-Agent Systems (MASs) approach as a mean of automating and facilitating the process of sustainable supplier selection and order allocation resulting in a more cooperative partnership.

3.3 Multi-criteria data envelopment analysis (MCDEA)

We presented a bi-objective output-oriented DEA model in section 2.2, i.e. model (2.1)-(2.7). In this section, however, we derive a bi-objective *input*-oriented DEA model because the inputs that are raw materials purchased from suppliers are more important in the context of supplier selection and order allocation problem. In doing so, we utilize the following DEA model initially developed by Weber et al. (1998):

$$Max \qquad h_0 = \sum_{r=1}^{s} \mu_r \tag{3.1}$$

s.t.

$$\sum_{r=1}^{s} \mu_r - \sum_{j=1}^{v} \lambda_j x_{jk} \leq 0; \qquad \forall k$$

$$\sum_{j=1}^{v} \lambda_j x_{j0} = 1$$

$$\mu_r, \lambda_j \geq \epsilon; \qquad \forall i, r$$

The abovementioned model evaluates the efficiency of DMU_0, where s and v represent the number of outputs and inputs, respectively. Moreover, x_{jk} is the amount of input j to DMU k, μ_r and λ_j are the weights given to output r and input j, respectively. In this model, optimal weights are determined in such a way that each DMU maximizes its own efficiency score defined as the ratio of sum of the weighted outputs to sum of the weighted inputs. If there are K DMUs to be evaluated, this model is solved K times in order to determine the efficiency score of all DMUs under study. In order to derive the efficiency score of all DMUs simultaneously, we propose the following simultaneous DEA model, which is based on the simultaneous DEA model proposed by Klimberg and Ratick (2008), where d_k is the inefficiency score of DMU k:

$$Max \qquad h = \sum_{k=1}^{K} (1 - d_k) \tag{3.2}$$

s.t.

$$\sum_{j=1}^{v} \lambda_{jk} x_{jk} = 1; \qquad \forall k$$

$$\sum_{r=1}^{s} \mu_{rk} + d_k = 1; \qquad \forall k$$

$$\sum_{r=1}^{s} \mu_{rk} - \sum_{j=1}^{v} \lambda_{jk} x_{j\bar{k}} \leq 0; \qquad \forall k, \bar{k}; k \neq \bar{k}$$

$$\mu_{rk}, \lambda_{jk} \geq \epsilon; \qquad \forall j, r, k$$

$$d_k \geq 0; \qquad \forall k$$

The initial DEA model suffers from two shortcomings including low discriminating power and unrealistic weights distribution. In order to overcome either of or both these problems, various approaches have been proposed in the literature such as cone ratio model (Charnes et al. 1990), assurance region (Thompson et al. 1986), super efficiency (Andersen and Petersen 1993), cross-evaluation (Sexton 1986) and multi-objective approach (Li and Reeves 1999), among others. The latter approach has several advantages over the former ones to be used in this chapter, as follows:

- Once integrated into the proposed SSSOA model, it results in fewer and more tractable nonlinear terms.
- It is a one-stage approach, i.e. it eliminates both aforementioned problems by solving only one multi-objective problem that produces unique solution for the weight of inputs and outputs.
- It does not need any priori information from decision maker, which may cause infeasibility of the problem.

However, recently Ghasemi et al. (2014) proposed a modification on Li and Reeves's model that provides better weight dispersion and discrimination power. Implementing this modification on model (2) results in:

$$Min \qquad ES_1 = \eta \tag{3.3}$$

$$Min \qquad ES_2 = \sum_{k=1}^{K} d_k \tag{3.4}$$

s.t.

$$\sum_{j=1}^{v} \lambda_{jk} x_{jk} = 1; \qquad \forall k \tag{3.5}$$

$$\sum_{r=1}^{s} \mu_{rk} - \sum_{j=1}^{v} \lambda_{jk} x_{j\bar{k}} \leq 0; \qquad \forall k, \bar{k}; k \neq \bar{k} \tag{3.6}$$

$$\sum_{r=1}^{s} \mu_{rk} + d_k = 1; \qquad \forall k \tag{3.7}$$

$$\eta - d_k \geq 0; \qquad \forall k \tag{3.8}$$

$$\mu_{rk}, \lambda_{jk} \geq \epsilon; \qquad \forall j, r, k \tag{3.9}$$

$$d_k \geq 0; \qquad \forall k \tag{3.10}$$

where objective functions (3.3) and (3.4) are to minimize the maximum quantity and summation of all deviation (inefficiency) variables $d_j; \forall j$, respectively. η serves as the maximum inefficiency score of all DMUs.

3.4 Model Development

In this section, we develop a novel multi-period, multi-perishable commodity and multi-objective MINLP model to SSSOA problem in the presence of price discount and shortage condition using the indexes, parameters and decision variables presented in Appendix.

3.4.1 Objective functions

Cost objective function: Objective function (3.11) minimizes the total cost over the time horizon, which includes the purchasing cost, fixed and variable transportation costs, holding cost of extra raw materials and shortage cost over all time periods.

$$\begin{aligned} Min \qquad TC = & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} ppr_{ijl}^{t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} fsc_{ijk}^{t}(X_{ijlk}^{t}/act) + \\ & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} vsc_{ijk}^{t} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} ILP_{jk}^{t} hoc_{jk}^{t} + \\ & \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} \overline{ILP}_{jk}^{t} shc_{jk}^{t} \end{aligned} \tag{3.11}$$

Emission objective function: Objective function (3.12) minimizes total CO2 emission over the time horizon, which consists of fixed and variable transportation emissions, processing emission of suppliers and holding emission of extra raw materials over consecutive periods.

$$\begin{aligned} Min \qquad TC = & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} fse_{ijk}^{t}(X_{ijlk}^{t}/act) + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} vse_{ijk}^{t} + \\ & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} pre_{ij} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} ILP_{jk}^{t} hoe_{jk}^{t} \end{aligned} \tag{3.12}$$

Social responsibility objective function: This objective function contains two components including job opportunities created and lost days caused from works damages, which are formulated as the first and second terms respectively in objective function (3.13), where ζ_1 and ζ_2 are normalizing weight factors given to job opportunities created and lost days caused from works damages, respectively. In formulating these terms, we modify the procedure proposed by Devika et al. (2014).

$$\begin{aligned} Max \qquad SR =& \zeta_1[\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} var_k X_{ijlk}^t/pca_k^t]- \\ & \zeta_2[\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} los_k X_{ijlk}^t/pca_k^t] \\ =& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K}(\zeta_1 var_k - \zeta_2 los_k) X_{ijlk}^t/pca_k^t \end{aligned} \tag{3.13}$$

DEA efficiency objective functions: Objective functions (3.14) and (3.15) characterize the DEA efficiency objective functions in this study:

$$Min \qquad ES_1 = \eta \tag{3.14}$$

$$Min \qquad ES_2 = \sum_{k=1}^{K} d_k \tag{3.15}$$

3.4.2 Constraints

Constraint (3.16) enforces the total rejected amount of each raw material in each purchasing firm be less than or equal to its corresponding desired quantity in each period.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^t rep_{ij}^t \leq \hat{dem}_{jk}^t apr_{jk}^t; \qquad \forall k, j, t \tag{3.16}$$

Constraint (3.17) satisfies the total amount of each raw material delivered on-time to each purchasing firm be greater than or equal to its corresponding desired quantity in each period.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^t otd_{ij}^t \geq \hat{dem}_{jk}^t apd_{jk}^t; \qquad \forall k, j, t \tag{3.17}$$

Since any purchasing firm has a limited space for holding any raw material in each period, constraint (3.18) satisfies that the area occupied by each raw material supplied by all suppliers

to each purchasing firm should be less than or equal to the respective total available space in each period.

$$[(1-per_{jk}^{t-1})ILP_{jk}^{t-1}+\sum_{i=1}^{m}\sum_{l=1}^{L_i}X_{ijlk}^{t}-\hat{dem}_{jk}^{t}-\overline{ILP}_{jk}^{t}]sph_j \leq Z_{jk}^{t}sts_{jk}^{t}; \qquad \forall k,j,t \quad (3.18)$$

Constraint (3.19) guarantees that, in each period, the amount of raw material j provided by supplier i in all price levels to all purchasing firms should be less than or equal to its respective capacity.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i}X_{ijlk}^{t} \leq cap_{ij}^{t}; \qquad \forall i,j,t \quad (3.19)$$

In practice, a purchasing firm would like to work with a specific number of suppliers in each period to provide a particular raw material. This helps a purchasing firm manage and monitor its suppliers more appropriately and control the supply risk. This is satisfied by constraint (3.20).

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i}Y_{ijlk}^{t} \leq nsu_{jk}^{t}; \qquad \forall j,k,t \quad (3.20)$$

Constraint (3.21) determines the amount of each raw material each purchasing firm holds, while constraint (3.22) derives the shortage amount for each raw material each purchasing firm faces at the end of each period, where $ILP_{jk}^{0}=\overline{LOI}_{jk}^{0}=0$:

$$ILP_{jk}^{t}=(1-per_{jk}^{t-1})ILP_{jk}^{t-1}+\sum_{i=1}^{m}\sum_{l=1}^{L_i}X_{ijlk}^{t}-\hat{dem}_{jk}^{t}-\overline{ILP}_{jk}^{t-1}; \qquad \forall k,j,t \quad (3.21)$$

$$\overline{ILP}_{jk}^{t}=\overline{ILP}_{jk}^{t-1}+\hat{dem}_{jk}^{t}-\sum_{i=1}^{m}\sum_{l=1}^{L_i}X_{ijlk}^{t}-(1-per_{jk}^{t-1})ILP_{jk}^{t-1}; \qquad \forall k,j,t \quad (3.22)$$

Constraint (3.23) requires the amount of material j that is perished at the end of period t in purchasing firm k should not exceed its corresponding maximum value.

$$[(1-per_{jk}^{t-1})ILP_{jk}^{t-1}+\sum_{i=1}^{m}\sum_{l=1}^{L_i}X_{ijlk}^{t}-\hat{dem}_{jk}^{t}-\overline{ILP}_{jk}^{t}]per_{jk}^{t} \leq Z_{jk}^{t}mpr_{jk}^{t}; \qquad \forall k,j,t \quad (3.23)$$

Constraint (3.24) guaranties that, at each period, the shortage amount of each raw material be less than or equal to a predetermined quantity.

$$\overline{ILP}_{jk}^{t}+\hat{dem}_{jk}^{t}-\sum_{i=1}^{m}\sum_{l=1}^{L_i}X_{ijlk}^{t}-(1-per_{jk}^{t-1})ILP_{jk}^{t-1} \leq U_{jk}^{t}msh_{jk}^{t}; \qquad \forall k,j,t \quad (3.24)$$

Constraint (3.25) requires a purchasing firm should either hold extra amount of each row material or lack for it in each period.

$$Z_{jk}^{t} + U_{jk}^{t} = 1; \qquad \forall k, j, t \tag{3.25}$$

Obviously when supplier i is not selected to procure raw material j at price level l for a purchasing firm in period t, i.e. $Y_{ijlk}^{t} = 0$, then nothing of that type of raw material in the specified price level should be shipped from supplier i to the purchasing firm, i.e. $X_{ijlk}^{t} = 0$. On the other hand, if $Y_{ijlk}^{t} = 1$, then X_{ijlk}^{t} should be within the respective purchase volume of price level l. Constraints (3.26) and (3.27) imply the current two conditions, where M denotes a big positive number.

$$Y_{ijlk}^{t}\ell_{ijl}^{t} \leq X_{ijlk}^{t} \leq Y_{ijlk}^{t}\ell_{ij,l+1}^{t}; \qquad \forall i, j, l = 1, ..., L_i - 1, k, t \tag{3.26}$$

$$Y_{ijL_ik}^{t}\ell_{ijL_i}^{t} \leq X_{ijL_ik}^{t} \leq Y_{ijL_ik}^{t}M; \qquad \forall i, j, k, t \tag{3.27}$$

In order to establish the DEA constraints for purchasing firms, it is first required determining their respective inputs. In this study, the inputs to each purchasing firm include the raw materials transported from all suppliers, total current expenses and labor forces. In each period of time, total amount of incoming raw material j shipped from all suppliers to purchasing firm k is equal to $\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t}$. Therefore, the input j of purchasing firm k, which is defined as total amount of incoming raw material j shipped from all suppliers to purchasing firm k in all periods, is equal to $\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t}$. Furthermore, total current expenses and labor forces for purchasing firm k in all periods are equal to $\sum_{t=1}^{T} cux_{k}^{t}$ and $\sum_{t=1}^{T} lab_{k}^{t}$, respectively. That the inputs are derived over all periods in this study is based on the aggregate model proposed by Kao and Liu (2014). Therefore, each purchasing firm k possesses $\upsilon + 2$ inputs. In this case, constraint (3.28) denotes the weighted sum of inputs to each purchasing firm is equal to one. This constraint is equivalent to constraint (3.5) in DEA model.

$$\sum_{j=1}^{v}\lambda_{jk}\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t} + \bar{\lambda}_{k}\sum_{t=1}^{T} cux_{k}^{t} + \bar{\bar{\lambda}}_{k}\sum_{t=1}^{T} lab_{k}^{t} = 1; \qquad \forall k \tag{3.28}$$

In addition, constraints (3.29) that is equivalent to constraint (3.6) in DEA model shows the efficiency score of each purchasing firm is less than or equal to one.

$$\sum_{r=1}^{s}\mu_{rk} - \sum_{j=1}^{v}\lambda_{jk}\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijl\bar{k}}^{t} - \bar{\lambda}_{k}\sum_{t=1}^{T} cux_{\bar{k}}^{t} - \bar{\bar{\lambda}}_{k}\sum_{t=1}^{T} lab_{\bar{k}}^{t} \leq 0; \qquad \forall k, \bar{k}; k \neq \bar{k} \tag{3.29}$$

Meanwhile, the remained DEA constraints (3.7)-(3.10) are unchanged and still formulated

as follows:

$$\sum_{r=1}^{s} \mu_{rk} + d_k = 1; \qquad \forall k \tag{3.30}$$

$$\eta - d_k \geq 0; \qquad \forall k \tag{3.31}$$

$$\mu_{rk}, \lambda_{jk} \geq \epsilon; \qquad \forall j, r, k \tag{3.32}$$

$$d_k \geq 0; \qquad \forall k \tag{3.33}$$

Finally, constraint (3.34) requires Y_{ijlk}^t, Z_{jk}^t and U_{jk}^t be binary and X_{ijlk}^t, ILP_{jk}^t and $\overline{ILP}_{jk}^t$ be nonnegative.

$$Y_{ijlk}^t, Z_{jk}^t, U_{jk}^t \in \{0,1\}, X_{ijlk}^t, ILP_{jk}^t, \overline{ILP}_{jk}^t \geq 0; \qquad \forall i, j, l, k, t \tag{3.34}$$

In the above-developed programming model, the hat symbol on the $\hat{dem}_{jk}^t$ parameter indicates that the demand of raw material j in purchasing firm k at time period t is stochastic following any arbitrary probability density function (PDF).

3.5 Solution procedure

In this section, we devise a solution procedure for the stochastic multi-objective mixed integer nonlinear SSSOA programming model developed in the preceding section. The nonlinearity issue is due to the presence of the bilinear term in the DEA efficiency constraints (3.28) and (3.29). In the current constraints, the bilinear term is formed by multiplication of the mass flow decision variable by its respective DEA weight. This makes the proposed programming problem nonconvex, for which a global optimum solution is not guaranteed to attain using the ordinary optimization methods. In order to deal with the non-convexity issue, the bilinear term is firstly relaxed in this chapter using PCM approach. Since relaxing the bilinear term by PCM requires adding some extra mixed integer linear constraints to the original SSSOA problem, a set of valid inequalities is then introduced to improve the performance of the proposed solution procedure in terms of its solution time. In addition, as indicated before, the demand of raw material j in purchasing firm k at time period t, i.e. $\hat{dem}_{jk}^t$, is stochastic with any arbitrary PDF. The chance constrained stochastic programming is utilized in the third step of the solution procedure to deal with stochastic demands. Another issue to encounter is the presence of multiple objective functions in the proposed programming problem. In the last step of the solution procedure, to tackle the current concern, a fuzzy programming approach is exploited in which a weighted sum fuzzy aggregation function is applied to derive a single objective function.

3.5.1 Bilinear relaxation by Piecewise McCormick envelopes

We elaborated Piecewise McCormick envelopes (PCM) in section 2.5.1 to deal with bilinear terms in constraints (2.78)-(2.92). The same procedure is utilized is applied in this section to linearize the DEA efficiency constraints (3.28) and (3.29). However, linearizing bilinear terms using PCM provides a lower (upper) bound for a minimization (maximization) programming model. Therefore, the PCMGO algorithm described in section 2.5.1 again assists with obtaining global optimum solution of a minimization programming model containing bilinear terms. A simple variant of it can also be utilized for a maximization problem.

Relaxing the bilinear term of the proposed SSSOA problem in the present study requires determining the variable on which partitioning is performed. In this study, the mass flow decision variable, i.e. X^t_{ijlk}, is selected to be partitioned mainly because we are able to derive its upper and lower bounds either by intuition or using mathematics of the proposed programming problem. While the lower bound of the mass flow variable is equal to zero, its corresponding upper bound is derived using constraint (3.19) to be cap^t_{ij}. However, there might be another upper bound for the mass flow decision variable that could have been obtained by more complicated arithmetic. The derived upper bound is the one that is easily accessible from constraint (3.19) of developed programming problem. Using this upper bound, the bilinear term in the DEA efficiency constraints (3.28) and (3.29) is relaxed as follows:

$$W^t_{ijlkk} \geq \sum_{n=1}^{N} \hat{\lambda}_{jkn} X^{t-L}_{ijlkn}; \qquad \forall i,j,l,k,t \tag{3.35}$$

$$W^t_{ijlkk} \geq \sum_{n=1}^{N} (\hat{\lambda}_{jkn} X^{t-U}_{ijlkn} + \lambda^U_{jk} \hat{X}^t_{ijlkn} - \lambda^U_{jk} X^{t-U}_{ijlkn} Q^t_{ijlkn}); \qquad \forall i,j,l,k,t \tag{3.36}$$

$$W^t_{ijlkk} \leq \sum_{n=1}^{N} (\hat{\lambda}_{jkn} X^{t-L}_{ijlkn} + \lambda^U_{jk} \hat{X}^t_{ijlkn} - \lambda^U_{jk} X^{t-L}_{ijlkn} Q^t_{ijlkn}); \qquad \forall i,j,l,k,t \tag{3.37}$$

$$W^t_{ijlkk} \leq \sum_{n=1}^{N} \hat{\lambda}_{jkn} X^{t-U}_{ijlkn}; \qquad \forall i,j,l,k,t \tag{3.38}$$

$$W^t_{ijlk\bar{k}} \geq \sum_{n=1}^{N} \hat{\lambda}_{jkn} X^{t-L}_{ijl\bar{k}n}; \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \tag{3.39}$$

$$W^t_{ijlk\bar{k}} \geq \sum_{n=1}^{N} (\hat{\lambda}_{jkn} X^{t-U}_{ijl\bar{k}n} + \lambda^U_{jk} \hat{X}^t_{ijl\bar{k}n} - \lambda^U_{jk} X^{t-U}_{ijl\bar{k}n} Q^t_{ijl\bar{k}n}); \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \tag{3.40}$$

$$W^t_{ijlk\bar{k}} \leq \sum_{n=1}^{N} (\hat{\lambda}_{jkn} X^{t-L}_{ijl\bar{k}n} + \lambda^U_{jk} \hat{X}^t_{ijl\bar{k}n} - \lambda^U_{jk} X^{t-L}_{ijl\bar{k}n} Q^t_{ijl\bar{k}n}); \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \tag{3.41}$$

$$W^t_{ijlk\bar{k}} \leq \sum_{n=1}^{N} \hat{\lambda}_{jkn} X^{t-U}_{ijl\bar{k}n}; \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \tag{3.42}$$

$$\lambda_{jk} = \sum_{n=1}^{N} \hat{\lambda}_{jkn}; \qquad \forall j,k \tag{3.43}$$

$$X^t_{ijlk} = \sum_{n=1}^{N} \hat{X}^t_{ijlkn}; \qquad \forall i,j,l,k,t \tag{3.44}$$

$$\sum_{n=1}^{N} Q^t_{ijlkn} = 1; \qquad \forall i,j,l,k,t \tag{3.45}$$

$$X^{t-L}_{ijlkn} Q^t_{ijlkn} \leq \hat{X}^t_{ijlkn} \leq X^{t-U}_{ijlkn} Q^t_{ijlkn}; \qquad \forall i,j,l,k,t,n \tag{3.46}$$

$$0 \leq \hat{\lambda}_{jkn} \leq \lambda^U_{jk} Q^t_{ijlkn}; \qquad \forall i,j,l,k,t,n \tag{3.47}$$

$$Q^t_{ijlkn} \in \{0,1\} \tag{3.48}$$

where, λ^U_{jk} denotes an arbitrary upper bound for λ_{jk}. In this case, the bilinear terms $\lambda_{jk} X^t_{ijlkn}$ and $\lambda_{jk} X^t_{ijl\bar{k}n}$ in constraints (3.28) and (3.29) are replaced by W^t_{ijlkk} and $W^t_{ijlk\bar{k}}$, respectively. It should be also noted that we derive the upper and lower bounds of the partitioned decision variable, i.e. X^t_{ijlkn}, in each disjoint partition n as follows:

$$X^{t-U}_{ijlkn} = \frac{n^2}{N} cap^t_{ij}; \qquad \forall i,j,l,k,t,n \in \{1,...,N\} \tag{3.49}$$

$$X^{t-L}_{ijlkn} = \frac{n(n-1)}{N} cap^t_{ij}; \qquad \forall i,j,l,k,t,n \in \{1,...,N\} \tag{3.50}$$

3.5.2 Introducing valid inequalities

Relaxing the bilinear term by PCM requires adding extra mixed integer linear constraints to the original SSSOA problem. Consequently, the solution time of the problem might increase to some degree. In order to accelerate the PCMGO algorithm, a set of valid inequalities is introduced to add to the relaxed MILP model.

In section 2.5.2, we introduce a set of valid inequalities to improve the developed solution procedure with respect to the solution time. Using the same approach, we obtain a couple of valid inequalities to accelerate PCMGO algorithm here.

Definition 3.1: Consider constraint (3.20) for the number of suppliers selected. Let $\bar{N} =$

$\{(i,l)|i \in \{1,...,m\}, l \in \{1,...,L_i\}\}$. A set $\dot{N} \subseteq \bar{N}$ is a *cover* if $nsu^t_{jk} < |\dot{K}|; \forall j,k,t$.

Proposition 3.1: If $\dot{N} \subseteq \bar{N}$ is a cover, the cover inequality $\sum_{i,l \in \dot{N}} Y^t_{ijlk} \leq |\dot{N}|-1; \forall j,k,t$ is valid for constraint (3.20).

Proof: The proof is straightforward based on Wolsey (1998); page: 147.

Proposition 3.2: If $\dot{N} \subseteq \bar{N}$ is a cover, then the valid inequality, then

$$\sum_{i,l \in \dot{N}} Y^t_{ijlk} + \sum_{i,l \notin \dot{N}} \theta^t_{ijlk} Y^t_{ijlk} \leq |\dot{N}|-1; \qquad \forall j,k,t \tag{3.51}$$

is non-redundant (facet-defining) and thus as strong as possible, where

$$\theta^t_{ijlk} = |\dot{N}| - nsu^t_{jk}; \qquad \forall j,k,t \& i,l \notin \dot{N}$$

Proof: Following the procedure to obtain a lifted cover inequality by Wolsey (1998); page: 149, the proof is straightforward.

In addition, constraint (3.52) implies that the selected suppliers provide sufficient capacity to all raw materials in purchasing firms.

$$\hat{dem}^t_{jk} \leq \sum_{i=1}^{m} \sum_{l=1}^{L_i} cap^t_{ij} Y^t_{ijlk}; \qquad \forall j,k,t \tag{3.52}$$

3.5.3 Chance-constrained programming

In order to deal with stochastic parameters in an optimization problem, the literature has widely evolved into application of stochastic programming approaches. One of the most commonly used methods is chance-constrained programming (CCP) initially proposed by Charnes and Cooper (1990). In CCP modeling of a stochastic decision system, the constraints will hold at least α percent of time satisfied. Here, α denotes the confidence level provided by the decision maker as a proper safety margin.

We consider the following mathematical programming problem with stochastic parameters:

$$Max \qquad f(\mathbf{x})$$

$$\text{s.t.}$$

$$g_j(\mathbf{x}, \xi) \leq 0; \qquad j = 1,...,p$$

where $\mathbf{x}$ is an n-dimensional decision vector, ξ is a stochastic vector, $f(\mathbf{x})$ and $g_j(\mathbf{x}, \xi) \leq 0; j = 1,...,p$ are non-stochastic objective and stochastic constraint functions, respectively. In order

to treat this type of problem correctly, Liu (1999) suggested using the following CCP model:

$$Max \qquad f(\mathbf{x}) \tag{3.53}$$

s.t.

$$Pr\{g_j(\mathbf{x},\xi) \leq 0\} \geq \alpha_j; \qquad j = 1, ..., p$$

where Pr{.} implies the probability of the event in {.} and α_j is a predetermined confidence level to the stochastic constraint j.

In the current chapter, $\hat{dem}_{kj}^{t}$ is a stochastic parameter that is appeared in several constraints. According to model (3.53), it might be possible to convert the stochastic constraints to their deterministic equivalents for the predetermined confidence levels. Then, the deterministic programming problem is solved by typical solution approaches. Although this procedure seems relatively difficult to employ and only successful for special cases, we explain how it is effectively utilized along with Lemma 3.1 to deal with the stochastic constraints in this study.

Lemma 3.1- Assuming $g(\mathbf{x},\xi) = h(\mathbf{x}) - \xi$, the deterministic equivalent of $Pr\{g(\mathbf{x},\xi) \leq 0\} \geq \alpha$ in CCP model is derived as $h(\mathbf{x}) \leq \kappa_\alpha$, where $\kappa_\alpha = \varphi^{-1}(1-\alpha)$ and φ^{-1} is the inverse of cumulative distribution function $\varphi(.)$.

Proof: Based on the theory of statistics, the proof is straightforward. □

Another variant of lemma (3.1) is as follows: assuming $g(\mathbf{x},\xi) = h(\mathbf{x}) - \xi$, the deterministic equivalent of $Pr\{g(\mathbf{x},\xi) \geq 0\} \geq \alpha$ in CCP model is derived as $h(\mathbf{x}) \geq \kappa_\alpha$, where $\kappa_\alpha = \varphi^{-1}(\alpha)$ and φ^{-1} is the inverse of cumulative distribution function $\varphi(.)$.

Based on the stochastic constraint in (3.53), we formulate the stochastic constraints (3.16) and (3.17) as follows:

$$Pr\{g(\mathbf{x},\hat{dem}) \leq 0\} = Pr\{\frac{\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t} rep_{ij}^{t}}{apr_{jk}^{t}} \leq \hat{dem}_{jk}^{t}\} \geq \alpha_1; \qquad \forall j,k,t \tag{3.54}$$

$$Pr\{g(\mathbf{x},\hat{dem}) \leq 0\} = Pr\{\frac{\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t} otd_{ij}^{t}}{apd_{jk}^{t}} \geq \hat{dem}_{jk}^{t}\} \geq \alpha_2; \qquad \forall j,k,t \tag{3.55}$$

Then, we derive the following deterministic equivalent constraints to these stochastic constraints:

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t} rep_{ij}^{t} \leq apr_{jk}^{t}\varphi^{-1}_{\hat{dem}_{jk}^{t}}(1-\alpha_1); \qquad \forall j,k,t \tag{3.56}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t} otd_{ij}^{t} \leq apd_{jk}^{t}\varphi^{-1}_{\hat{dem}_{jk}^{t}}(\alpha_2); \qquad \forall j,k,t \tag{3.57}$$

where $\varphi^{-1}_{\hat{dem}^{t}_{jk}}(.)$ denotes the inverse cumulative density function of $\hat{dem}^{t}_{jk}$.

Following the same procedure to derive the deterministic equivalents for remained stochastic constraints, we obtain the following deterministic equivalent to the stochastic relaxed MILP model:

$$(11)\text{-}(15)$$

$$\text{s.t.}$$

$$(19)\text{-}(20),(25)\text{-}(27),(43)\text{-}(44),(46)\text{-}(47),(30)\text{-}(31),(32)\text{-}(34),(48)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk} rep^t_{ij} \le apr^t_{jk}\varphi^{-1}_{\hat{dem}^{t}_{jk}}(1-\alpha_1); \qquad \forall k,j,t \tag{3.58}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk} otd^t_{ij} \ge apd^t_{jk}\varphi^{-1}_{\hat{dem}^{t}_{jk}}(\alpha_2); \qquad \forall k,j,t \tag{3.59}$$

$$[(1-per^{t-1}_{jk})ILP^{t-1}_{jk}+\sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk}-\varphi^{-1}_{\hat{dem}^{t}_{jk}}(1-\alpha_3)-\overline{ILP}^{t}_{jk}]sph_j \le Z^t_{jk} sts^t_{jk}; \qquad \forall k,j,t \tag{3.60}$$

$$ILP^t_{jk} = (1-per^{t-1}_{jk})ILP^{t-1}_{jk} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk} - \varphi^{-1}_{\hat{dem}^{t}_{jk}}(\alpha_4) - \overline{ILP}^{t-1}_{jk}; \qquad \forall k,j,t \tag{3.61}$$

$$\overline{ILP}^{t}_{jk} = \overline{ILP}^{t-1}_{jk} + \varphi^{-1}_{\hat{dem}^{t}_{jk}}(\alpha_5) - \sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk} - (1-per^{t-1}_{jk})ILP^{t-1}_{jk}; \qquad \forall k,j,t \tag{3.62}$$

$$[(1-per^{t-1}_{jk})ILP^{t-1}_{jk} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk} - \varphi^{-1}_{\hat{dem}^{t}_{jk}}(1-\alpha_6) - \overline{ILP}^{t}_{jk}]per^t_{jk} \le Z^t_{jk} mpr^t_{jk}; \forall k,j,t \tag{3.63}$$

$$\overline{ILP}^{t}_{jk} + \varphi^{-1}_{\hat{dem}^{t}_{jk}}(\alpha_7) - \sum_{i=1}^{m}\sum_{l=1}^{L_i} X^t_{ijlk} - (1-per^{t-1}_{jk})ILP^{t-1}_{jk} \le U^t_{jk} msh^t_{jk}; \qquad \forall k,j,t \tag{3.64}$$

$$\sum_{j=1}^{v}\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i} W^t_{ijlkk} + \bar{\lambda}_k \sum_{t=1}^{T} cux^t_k + \bar{\bar{\lambda}}_k \sum_{t=1}^{T} lab^t_k = 1; \qquad \forall k \tag{3.65}$$

$$\sum_{r=1}^{s}\mu_{rk} - \sum_{j=1}^{v}\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i} W^t_{ijl\bar{k}k} - \bar{\lambda}_k \sum_{t=1}^{T} cux^t_{\bar{k}} - \bar{\bar{\lambda}}_k \sum_{t=1}^{T} lab^t_{\bar{k}} \le 0; \qquad \forall k,\bar{k}; k \ne \bar{k} \tag{3.66}$$

$$W^t_{ijlkk} \ge \frac{cap^t_{ij}}{N}\sum_{n=1}^{N} n(n-1)\hat{\lambda}_{jkn}; \qquad \forall i,j,l,k,t \tag{3.67}$$

$$W^t_{ijlkk} \ge \sum_{n=1}^{N}(\frac{n^2}{N}\hat{\lambda}_{jkn} cap^t_{ij} + \lambda^U_{jk}\hat{X}^t_{ijlkn} - \frac{n^2}{N}\lambda^U_{jk} cap^t_{ij} Q^t_{ijlkn}); \qquad \forall i,j,l,k,t \tag{3.68}$$

$$W^t_{ijlkk} \leq \sum_{n=1}^{N}(\frac{n(n-1)}{N}\hat{\lambda}_{jkn}cap^t_{ij} + \lambda^U_{jk}\hat{X}^t_{ijlkn} - \frac{n(n-1)}{N}\lambda^U_{jk}cap^t_{ij}Q^t_{ijlkn}); \qquad \forall i,j,l,k,t \quad (3.69)$$

$$W^t_{ijlkk} \leq \frac{cap^t_{ij}}{N}\sum_{n=1}^{N} n^2\hat{\lambda}_{jkn}; \qquad \forall i,j,l,k,t \quad (3.70)$$

$$W^t_{ijlk\bar{k}} \geq \frac{cap^t_{ij}}{N}\sum_{n=1}^{N} n(n-1)\hat{\lambda}_{jkn}; \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \quad (3.71)$$

$$W^t_{ijlk\bar{k}} \geq \sum_{n=1}^{N}(\frac{n^2}{N}\hat{\lambda}_{jkn}cap^t_{ij} + \lambda^U_{jk}\hat{X}^t_{ijl\bar{k}n} - \frac{n^2}{N}\lambda^U_{jk}cap^t_{ij}Q^t_{ijl\bar{k}n}); \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \quad (3.72)$$

$$W^t_{ijlk\bar{k}} \leq \sum_{n=1}^{N}(\frac{n(n-1)}{N}\hat{\lambda}_{jkn}cap^t_{ij} + \lambda^U_{jk}\hat{X}^t_{ijl\bar{k}n} - \frac{n(n-1)}{N}\lambda^U_{jk}cap^t_{ij}Q^t_{ijl\bar{k}n}); \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \quad (3.73)$$

$$W^t_{ijlk\bar{k}} \leq \frac{cap^t_{ij}}{N}\sum_{n=1}^{N} n^2\hat{\lambda}_{jkn}; \qquad \forall i,j,l,k,\bar{k},t; k \neq \bar{k} \quad (3.74)$$

$$\frac{n(n-1)}{N}cap^t_{ij}Q^t_{ijlkn} \leq \hat{X}^t_{ijlkn} \leq \frac{n^2}{N}cap^t_{ij}Q^t_{ijlkn}; \qquad \forall i,j,l,k,t,n \quad (3.75)$$

$$\sum_{i,l\in\dot{N}} Y^t_{ijlk} + \sum_{i,l\notin\dot{N}}(|\dot{N}| - nsu^t_{jk})Y^t_{ijlk} \leq |\dot{N}| - 1; \qquad \forall j,k,t \quad (3.76)$$

$$\varphi^{-1}_{\hat{dem}^t_{jk}}(\alpha_8) \leq \sum_{i=1}^{m}\sum_{l=1}^{L_i} cap^t_{ij}Y^t_{ijlk}; \qquad \forall j,k,t \quad (3.77)$$

The CCP approach can easily incorporate more analysis and subjective assessments rather than other stochastic programming routines. In contrast to the two-stage stochastic programming approach in which violation of the constraints is allowed but penalized by a penalty term in the objective function, the CCP approach maintains a high level of reliability by expressing a minimum requirement on the probability of satisfying constraints. In other words, the resulting decision ensures the probability of complying with constraints, i.e. the confidence level of being feasible (Li et al. 2008). Since our framework involves principal decisions on sustainable supplier selection and order allocation, any violation of the organization's constraints may have critical consequences in terms of organizational budgets and operational-ability. Furthermore, the solution of a problem obtained by CCP provides comprehensive information on the economical outcome as a function of the desired confidence level of satisfying constraints, which is crucial for decision-making (Li et al. 2008). Hence, the CCP approach appears to be a better choice to treat uncertainty associated with the SSSOA problem.

3.5.4 Fuzzy multi-objective programming

The deterministic relaxed multi-objective SSSOA model obtained in the preceding section is converted to a single objective programming model in this section using a fuzzy multi-objective programming. As explained in section 2.5.3, fuzzy mathematical programming is an efficient approach to derive a Pareto-optimal solution for a multi-objective optimization problem in which each objective function is modeled as a fuzzy set. The membership function of the latter fuzzy set characterizes the degree of satisfaction of its corresponding objective function. Zimmermann (1978) firstly introduced the max-min operator to aggregate the fuzzy objective functions for making a compromise decision. However, Zimmermann's approach has a couple of shortcomings; i) a Pareto optimal solution is not guaranteed to obtain and ii) it does not allow for compensation among objective functions. In order to overcome these pitfalls, we exploit an interactive possibilistic programming approach proposed by Torabi and Hassini (2008), which ensures the achievement of efficient solution.

In order to implement the approach developed by Torabi and Hassini (2008), we proceed as follows:

Step 1: Obtain the global optimum solution of each objective function over the constraints of the relaxed MILP problem by the means of global optimization algorithm in section 2.5.1. Let $\mathbb{X} = \{X^*_{TC}, X^*_{TE}, X^*_{SR}, X^*_{ES_1}, X^*_{ES_2}\}$, where X^*_{TC}, X^*_{TE}, X^*_{SR}, $X^*_{ES_1}$ and $X^*_{ES_1}$ are the obtained global optimum solutions corresponding to TC, TE, SR, ES_1 and ES_2 objective functions.

Step 2: Find the values of the objective functions TC, TE, SR, ES_1 and ES_2 at each point of $\mathbb{X}$.

Step 3: Derive the minimum value of objective functions as $TC^{min} = min\{TC(x), x \in \mathbb{X}\}$, $TE^{min} = min\{TE(x), x \in \mathbb{X}\}$, $SR^{min} = min\{SR(x), x \in \mathbb{X}\}$, $ES_1^{min} = min\{SE_1(x), x \in \mathbb{X}\}$ and $ES_2^{min} = min\{SE_2(x), x \in \mathbb{X}\}$, and the maximum value of them as follows: $TC^{max} = max\{TC(x), x \in \mathbb{X}\}$, $TE^{max} = max\{TE(x), x \in \mathbb{X}\}$, $SR^{max} = max\{SR(x), x \in \mathbb{X}\}$, $ES_1^{max} = max\{SE_1(x), x \in \mathbb{X}\}$ and $ES_2^{max} = max\{SE_2(x), x \in \mathbb{X}\}$.

Step 4: Construct the membership function for each objective function as follows:

$$\vartheta^{TC}(x) = \begin{cases} 1; & TC < TC^{min} \\ \dfrac{TC^{max} - TC}{TC^{max} - TC^{min}}; & TC^{min} \leq TC \leq TC^{max} \\ 0; & TC > TC^{max} \end{cases} \tag{3.78}$$

$$\vartheta^{TE}(x) = \begin{cases} 1; & TE < TE^{min} \\ \dfrac{TE^{max} - TE}{TE^{max} - TE^{min}}; & TE^{min} \leq TE \leq TE^{max} \\ 0; & TE > TE^{max} \end{cases} \tag{3.79}$$

$$\vartheta^{SR}(x) = \begin{cases} 1; & SR > SR^{max} \\ \dfrac{SR - SR^{min}}{SR^{max} - SR^{min}}; & SR^{min} \leq SR \leq SR^{max} \\ 0; & SR < SR^{min} \end{cases} \tag{3.80}$$

$$\vartheta^{ES_1}(x) = \begin{cases} 1; & ES_1 < ES_1^{min} \\ \dfrac{ES_1^{max} - ES_1}{ES_1^{max} - ES_1^{min}}; & ES_1^{min} \leq ES_1 \leq ES_1^{max} \\ 0; & ES_1 > ES_1^{max} \end{cases} \tag{3.81}$$

$$\vartheta^{ES_2}(x) = \begin{cases} 1; & ES_2 < ES_2^{min} \\ \dfrac{ES_2^{max} - ES_2}{ES_2^{max} - ES_2^{min}}; & ES_2^{min} \leq ES_2 \leq ES_2^{max} \\ 0; & ES_2 > ES_2^{max} \end{cases} \tag{3.82}$$

In the proposed relaxed SSSOA model, social responsibility objective function is to be maximized. In this case, the decision maker's satisfaction increases as the solution moves toward upper bound of this objective function. Therefore, the decision maker is completely satisfied if social responsibility objective function reaches to its upper bound. On the other hand, all other objective functions are to be minimized. Under this condition, the level of decision makers satisfaction increases if the obtained solution approaches the lower bound of each objective function. In this case, the decision maker is fully satisfied if the objective functions reach their respective lower bounds. In fact, the aforementioned membership functions denote the satisfaction degrees of their corresponding objective functions for the given solution vector $\mathbf{x}$.

Step 5: Aggregate the above-mentioned membership functions to formulate the following single objective programming model:

$$\begin{aligned} Max \qquad \kappa(x) =& \gamma\kappa_0 + (1-\gamma)[\omega^{TC}\vartheta^{TC}(x) + \omega^{TE}\vartheta^{TE}(x) + \omega^{SR}\vartheta^{SR}(x) + \\ & \omega^{ES_1}\vartheta^{ES_1}(x) + \omega^{ES_2}\vartheta^{ES_2}(x)] \end{aligned} \tag{3.83}$$

$$\text{s.t.}$$

$$x \in \mathbb{A}$$

$$\kappa_0 \leq \vartheta^{TC}(x)$$

$$\kappa_0 \leq \vartheta^{TE}(x)$$

$$\kappa_0 \leq \vartheta^{SR}(x)$$

$$\kappa_0 \leq \vartheta^{ES_1}(x)$$

$$\kappa_0 \leq \vartheta^{ES_2}(x)$$

$$\kappa_0, \gamma \in [0, 1]$$

where, ω^{TC}, ω^{TE}, ω^{SR}, ω^{ES_1} and ω^{ES_2} are the weights (relative importance) of the corresponding objective functions such that $\omega^{TP} + \omega^{TE} + \omega^{SR} + \omega^{ES_1} + \omega^{ES_2} = 1$. Furthermore, $\mathbb{A}$ and γ characterize the set of all constraints in the proposed deterministic relaxed SSSOA model and the coefficient of compensation, respectively. Notably, the optimal value of variable $\kappa_0 = min\{\vartheta^{TC}(x), \vartheta^{TE}(x), \vartheta^{SR}(x), \vartheta^{ES_1}(x), \vartheta^{ES_2}(x)\}$ indicates the minimum satisfaction degree of objective functions. In fact, aggregation function looks for a compromise value between the min operator and the weighted sum operator based on the value of γ. In other words, the decision maker can obtain both balanced and unbalanced compromised solution via manipulating the value of weights and γ, based on his/her preferences.

3.6 Empirical study

An empirical study is presented in this section to elucidate how to implement SSSOA model and its solution procedure in a practical context. In this regard, the proposed approach is employed for a manufacturing and engineering company in automotive industry, which is a sole manufacturer of three types of automobile transmission systems in Iran, i.e. $K = 1$ and $s = 3$. The purpose of this empirical study is to evaluate six potential suppliers for two required items in time horizon of two years, i.e. $T = 2$, in such a way that not only the company's total cost, CO2 emission and social responsibility are optimized, but also it operates in its most efficient condition. Because of the economic troubles encountered recently and lack of the required manufacturing resources, the issue of efficiency is of great importance for this company. In such a circumstance, being in the efficient state aggrandizes the company's capability to corrival with any likely competing company in the near future.

The required items are in fact the forged parts of two shafts called primary shaft (PS) and secondary shaft (SS), i.e. $v = 2$. Due to the complex chemical combinations and high-tech manufacturing processes required for these items, the company succeeded in identifying only six potential domestic suppliers across the country, each of which has two price levels for each shaft, i.e. $m = 6$ and $L_i = 2$ for $i = 1, ..., 6$. This manufacturing company produces three types of transmission systems; automatic transmission (AT), manual transmission (MT) and continuous variable transmission (CVT). In the manufacturing process, one primary shaft and one secondary shaft are assembled together in each type of transmission systems. All parameter values are presented in the file uploaded into Scribd (https://www.scribd.com/document/347776772/Empirical-Study-Data). These data are derived from historical records, which are mainly available in production planning and control department, purchasing department and quality control department of this manufacturing company.

We also use log-normal density function to model the uncertain demand parameters of this manufacturer. The non-negativity of the demand values is preserved by utilizing log-normal distribution. Furthermore, Kamath and Pakkala (2002) provided evidence that log-normal distribution is well-suited for modeling economic stochastic variables such as demand. In this study, using the Chi-square test, we derive the parameters of log-normal density function as follows: both $\hat{dem}^{1}_{PS,1}$ and $\hat{dem}^{2}_{PS,1} \sim lognormal(11.1, 0.15)$, and both $\hat{dem}^{1}_{SS,1}$ and $\hat{dem}^{2}_{SS,1} \sim lognormal(11.244, 0.15)$. In addition, we set all the confidence levels of stochastic constraints to 0.95.

3.6.1 Results

As the first step in the proposed solution procedure, the bilinear term in DEA constraints (3.28)-(3.29) need to be relaxed by PCM approach. We presented how to implement such a relaxation on the bilinear terms $\lambda_{kj}X^t_{ijlk}$ and $\lambda_{kj}X^t_{ijl\bar{k}}$ before. The only point remained in developing the relaxed SSSOA model, which needs to be considered in the relaxation step, is to set appropriate upper bounds for un-partitioned decision variables, namely, the DEA weights. The mathematical structure of the proposed original SSSOA problem does not yield an analytical upper bound for a DEA weight. Hence, we define an arbitrary constant but appropriately large upper bound for all DEA weights. Currently, we initially set the upper bound of all DEA weights to 50 in the relaxation procedure.

According to the definition of a cover inequality, we can derive a strong valid inequality for constraint (3.20) as inequality (3.76). In this regard, we define $\dot{N}$ as a subset of $\bar{N}$ with $|\dot{N}|= 5$. Although we can derive 792 of such subsets, i.e. C^{12}_5, we just arbitrarily pick 4 cover inequalities associated with constraint (3.20) as follows:

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} \leq 4 \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j2k} \leq 4 \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{4j1k} \leq 4 \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{4j2k} \leq 4 \qquad \forall j,k,t$$

Therefore, strong valid inequality for constraint (3.20) are defined as follows:

$$Y^t_{1j1k}+Y^t_{1j2k}+Y^t_{2j1k}+Y^t_{2j2k}+Y^t_{3j1k}+2(Y^t_{3j2k}+Y^t_{4j1k}+Y^t_{4j2k}+Y^t_{5j1k}+Y^t_{5j2k}+Y^t_{6j1k}+Y^t_{6j2k}) \leq 4; \forall j,k,t$$

$$Y^t_{1j1k}+Y^t_{1j2k}+Y^t_{2j1k}+Y^t_{2j2k}+Y^t_{3j2k}+2(Y^t_{3j1k}+Y^t_{4j1k}+Y^t_{4j2k}+Y^t_{5j1k}+Y^t_{5j2k}+Y^t_{6j1k}+Y^t_{6j2k}) \leq 4; \forall j,k,t$$

$$Y^t_{1j1k}+Y^t_{1j2k}+Y^t_{2j1k}+Y^t_{2j2k}+Y^t_{4j1k}+2(Y^t_{3j1k}+Y^t_{3j2k}+Y^t_{4j2k}+Y^t_{5j1k}+Y^t_{5j2k}+Y^t_{6j1k}+Y^t_{6j2k}) \leq 4; \forall j,k,t$$

$$Y^t_{1j1k}+Y^t_{1j2k}+Y^t_{2j1k}+Y^t_{2j2k}+Y^t_{4j2k}+2(Y^t_{3j1k}+Y^t_{3j2k}+Y^t_{4j1k}+Y^t_{5j1k}+Y^t_{5j2k}+Y^t_{6j1k}+Y^t_{6j2k}) \leq 4; \forall j,k,t$$

Using all defined data and the derived strong valid inequalities, we optimize each of five objective functions in the relaxed SSSOA model by following the PCMGO algorithm presented in section 5.1. In this regard, we first program the original SSSOA model in GAMS 24.1.2 and use BARON solver on a computer with CPU of 2.5 GHz and 6.0 GB RAM to derive a *local* optimum solution of each objective function. For *TC*, *TE*, ES_1 and ES_2 objective functions, such a local optimum solution provides an upper bound, while it is a lower bound for *SR* objective function. Furthermore, we define $\Delta = 1$ as the step size to increase the accuracy of partitioned decision variables and $\varepsilon = 10^{-3}$ as the maximum acceptable optimality gap. In this case, Table 3.1 presents the global optimum solution of each objective function alongside its corresponding number of partitions used for algorithm convergence. It also gives the local optimum solution of each objective function. We exploit GMAS 24.1.2 with CPLEX solver to solve these five relaxed MILP models. For *TC*, *TE* and *SR* objective functions, it appears that the PCMGO algorithm terminates with $N = 2$ while their respective actual optimality gaps are greater than $\varepsilon = 10^{-3}$. That is because the problem becomes integer infeasible for N being greater than 2. However, for ES_1 and ES_2 objective functions, the desired optimality gap is satisfied. Table 3.1 also presents both minimum and maximum values of each objective function as they are required to derive the single objective function based on step 4 of fuzzy multi-objective programming.

Table 3.1: Solution report of each objective function in the computational problem.

Obj.	MINLP model		Relaxed MINLP model			Optimality	Minimum	Maximum
Fun.	Bound type (value)	CPU time (sec)	Optimum value	CPU time (sec)	N	gap	value	value
TC	Upper (80,901,588)	68.4	80,154,410	1.03	2	0.0093	80,154,410	94,996,960
TE	Upper (157,412)	86.3	152,373	3.21	2	0.033	152,373	206,992.9
SR	Lower (3.552)	18.19	3.472	0.98	2	0.023	2.5	3.472
ES_1	Upper (0)	2.12	0	0.31	1	0	0	1
ES_2	Upper (0)	1.19	0	0.25	1	0	0	0.997

The dimensions of original and relaxed SSSOA problems with respect to (# of integer variables, # of total variables, # of constraints) are (56,133,185) and (96,247,656), respectively. It appears that the dimension of original programming model is smaller than that of relaxed model. However, according to Table 3.1, the substantially large dimension of relaxed SSSOA model does not have a bad effect on solution time of each objective function. That is because the relaxed programming model can be effectively solved using the available heuristics and that, as

presented latter, the valid inequalities are effective in reducing the solution time of the relaxed SSSOA model.

Using the minimum and maximum values of each objective function, we can construct the respective membership functions, which themselves are used to formulate the following single objective (aggregated) programming model:

$$
\begin{aligned}
Max \quad & 0.4\kappa_0 + \{\omega^{TC}(\frac{94,996,960 - TC}{14,842,550}) + \omega^{TE}(\frac{206,992.9 - TE}{54,619.9}) + \omega^{SR}(\frac{SR - 3.184}{0.288}) + \\
& \omega^{ES_1}(\frac{0.997 - ES_1}{0.997}) + \omega^{ES_2}(\frac{0.997 - ES_2}{0.997})\} \\
& \text{s.t.} \\
& x \in \mathbb{A} \\
& \kappa_0 \leq \vartheta^{TC}(x) \\
& \kappa_0 \leq \vartheta^{TE}(x) \\
& \kappa_0 \leq \vartheta^{SR}(x) \\
& \kappa_0 \leq \vartheta^{ES_1}(x) \\
& \kappa_0 \leq \vartheta^{ES_2}(x) \\
& \kappa_0 \in [0,1]
\end{aligned}
$$

where γ is set to 0.4 as Torabi and Hassini (2008) found this value more appropriate. We again utilize GMAS 24.1.2 with CPLEX solver to solve the current single objective relaxed SSSOA model.

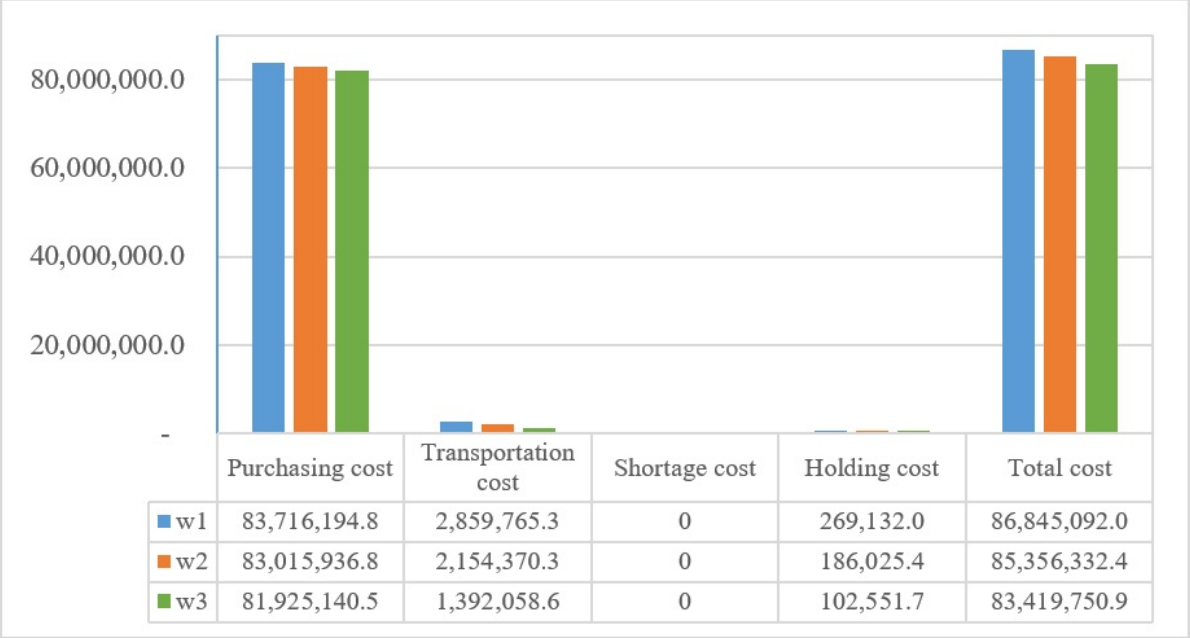
In order to examine the effect of incorporation of DEA efficiency, we consider various values for ω^{TC}, ω^{TE}, ω^{SR}, ω^{ES_1} and ω^{ES_2}. Particularly, we evaluate three instances in which we set the importance weight of objective functions as follows:

1. $\omega_1 = (\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = ((1-2\varepsilon)/3, (1-2\varepsilon)/3, (1-2\varepsilon)/3, \varepsilon, \varepsilon)$. In this instance, we assign extremely small positive importance weights to DEA objective functions to ignore their effects on the obtained optimum solution. We notice that one should not assign zero to the importance weight of DEA objective functions in this instance because he/she needs to assure non-dominated solutions. However, since ε is a very small quantity that is close to zero, this instance appropriately represents a situation in which DEA efficiency score is not taken into account when planning the supply network.

2. $\omega_2 = (\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = (0.2, 0.2, 0.2, 0.2, 0.2)$. In the current instance, we consider all objective functions that are equally weighted.

3. $\omega_3 = (\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = (\varepsilon, \varepsilon, \varepsilon, (1-3\varepsilon)/2, (1-3\varepsilon)/2)$. In this instance, we set the importance weight of total profit, total CO2 emission and social responsibility objective functions to a diminutive quantity. Hence, this instance properly models a situation in which we disregard the aforementioned objective functions while assuring non-dominated solutions.

The DEA efficiency of purchasing firm under each instance defined above is 0.6154, 0.8074 and 1, respectively. It appears that as ES_1 and ES_2 increase in instances ω_2 and ω_3, the efficiency score of purchasing firm raises as well. Since ω_1 reflects a situation in which we do not take DEA efficiency into account when planning the supply network, we observe that the purchasing firm is operating very inefficiently in this instance. However, it reaches its most efficient condition in instance ω_3, where we maximize its respective DEA efficiency score while disregarding sustainability objective functions. Therefore, it turns out that the DEA efficiency score of purchasing firm grows up by 62.5%. Figure 3.1 demonstrates cost components of total

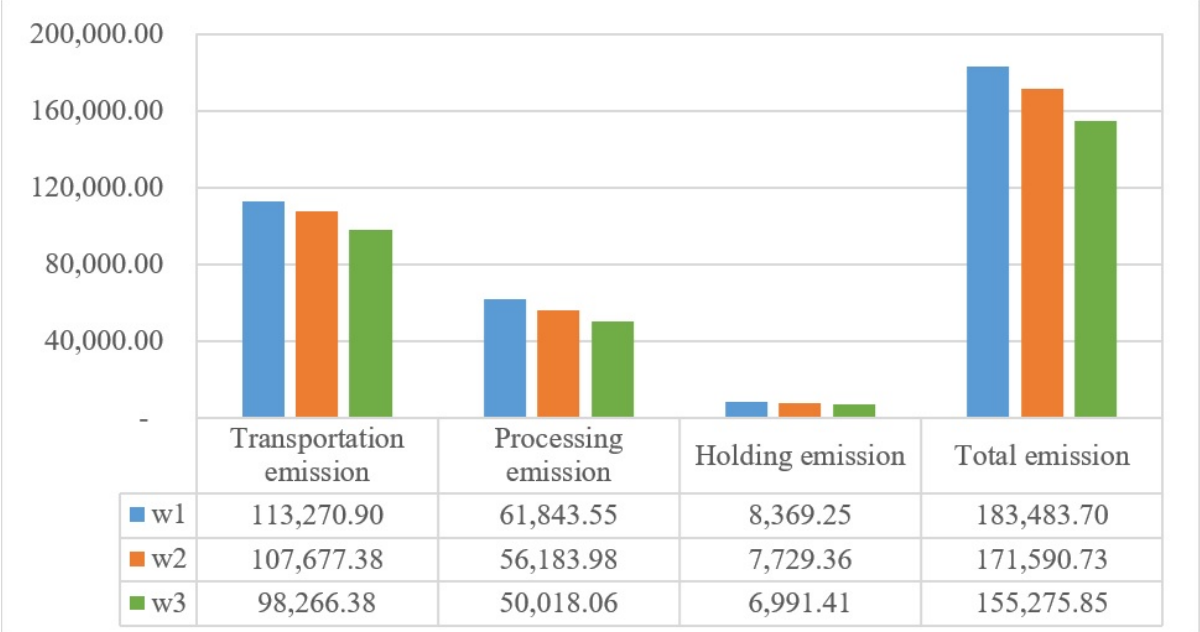


	Purchasing cost	Transportation cost	Shortage cost	Holding cost	Total cost
w1	83,716,194.8	2,859,765.3	0	269,132.0	86,845,092.0
w2	83,015,936.8	2,154,370.3	0	186,025.4	85,356,332.4
w3	81,925,140.5	1,392,058.6	0	102,551.7	83,419,750.9

Figure 3.1: The components of total cost objective function.

cost objective function (3.11). It turns out that total purchasing cost decreases when the importance weights ω^{ES_1} and ω^{ES_2} grow up in instances ω_2 and ω_3. It follows because the bi-objective DEA model (3.3)-(3.10) has input orientation. To improve efficiency in an input-oriented DEA model, we need to decrease the input levels of a DMU while retaining its output levels un-

changed. Hence, we conclude the level of incoming raw materials to purchasing firm reduces as its DEA efficiency score raises from instance ω_1 to instance ω_3. It implies the purchasing firm purchases less raw materials to satisfy its demands. Hence, as depicted in Figure 3.1, total purchasing cost diminishes while increasing the DEA score of purchasing firm. Moreover, the less raw materials to purchase, the smaller amount of raw materials needs to be transported from suppliers to the purchasing firm. Hence, total transportation cost also decreases with increasing DEA efficiency score of purchasing firm. There is no shortage cost incurred in this empirical study. However, the holding cost lessens as the DEA score increases from instance ω_1 to instance ω_3. It follows because the amount of incoming raw materials is decreasing from instance ω_1 to instance ω_3, whereas demand is fixed. Based on constraint (3.21), since the amount of raw materials held (or the amount of shortage) at the end of the former period is also fixed at the current period, it turns out the amount of raw materials held at the end of current period reduces from instance ω_1 to instance ω_3, which itself results in diminishing holding cost. Finally, we observe that total cost of the supply network decreases as the efficiency of purchasing firm grows up. This is a naturally true conclusion that the more efficiency, the less cost incurred (Nuti et al. 2011, Gok and Altndag 2015).



	Transportation emission	Processing emission	Holding emission	Total emission
w1	113,270.90	61,843.55	8,369.25	183,483.70
w2	107,677.38	56,183.98	7,729.36	171,590.73
w3	98,266.38	50,018.06	6,991.41	155,275.85

Figure 3.2: The components of total cost objective function.

Figure 3.2 represents the components of emission objective function (3.12). The transporta-

tion emission reduces as the amount of incoming raw materials decreases with increasing DEA efficiency score of purchasing firm from instance ω_1 to instance ω_3. Moreover, that incoming raw materials reduce with increasing DEA efficiency score implies the suppliers produce less raw materials from instance ω_1 to instance ω_3, which results in decreasing total processing emission. The holding emission also diminishes while increasing the DEA efficiency score. The reason for decreasing holding emission is exactly the same as that for reducing holding cost explicated before. Finally, total emission of the supply network diminishes once the DEA efficiency of purchasing firm grows up.

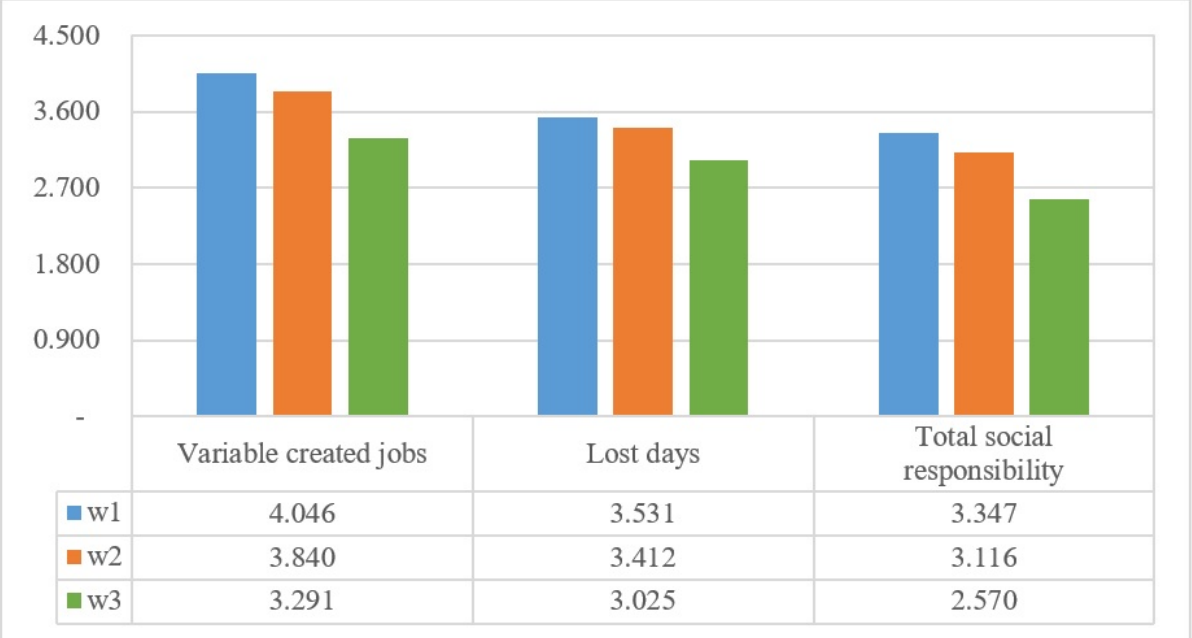


Figure 3.3: The components of total cost objective function.

Figure 3.3 shows the components of social responsibility objective function (3.13). It appears that the number of variable created jobs in the purchasing firm decreases as its DEA efficiency score grows up. Based on how we formulated the number of variable jobs created in section 3.4.1, it turns out that as the amount of incoming raw materials diminishes with increasing DEA efficiency, the number of variable jobs created decreases. In other words, the less incoming raw materials, the smaller number of employees the purchasing firm needs to process them. Furthermore, the number of days lost also slightly decreases with increasing DEA efficiency. Again, according to how we derived the number of lost days in section 3.4.1, it follows that the less incoming raw materials, the less lost days we face. Finally, as the DEA efficiency of

purchasing firm increases, Figure 3.3 depicts that total social responsibility decreases, which implies a trade-off between efficiency and social responsibility aspect

3.6.2 Sensitivity analysis

In this section, we implement a sensitivity analysis on some controllable parameter of developed programming model and its solution procedure. To do so, we assume the weight of objective functions is the same as instance ω_2, i.e. $(\omega^{TP}, \omega^{TE}, \omega^{SR}, \omega^{ES_1}, \omega^{ES_2}) = (0.2, 0.2, 0.2, 0.2, 0.2)$. Since α can be defined by the decision maker, it is possible to select different levels and make a

Table 3.2: Sensitivity analysis over confidence level of stochastic constraints.

Objective function	Confidence levels			
	0.95	0.75	0.5	0.25
TC	85,356,332.4	78,873,380	60,446,770	54,554,520
TE	171,590.73	124,487	89,209.39	80,385.15
SR	3.116	3.384	3.490	3.541
Efficiency	0.8074	0.8345	0.8887	0.9106

compromise between the objective function value and risk of constraints violation. A high value for the confidence level can be specified if holding the stochastic constraints are strongly desired. However, a feasible solution cannot be found if the predefined confidence level is higher than reachable. On the other hand, a small value for the confidence level might result in violation in some stochastic constraints. In the current circumstance, one might achieve a solution that is not implementable in practice since some stochastic constraints are violated. We intuitively set the confidence level in stochastic constraints to 0.95. In this case, the risk of having the stochastic constraints unsatisfied is equal to 0.05. Table 3.2 represents how the value of objective functions changes while the confidence level decreases. It appears that each objective function improves as the confidence level of stochastic constraints reduces. It follows because the cumulative distribution function is a non-decreasing function. It implies that decreasing the confidence level results in decreasing $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(\alpha)$ and increasing $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(1-\alpha)$. Having any stochastic constraint rearranged, we observe that $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(\alpha)$ and $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(1-\alpha)$ provide a lower bound and upper bound for the respective stochastic constraints. In other words, $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(\alpha)$ and $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(1-\alpha)$ appear in the right-hand-side of a stochastic constraint whose sign is $\geq$ and $\leq$, respectively. Therefore, decreasing $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(\alpha)$ and increasing $\varphi^{-1}_{\hat{dem}_{jk}^{\,t}}(1-\alpha)$ lead to expanding the feasibility area of the programming model, which itself results in improving the objective functions as

presented in Table 3.2. This conclusion can also be drawn intuitively, i.e. once the confidence level of a constraint decreases, then the corresponding constraint is less binding, which leads to improving the objective functions. However, determining a small confidence level to obtain an improved value of the objective functions is not recommended in practice because, in this case, the risk of having the stochastic constraints unsatisfied accordingly increases.

3.6.3 Valid inequalities performance

As explained earlier, the purpose of adding valid inequalities to the relaxed SSSOA model is to accelerate the solution procedure in terms of solution time. Hence, it is worth evaluating the performance of the added valid inequalities in achieving this target. Meanwhile, the empirical study explored in this chapter does not have significantly high dimension. Accordingly, the effect of valid inequalities in decreasing solution time might not be completely caught for this study. Hence, in this section, we generate two substantially larger instances of SSSOA problem for which the precise quantity of most parameters is obtained by Monte Carlo sampling method implemented on particular uniform distribution functions with predetermined intervals. Table 3.3 present the dimension of generated instances. Since the purpose of this evaluation is only to examine the power of valid inequalities, we set N to one.

Table 3.3: Dimension of the studied test problems.

Instance	m	n	s	K	L_i	T	N	# of integer variables	# of total variables	# of constraints
1	15	8	6	4	2	6	1	6,144	41,709	129,601
2	25	12	6	4	2	12	1	29,952	205,505	643,297

The uniform distribution functions are defined in such a way that the proposed model becomes feasible and the obtained results sound reasonable. In this regard, if $U \sim (a, b)$ denotes a uniform random variable in the interval (a, b), Table 3.4 presents the uniform distribution functions that are used to generate the quantity of the corresponding parameters. In both instances, we also assume all $nsu^t_{jk} = 5$, $\ell^t_{ij1} = 80$, $\ell^t_{ij2} = 80$ and that the demand parameter $\hat{dem}^t_{jk}$ follows a log-normal distribution function with parameters $4.7 + 0.02j + 0.05k + 0.02t$ and 0.1. For instance, log-normal parameters of $\hat{dem}^1_{32}$ are 4.88 and 0.1. In addition, we set all the confidence levels of stochastic constraints to 0.95. Finally, a cover is defined in such a way that $\dot{N} = 7$ in constraint (3.76) for both instances.

We solve each instance of the relaxed SSSOA problem 5 times with each objective function as well as the aggregated single objective function with equally weighted objective functions

Table 3.4: Characterization of various parameters in test problems.

Parameter	Value	Parameter	Value
rep_{ij}^{t} (%)	$U \sim (4, 9)$	$pca_{k}^{t}(kg)$	$U \sim (380, 500)$
$otd_{ij}^{t}(\%)$	$U \sim (92, 100)$	$shc_{jk}^{t}(\$)$	$U \sim (660, 750)$
$cap_{ij}^{t}(kg)$	$U \sim (420, 820)$	$apr_{jk}^{t}(\%)$	$U \sim (8, 12.5)$
$fsc_{ijk}^{t}(\times 10^{2}\$)$	$U \sim (1.8, 2.2)$	$apd_{jk}^{t}(\%)$	$U \sim (58, 60)$
$vsc_{ijk}^{t}(\$)$	$U \sim (5, 10)$	$cux_{k}^{t}(\times 10^{4}\$)$	$U \sim (2, 3)$
$fse_{ijk}^{t}(kg)$	$U \sim (160, 430)$	$hoe_{jk}^{t}(\times 10^{-5}kg)$	$U \sim (31, 41)$
$vse_{ijk}^{t}(kg)$	$U \sim (0.07, 0.4)$	$per_{jk}^{t}(\%)$	$U \sim (0.9, 1)$
$msh_{jk}^{t}(kg)$	$U \sim (55, 85)$	$mpr_{jk}^{t}(kg)$	$U \sim (85, 125)$
los_{k}	$U \sim (0.4, 1)$	$ppr_{ijl}^{t}(\$)$	$U \sim (210, 270)$
$sts_{jk}^{t}(m3)$	$U \sim (210, 320)$	var_{k}	$U \sim (0.5, 1.1)$
$hoc_{jk}(\$)$	$U \sim (210, 230)$	lab_{k}^{t}	$U \sim (25, 48)$
$sph_{j}(m3)$	$U \sim (0.001, 0.002)$	$pre_{ij}(kg)$	$U \sim (0.16, 0.2)$

under four different conditions. In the first condition represented as No Cut in Table 3.5, we apply neither constraint (3.76) nor constraint (3.77) and solve the relaxed problem with no valid inequality. Then, we apply only one individual valid inequality, first constraint (3.76) and then constraint (3.77), into the relaxed problem and solve it with each objective function represented as Constraint (3.76) and Constraint (3.77), respectively, in Table 3.5. Finally, we employ both constraints (3.76) and (3.77) into the relaxed model, corresponding results of which are given as Constraint (3.76) & (3.77) in Table 3.5. Table 3.5 reports the mean and standard deviation of solution times in hour:minute:second format.

Table 3.5: Comparative evaluation of different valid inequalities.

Objective function	Instance	No Cut		Constraint (76)		Constraint (77)		Constraint (76)&(76)	
		Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev
TC	1	04:18:29	00:12:09	00:42:57	00:06:35	02:03:18	00:09:47	00:17:49	00:03:12
	2	07:42:36	00:17:56	02:14:29	00:11:08	03:48:52	00:18:23	01:38:08	00:14:29
TE	1	03:51:14	00:10:43	00:36:23	00:07:44	01:46:31	00:10:51	00:14:50	00:02:47
	2	07:02:24	00:17:04	02:06:51	00:13:24	03:12:19	00:16:22	01:22:38	00:13:49
SR	1	04:52:31	00:14:41	00:53:18	00:08:13	02:32:47	00:11:53	00:39:34	00:06:13
	2	08:29:18	00:16:12	02:31:56	00:14:38	04:01:26	00:17:02	01:54:19	00:14:51
ES_1	1	00:56:28	00:10:37	00:11:52	00:04:38	00:23:15	00:03:18	00:08:53	00:02:34
	2	01:19:37	00:08:26	00:17:24	00:04:52	00:31:49	00:05:17	00:14:25	00:03:05
ES_2	1	00:50:13	00:07:46	00:14:27	00:05:02	00:26:19	00:06:11	00:12:18	00:02:46
	2	01:12:16	00:11:30	00:33:57	00:05:43	00:44:51	00:05:22	00:23:47	00:04:09
Agg.	1	05:08:25	00:16:39	01:44:18	00:15:42	02:19:03	00:13:35	01:53:27	00:11:14
	2	09:43:56	00:19:41	02:38:40	00:11:09	03:22:13	00:16:53	02:12:50	00:13:01

First, it appears that the solution time of instance 1 is far less than that of instance 2 for all objective functions because its dimension, in terms of the number of variables and constraints, is smaller as presented in Table 3.3. Moreover, for all objective functions, comparing the solution times mean of instances reveals that both constraint (3.76) and constraint (3.77) are very powerful in reducing the solution time when applied into the model individually. However, if we compare constraint (3.76) with constraint (3.77) in terms of solution times mean, it turns out that constraint (3.76) is more effective in decreasing solution time of both instances for all objective functions. It follows because, as mentioned in Section 3.5.2, constraint (3.76) is a non-redundant (facet-defining) valid inequality that is as strong as possible. In other words, no other valid inequality can be found that is more individually powerful than constraint (3.76) for the proposed relaxed SSSOA problem. Meanwhile, when both constraints (3.76) and (3.77) are jointly applied into the problem, their power in reducing solution times intensifies as presented in Table 3.5.

3.7 Appendix

Indexes

i: Index of suppliers; $i = 1, ..., m$

j: Index of raw materials; $j = 1, ..., v$

k: Index of purchasing firms; $k = 1, ..., K$

r: Index of end products; $r = 1, ..., s$

l: Index of price levels by suppliers; $l = 1, ..., L_i$

t: Index of period; $t = 1, ..., T$

Parameters

dem^t_{jk}: required demand of raw material j in purchasing firm k in period t.

rep^t_{ij}: rejection percentage of raw material j provided by supplier i in period t.

ppr^t_{ijl}: unit purchasing price of raw material j at price level l provided by supplier i in period t.

fsc^t_{ijk}: fixed shipment cost of raw material j shipped from supplier i to purchasing firm k in period t.

vsc^t_{ijk}: variable shipment cost of unit raw material j shipped from the supplier i to purchasing firm k in period t.

msh^t_{jk}: maximum amount of shortage for raw material j at purchasing firm k in period t.

sts^t_{jk}: maximum available storing space for raw material j in purchasing firm k in period t.

nsu^t_{jk}: maximum number of suppliers hired by purchasing firm k to provide raw material j in period t.

otd^t_{ij} : on-time delivery percentage of raw material j provided by supplier i in period t.

apd^t_{jk}: minimum acceptable percentage for on-time delivery of raw material j to purchasing firm k in period t.

apr^t_{jk}: maximum acceptable percentage for rejecting raw material j in purchasing firm k in period t.

cap^t_{ij}: capacity of supplier i to provide raw material j in period t.

fse^t_{ijk}: fixed shipment CO2 emission of raw material j shipped from supplier i to purchasing firm k in period t.

vse^t_{ijk}: variable shipment CO2 emission of raw material j shipped from supplier i to purchasing firm k in period t.

hoe^t_{jk}: holding CO2 emission of each unit of raw material j in purchasing firm k in period t.

mpr^t_{jk}: maximum perished amount of material j in purchasing firm k in period t.

hoc^t_{jk}: holding cost of each unit of raw material j in purchasing firm k in period t.

sph_j: required space for holding a unit of raw material j.

ℓ^t_{ijl}: upper limit of purchase volume for raw material i in supplier j at price level l in period t; $\ell^t_{ij1} \leq \ell^t_{ij2} \leq ...\ell^t_{ijL_i}$.

shc^t_{jk}: shortage cost of each unit of raw material j in purchasing firm k in period t.

per^t_{jk}: perishability rate of material j in purchasing firm k in period t.

act: average capacity of transportation vehicles.

pca^t_k : production capacity of purchasing firm k in period t.

L_i: number of price levels offered by supplier i.

pre_{ij}: processing CO2 emission of material j by supplier i.

cux^t_k: current expenses for purchasing firm k in period t.

lab^t_k: labor forces needed in purchasing firm k in period t.

los_k: lost days caused by works damages in purchasing firm k.

var_k: variable number of created jobs at purchasing firm k.

M: a big positive number.

Decision variables

X^t_{ijlk}: order quantity of raw material j at price level l provided by supplier i for purchasing firm k in period t.

Y^t_{ijlk}: 1 if supplier i provides raw material j at price level l to purchasing firm k in period t; Otherwise 0.

Z^t_{jk}: 1 if raw material j is stored by purchasing firm k in period t; Otherwise 0.

U^t_{jk}: if raw material j is lacked by purchasing firm k in period t; Otherwise 0.

ILP^t_{jk}: inventory level of material j at the end of period t in purchasing firm k.

$\overline{ILP}^t_{jk}$: shortage level of material j at the end of period t in purchasing firm k.

λ_{jk}: DEA weight of material j in purchasing firm k.

$\bar{\lambda}_k$: DEA weight of current expenses in purchasing firm k.

$\bar{\bar{\lambda}}_k$: DEA weight of labor force in purchasing firm k.

μ_{rk}: DEA weight of end-product r in purchasing firm k.

d_k: inefficiency of purchasing firm k.

η: maximum inefficiency.

Chapter 4

Sustainable Supplier Selection and Order Allocation: A Novel Multi-Objective Programming Model with a Hybrid Solution Approach

Sustainable supplier selection is the process of identifying the appropriate supply partners of an organization that produces the best mutual value, while diminishing the various effects of negative operational impacts on society and the environment. As more organizations outsource supply chain activities to third parties, supplier selection plays a significant role in impacting an organizations sustainability goals. Alongside sustainable supplier selection, assigning product orders to suppliers while satisfying sustainability criteria creates a dual set of targeted objectives. Decision support methodologies that support the dual objectives of sustainable supplier selection and order allocation are needed, yet there is a paucity of research in the literature on this subject. Our research seeks to develop an inclusive multi-objective mixed integer linear programming model, which accounts for multiple periods, multiple products and multimodal transportation, to evaluate suppliers and allocate order quantities. In addition, the model takes sustainability criteria, as well as both shortage and discount conditions into consideration. A hybrid three-step solution methodology is presented, by first converting the original multi-objective problem to a single objective model using the -constraint method. In the second step, the single objective programming model is solved using the Benders decomposition algorithm that in turn is accelerated by a variety of algorithmic enhancements. Finally, among all the

Pareto optimal solutions of the original multi-objective programming problem, the preferable solution is intelligently selected based on the DEA super efficiency score of all purchasing firms as a decision support tool. The applicability of the proposed approach is illustrated using an empirical case study from the automotive sector.

4.1 Introduction

The supplier selection decision process is an important determinant of improved supply chain performance. This decision involves evaluating and selecting the best suppliers for a sourcing requirement, and assigning production volumes to selected suppliers. The decision results in reduced purchasing cost and improved organizational competitiveness. For these reasons, supplier selection is considered a strategic decision in the field of supply chain management (Monczka et al., 2014; Willis et al. 1993). Multiple factors must be considered in evaluating suppliers during the selection decision. Dickson (1966) identified 23 criteria determined by purchasing managers for different types of supplier selection problems. A review of methods and criteria of supplier selection by Weber et al. (1991) revealed that 47 out of 76 reviewed papers considered multiple criterion during the evaluation process. This research ascertains that the supplier selection problem is intrinsically a multi-criteria problem, and that it is important to consider trade-offs between tangible and intangible factors in selecting suppliers. Supplier selection can be made more complicated by the fact that different and sometimes conflicting criteria must be factored into the decision.

As the result of increased customer knowledge and ecological pressures from markets and various stakeholders, organizations must consider environmental and social impacts of their operations, in addition to traditional economic profits. The concern around environmental and social impacts of business activities has led to a new paradigm called sustainable development. Sustainable development is defined as an organizational development that meets the needs of the present without compromising the ability of future generations to meet their own needs (WCED 1987). While diverse interpretations of sustainability exist, a central concept employed in operationalizing sustainability is the triple bottom line approach, which stipulates that performance must be considered in the three dimensions of economic, environmental and social performance (Elkington 1998). Dyllick and Hockerts (2002) framed the three dimensions of sustainability as the business case (economic), the natural case (environmental), and the societal case (social). Accordingly, practitioners and academia alike have emphasized the field of sustainable supply chain management (Brandenburg et al. 2014; Seuring and Muller 2008).

In the context of the increased focus on sustainability, it is therefore critical that organizations source from economically, environmentally and socially responsible suppliers (Buyukozkan and Cifci 2012). However, there are multiple challenges that arise in simultaneously seeking to

optimize economic, environmental and social dimensions of performance within supplier selection processes (Govindan et al. 2015; Zimmer et al. 2016). This has given rise to the sustainable supplier selection problem, which considers both environmental and social elements of suppliers performance (Zimmer et al. 2016). Prior research has included economic and environmental aspects of sustainability in supplier selection (Govindan et al. 2015), however the social responsibility dimension is often considered separately or overlooked altogether (Kuo et al. 2010; Bai and Sarkis 2010; Punniyamoorthy et al. 2011; Govindan et al. 2013). In addition, there is a dearth of research that considers all three dimensions of sustainable supplier selection and order allocation simultaneously. In sum, research that simultaneously considers both sustainable supplier selection criterion and order allocation is nascent (Azadnia et al. 2015).

In addressing this gap, we propose a novel and comprehensive multi-objective mixed integer linear programming (MILP) model, which considers all sustainability aspects (economic, environmental and social responsibility) aligned with the triple bottom line approach. The proposed model models the decision across multiple periods, multiple products and multimodal transportation options in the presence of shortage and discount conditions.

As the first step of the solution procedure, the ε-constraint method is used in this chapter in order to deal with the multiple objective functions and derive a single objective MILP model. In this model the objective function considers the economic aspect of sustainability as the primary objective function of the model. Accordingly, the objective functions associated with the environmental and social responsibility aspects are treated as appropriately defined constraints. The resulting single objective MILP model is a very complex large-scale sustainable supplier selection and order allocation models, due to the numerous raw materials considered, a long list of suppliers for each raw material and various types of transportation modes. In the second step of the solution procedure, an effective solution methodology based on Benders decomposition algorithm (BDA) is developed to solve the single objective MILP model and consequently derive a Pareto optimal solution for the original multi-objective MILP problem. In addition, a variety of enhancements are deployed to accelerate the Benders decomposition algorithm, including: i) adding valid inequalities to the master problem to reduce the number of feasibility cuts, ii) generating enhanced Pareto optimal cuts to exclude a larger space of the master problem, iii) specializing the local branching search to the derived single objective MILP model to concurrently improve both lower and upper bounds during the execution of Benders decomposition, and iv) generating knapsack cuts for master problem.

Once a set of Pareto optimal solutions are derived, the preferred solution is selected using data envelopment analysis (DEA) super efficiency in the final step of the developed solution procedure. DEA super efficiency is applied in an innovative approach, where physical inputs and outputs are derived to acquire DEA super efficiency of purchasing firms. The physical inputs (raw materials) purchased from suppliers are attained from Pareto optimal solutions. On the

other hand, physical outputs (end products) manufactured by purchasing firms are estimated using properly defined production functions. Afterwards, the DEA super efficiency of purchasing firms is derived using physical inputs and outputs. Therefore, the preferable solution is chosen among all Pareto optimal solutions in such a way that the purchasing firms are as efficient as possible. In this case, the decision maker is not required to select from numerous Pareto optimal solutions for the preferable solution based on individual (and thus subjective) criteria. Instead, an analytical decision support tool is developed to assist decision makers in selecting the preferable solution intelligently. The proposed methodology presents several unique advantages over previous studies: i) our literature review suggests that the majority of former studies have applied DEA only in supplier selection problem, and ii) in all reviewed papers, the primary resources and performance measures were inputs and outputs of DEA model, respectively. This standpoint was initially proposed by Boussofiane et al. (1991). In these studies, purchasing cost was the primary input while quality was the primary output of the DEA model. These criteria are not in seeking to understand the overall efficiency of purchasing firms relative to other forms of sustainable value defined earlier. The proposed framework in this chapter to sustainable supplier selection and order allocation is depicted in Figure 4.1. In summary, the contributions of our approach are as follows:

- A novel and comprehensive multi-objective MILP model for the sustainable supplier selection and order allocation problem is developed, which accounts for multiple periods, multiple products and multimodal transportation. The developed model, which includes all sustainability performance metrics, also considers both shortage and discount conditions into account.

- The original multi-objective MILP model is converted to a single objective programming model using -constraint method.

- A Benders decomposition algorithm that is computationally accelerated is developed to solve the single objective programming model and derive a Pareto optimal solution.

- Using physical inputs and outputs, DEA super efficiency score of purchasing firms is derived to assist the decision maker with intelligently choosing the preferable solution among all Pareto optimal solutions. Furthermore, the efficiency score of purchasing firms themselves are attained.

- The impact of suppliers' performance on the purchasing firms with respect to their DEA super efficiency scores can be clearly explored.

To the best of the authors knowledge, our approach to the sustainable supplier selection and order allocation problem is unique as prior studies have not addressed this problem.

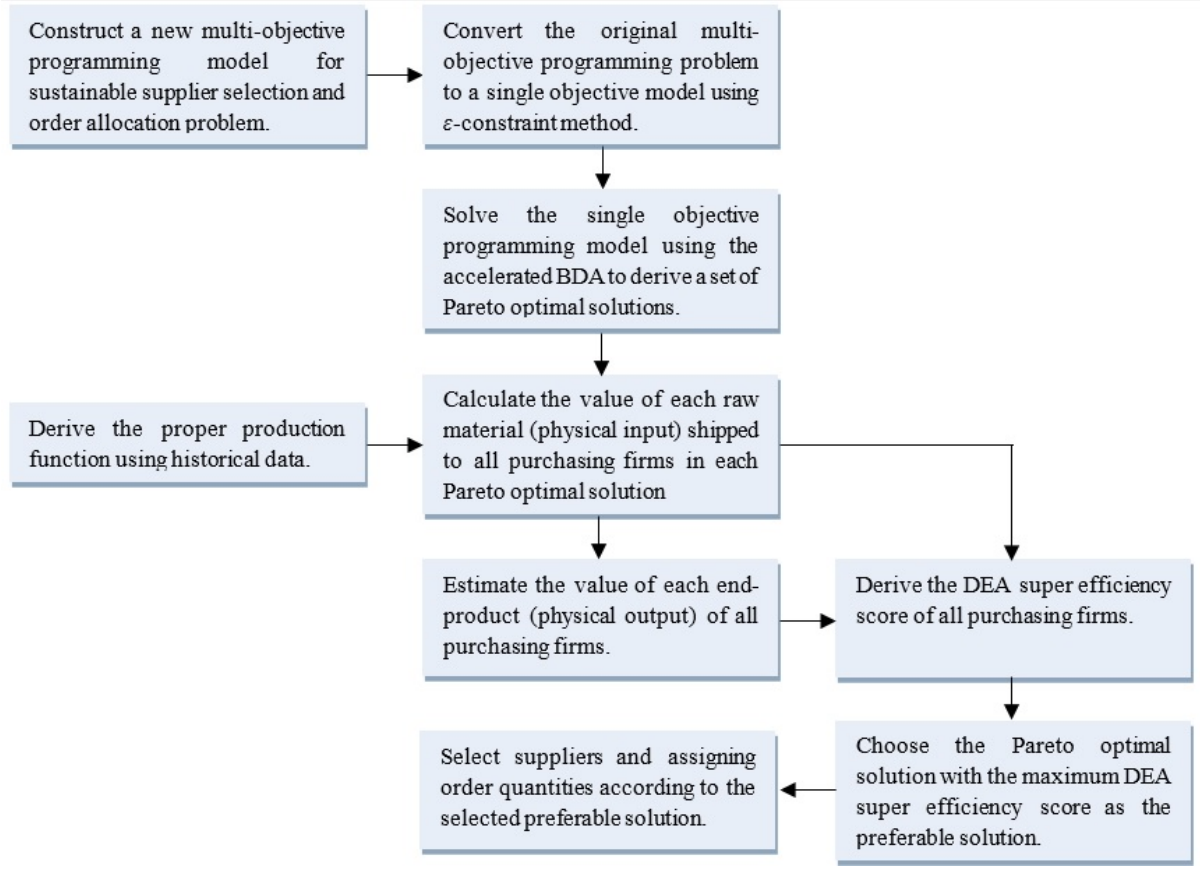


Figure 4.1: The closed loop supply chain network examined in this chapter.

The remainder of this chapter is organized as follows: Section 4.2 introduces a new multi-objective MILP model to sustainable supplier selection and order allocation. The proposed solution procedure is elaborated in Section 4.3. Section 4.4 explains how to implement the developed model and its solution procedure using an empirical study. Finally, Section 4.5 is devoted to conclusions and future works.

4.2 Literature review

All papers reviewed in sections 3.2.1, 3.2.2 and 3.2.3 can be recited here as relevant literature. Comparing the developed model and its solution procedure with the previous studies reviewed in the above-mentioned section, we understand how the current literature in the area of sustainable supplier selection and order allocation is expanded by our developed model in current chapter.

4.3 Model development

A novel multi-objective MILP model for sustainable supplier selection and order allocation problem with multiple periods, multiple products and multimodal transportation in the presence of price discount and shortage conditions, which henceforth is named as SSSOA model, is developed in this section using the the indexes, parameters and decision variables given in the Appendix.

4.3.1 Objective functions

Cost objective function: Objective function (4.1) that is associated with the economic aspect of sustainability minimizes total cost over the time horizon. It includes the purchasing cost, fixed and variable transportation costs, holding cost of extra raw materials, shortage cost and fixed contract cost over all time periods.

$$\begin{aligned} Min \qquad TC = & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} ppr_{ijl}^{t} + \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Q_{ijkp}^{t} vsc_{ijkp}^{t} + \\ & \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} fsc_{ijkp}^{t}(X_{ijlk}^{t}/tca_{ijkp}^{t}) + \\ & \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} ILP_{jk}^{t} hoc_{jk}^{t} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} \overline{ILP}_{jk}^{t} shc_{jk}^{t} + \\ & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i}\sum_{k=1}^{K} Y_{ijlk}^{t} con_{ijk}^{t} \end{aligned} \tag{4.1}$$

Emission objective function: Objective function (4.2) minimizes total CO2 emission, which consists of transportation emission, processing emission of suppliers and holding emission of extra raw materials over all time periods. This objective function is related to the environmental aspect of sustainability.

$$\begin{aligned} Min \qquad TE = & \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i}\sum_{k=1}^{K} fse_{ijkp}^{t}(X_{ijlk}^{t}/tca_{ijkp}^{t}) + \\ & \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{k=1}^{K} X_{ijkp}^{t} vse_{ijkp}^{t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} pre_{ij} + \\ & \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} ILP_{jk}^{t} hoe_{jk}^{t} \end{aligned} \tag{4.2}$$

Social responsibility objective function: This objective function that is associated with social responsibility aspect of sustainability contains two components including job opportunities created and lost days caused from works damages. These components are formulated as the first and second terms in objective function (4.3), respectively, where ζ_1 and ζ_2 are normalizing weights. The procedure proposed by Devika et al. (2014) is adapted in formulating the current terms.

$$\begin{aligned} Max \qquad SR =& \zeta_1[\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} var_k X_{ijlk}^t/pca_k^t]- \\ & \zeta_2[\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} los_k X_{ijlk}^t/pca_k^t] \\ =& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K}(\zeta_1 var_k - \zeta_2 los_k) X_{ijlk}^t/pca_k^t \end{aligned} \tag{4.3}$$

4.3.2 Constraints

Constraint (4.4) determines the amount of each raw material held by each purchasing firm at the end of each time period, while constraint (4.5) derives the shortage amount for each raw material faced by each purchasing firm at the end of each period, where $ILP_{jk}^0 = \overline{LOI}_{jk}^0 = 0$:

$$ILP_{jk}^t = ILP_{jk}^{t-1} + \sum_{i=1}^{m}\sum_{p=1}^{P} Q_{ijkp}^t - dem_{jk}^t - \overline{ILP}_{jk}^{t-1}; \qquad \forall k, j, t \tag{4.4}$$

$$\overline{ILP}_{jk}^t = \overline{ILP}_{jk}^{t-1} + dem_{jk}^t - \sum_{i=1}^{m}\sum_{p=1}^{P} Q_{ijkp}^t - ILP_{jk}^{t-1}; \qquad \forall k, j, t \tag{4.5}$$

Constraint (4.6) requires the total rejected amount of each raw material in each purchasing firm be less than or equal to its respective pre-determined quantity at each time period.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^t rep_{ij}^t \leq dem_{jk}^t apr_{jk}^t; \qquad \forall k, j, t \tag{4.6}$$

That the total amount of each raw material delivered on-time to each purchasing firm at each time period should be greater than or equal to its corresponding desired quantity is satisfied by constraint (4.7).

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^t otd_{ij}^t \geq dem_{jk}^t apd_{jk}^t; \qquad \forall k, j, t \tag{4.7}$$

Constraint (4.8) implies the total weighted amount of each raw material purchased from

all suppliers to each purchasing firm should exceed the respective pre-specified lower bound of rating value at each time period.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} X_{ijlk}^{t} rwe_{ij} \geq dem_{jk}^{t} lbr_{jk}; \qquad \forall k, j, t \tag{4.8}$$

Any purchasing firm has a limited space for holding any raw material. Therefore, constraint (4.9) satisfies that the area occupied by each raw material supplied by all suppliers to each purchasing firm at each time period should be less than or equal to the respective total available space.

$$[ILP_{jk}^{t-1} + \sum_{i=1}^{m}\sum_{p=1}^{P} Q_{ijkp}^{t} - dem_{jk}^{t} - \overline{ILP}_{jk}^{t}] sph_j \leq H_{jk}^{t} sts_{jk}^{t}; \qquad \forall k, j, t \tag{4.9}$$

Constraint (4.10) implies that the shortage amount of each raw material should be less than or equal to a predetermined quantity at each period.

$$\overline{ILP}_{jk}^{t} + dem_{jk}^{t} - \sum_{i=1}^{m}\sum_{p=1}^{P} Q_{ijkp}^{t} - ILP_{jk}^{t-1} \leq (1 - H_{jk}^{t}) msh_{jk}^{t}; \qquad \forall k, j, t \tag{4.10}$$

The amount of raw material j provided by supplier i in all price levels for all purchasing firms should be less than or equal to the capacity of that supplier for raw material j, which is guaranteed by constraint (4.11).

$$\sum_{l=1}^{L_i}\sum_{k=1}^{K} X_{ijlk}^{t} \leq cap_{ij}^{t}; \qquad \forall i, j, t \tag{4.11}$$

In order to help a purchasing organization manage its supply network more appropriately and control the associated risks, constraint (4.12) satisfies that, for each raw material, a particular number of suppliers is chosen for each purchasing firm at each time period.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} Y_{ijlk}^{t} \leq nsu_{jk}^{t}; \qquad \forall j, k, t \tag{4.12}$$

Constraint (4.13) indicates, at each time period, all purchased raw materials from suppliers should be transported to purchasing firms, i.e. no purchased raw materials are remained in suppliers.

$$\sum_{l=1}^{L_i} X_{ijlk}^{t} = \sum_{p=1}^{P} Q_{ijkp}^{t}; \qquad \forall i, j, k, t \tag{4.13}$$

Constraint (4.14) requires that the amount of each raw material that is transported by a

transportation mode from a supplier to a purchasing firm should be less than or equal to the respective transportation capacity at each time period.

$$Q^t_{ijkp} \leq U^t_{ijkp} tca^t_{ijkp}; \qquad \forall i, j, k, p, t \tag{4.14}$$

Constraint (4.15) satisfies that if a raw material cannot be transported by a transportation mode, then no supplier should use that transportation mode to send that type of raw material to any purchasing firm.

$$\sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{k=1}^{K} U^t_{ijkp} \leq z^p_j TmK; \qquad \forall j, p \tag{4.15}$$

Constraints (4.16) and (4.17) imply that if $Y^t_{ijlk} = 1$, then X^t_{ijlk} should be within the respective purchase volume of price level l at each time period. Conversely, if $Y^t_{ijlk} = 0$, then they guarantee that no raw material j at price level l is purchased from supplier i for purchasing firm k at time period t., i.e. $X^t_{ijlk} = 0$. In addition, constraint (4.18) guarantees that at most one discount interval can be selected for each raw material provided by each supplier for a purchasing firm in a particular period of time.

$$Y^t_{ijlk} \ell^t_{ijl} \leq X^t_{ijlk} \leq Y^t_{ijlk} \ell^t_{ij,l+1}; \qquad \forall i, j, l = 1, ..., L_i - 1, k, t \tag{4.16}$$

$$Y^t_{ijL_ik} \ell^t_{ijL_i} \leq X^t_{ijL_ik} \leq Y^t_{ijL_ik} M; \qquad \forall i, j, k, t \tag{4.17}$$

$$\sum_{l=1}^{L_i} Y^t_{ijlk} \leq 1; \qquad \forall i, j, k, t \tag{4.18}$$

Finally, constraint (4.19) requires Y^t_{ijlk}, H^t_{jk} and U^t_{ijkp} be binary, and X^t_{ijlk}, Q^t_{ijkp}, ILP^t_{jk} and $\overline{ILP}^t_{jk}$ be nonnegative variables.

$$Y^t_{ijlk}, H^t_{jk}, U^t_{ijkp} \in \{0, 1\}, X^t_{ijlk}, Q^t_{ijkp}, ILP^t_{jk}, \overline{ILP}^t_{jk} \geq 0; \qquad \forall i, j, l, k, p, t \tag{4.19}$$

For the above multi-objective programming model, ψ is called a Pareto optimal solution if it is impossible to improve all objective functions simultaneously by any other solution. Among all Pareto optimal solutions, the one selected by a decision maker is called the preferable solution (Branke et al. 2008).

4.4 Solution methodology

In this section, a solution procedure for the SSSOA model developed in the preceding section is presented. In the first step of the proposed solution methodology, the multiple objective functions are transformed into one single objective function using the -constraint method. The

obtained single objective programming model is then solved using the Benders decomposition algorithm, which itself is accelerated by several computational enhancements, to attain a set of Pareto optimal solutions. Having Pareto optimal solutions derived, the preferable solution is selected using DEA super efficiency of purchasing firms in the final step of the proposed solution procedure based on physical inputs (raw materials) and outputs (end products). In other words, among all the Pareto optimal solutions, the one maximizing DEA super efficiency of purchasing firms is chosen as the preferable solution.

4.4.1 The ε-constraint method

In the developed SSSOA multi-objective programming model, the goal is to provide a solution compromising among economic, environmental and social responsibility aspects of sustainability. Among the available methods to deal with a multi-objective programming problem, the ε-constraint method is selected in the present chapter due to the simplicity and applicability of its implementation. It has been also applied to various multi-objective problems in the literature (e.g. Felfel et al. 2016, Fahimnia et al. 2017, Paydar et al. 2017, Mohammed et al. 2017).

The proposed SSSOA model, in which total cost and CO2 emission are minimized and social responsibility is maximized, can be summarized as follows:

$$\begin{aligned} Min \quad & \{TC, TE\} \\ Max \quad & SR \\ s.t. \quad & \mathbf{x} \in S \end{aligned} \tag{4.20}$$

where $\mathbf{x}$ is the vector of decision variables, TC, TE and SR are total cost, CO2 emission and social responsibility objective functions, respectively, and S denotes the space of feasible solutions. In the ε-constraint method, the original multi-objective problem is converted into a single objective programming model by retaining one of the objective functions as the primary objective function in the model. Accordingly, the remaining objective functions are expressed as the properly defined constraints with enforcing upper and/or lower bounds. Therefore, if total cost objective function (4.1) is chosen as the primary objective function, the following single objective programming model is attained where total CO2 emission and social responsibility objective functions are treated as model constraints:

$$\begin{aligned} Min \quad & TC \\ s.t. \quad & TE \leq TE_{min} + v\Delta\varepsilon_{TE} \\ & SR \geq SR_{min} + v\Delta\varepsilon_{SR} \\ & \mathbf{x} \in S \end{aligned} \tag{4.21}$$

where $v = 0, 1, ..., \vartheta$, $\Delta\varepsilon_{TE} = \dfrac{TE^{max} - TE^{min}}{\vartheta}$ and $\Delta\varepsilon_{SR} = \dfrac{SR^{max} - SR^{min}}{\vartheta}$. In order to derive the maximum and minimum values of total CO2 emission and social responsibility objective functions, the following steps are taken: Step 1: Obtain the global optimum solution of each objective function over S. Then, let $\mathbb{X} = \{X^*_{TC}, X^*_{TE}, X^*_{SR}\}$, where X^*_{TC}, X^*_{TE} and X^*_{SR} are the obtained global optimum solutions corresponding to TC, TE and SR objective functions. Step 2: Find the values of the objective functions TE and SR at each point of $\mathbb{X}$. Step 3: Derive the minimum value of TE and SR objective functions as $TE^{min} = min\{TE(x), x \in \mathbb{X}\}$ and $SR^{min} = min\{SR(x), x \in \mathbb{X}\}$, and their maximum value as $TE^{max} = max\{TE(x), x \in \mathbb{X}\}$ and $SR^{max} = max\{SR(x), x \in \mathbb{X}\}$.

However, to guarantee an optimum solution of (4.21) is a Pareto optimal solution of the original multi-objective problem (4.20), the modification proposed by Mavrotas (2009) is applied in this paper, where the constraints associated with the added objective functions are transformed to equality by explicitly incorporating the appropriate slack or surplus variables (S_1 and S_2) and then penalizing the current new variables at the single objective function. In this case, model (4.21) becomes:

$$\begin{aligned} Min \qquad TC' = & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X^t_{ijlk} ppr^t_{ijl} + \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Q^t_{ijkp} vsc^t_{ijkp} + \\ & \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} fsc^t_{ijkp}(X^t_{ijlk}/tca^t_{ijkp}) + \\ & \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} ILP^t_{jk} hoc^t_{jk} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} \overline{ILP}^t_{jk} shc^t_{jk} + \\ & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i}\sum_{k=1}^{K} Y^t_{ijlk} con^t_{ijk} + \theta(S_1 + S_2) \end{aligned} \tag{4.22}$$

s.t. (4)-(18)

$$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i}\sum_{k=1}^{K} fse^t_{ijkp}(X^t_{ijlk}/tca^t_{ijkp}) + \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{k=1}^{K} X^t_{ijkp} vse^t_{ijkp} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} X^t_{ijlk} pre_{ij} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{v} ILP^t_{jk} hoe^t_{jk} + S_1 = TE^{min} + v\Delta\varepsilon_{TE} \tag{4.23}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} (\zeta_1 var_k - \zeta_2 los_k) X^t_{ijlk}/pca^t_k - S_2 = SR^{min} + v\Delta\varepsilon_{SR} \tag{4.24}$$

$$Y^t_{ijlk}, H^t_{jk}, U^t_{ijkp} \in \{0,1\}, X^t_{ijlk}, Q^t_{ijkp}, ILP^t_{jk}, \overline{ILP}^t_{jk}, S_1, S_2 \geq 0; \qquad \forall i, j, l, k, p, t \tag{4.25}$$

where θ is an adequately small value (usually between 10^{-3} and 10^{-6}) that does not affect the objective function. For a particular value of v, a Pareto optimal solution for the original multi-objective problem (4.20) is attained by solving the current single objective programming model. Hence, a set of $\vartheta + 1$ Pareto optimal solutions is acquired that defines a discretized Pareto front.

4.4.2 Benders decomposition algorithm

The single objective programming problem (4.22)-(4.25) is a MILP model, which typically needs to be solved in practice with numerous suppliers, raw materials and transportation modes in order to achieve one Pareto optimal solution associated with a particular value of v. The large-scale property of the developed model in real world yields substantially computational difficulty that cannot be alleviated by utilizing commercial optimization software such as CPLEX, GAMS and LINGO. Such a difficulty is intensified once one notes that the aforementioned model should be solved $\vartheta + 1$ times to obtain a set of $\vartheta + 1$ Pareto optimal solutions. Hence, in order to deal with such a computational complexity, a Benders decomposition algorithm (BDA) is developed in this section, which is accelerated by: i) adding valid inequalities to the master problem to reduce the number of feasibility cuts, ii) generating enhanced Pareto-optimal cuts to exclude a larger space of the master problem, iii) specializing the local branching search to the derived single objective MILP model to concurrently improve both lower and upper bounds during the execution of Benders decomposition, and iv) generating knapsack cuts for the master problem.

BDA initially proposed by Benders (1962) is currently known as an efficient algorithm to solve large-scale mixed integer programming problems. In this algorithm, the original complicated MILP problem is not directly solved and is instead decomposed into a pure integer programming problem called the master problem and a linear programming problem called the subproblem. The optimal solutions of the latter problems are used in an iterative fashion toward acquiring the optimum solution of the original MILP problem. BDA has significant advantages compared to other solution methods, e.g. metaheuristic methods, such as: 1) it relies on strong algebra concepts, 2) the convergence of this algorithm and achievement of optimum solution is analytically proven, 3) the decision maker can adjust the optimality gap precisely when it is needed, and 4) other efficient solution methods can be employed while solving the decomposed problems, i.e. master problem and subproblem (Pishvaee et al., 2014). BDA has been successfully exploited in various contexts such as energy management, supply chain management, etc. (Rahmaniani et al. 2017).

To develop BDA for the proposed single objective model (4.22)-(4.25), dual subproblem (DSP) and master problem (MP) should be firstly formulated. Fixing the binary variables to particular given values, i.e. $Y^t_{ijlk} = \bar{y}^t_{ijlk}$, $H^t_{jk} = \bar{h}^t_{jk}$ and $U^t_{ijkp} = \bar{u}^t_{ijkp}$, Benders primal

subproblem (PSP) is formulated as follows:

$$Min \qquad TC' \tag{4.26}$$

s.t. (4)-(8),(11),(13),(23),(24)

$$[ILP_{jk}^{t-1} + \sum_{i=1}^{m}\sum_{p=1}^{P} Q_{ijkp}^{t} - dem_{jk}^{t} - \overline{ILP}_{jk}^{t}]sph_j \leq \bar{h}_{jk}^{t} sts_{jk}^{t}; \qquad \forall k,j,t \tag{4.27}$$

$$\overline{ILP}_{jk}^{t} + dem_{jk}^{t} - \sum_{i=1}^{m}\sum_{p=1}^{P} Q_{ijkp}^{t} - ILP_{jk}^{t-1} \leq (1-\bar{h}_{jk}^{t}) msh_{jk}^{t}; \qquad \forall k,j,t \tag{4.28}$$

$$Q_{ijkp}^{t} \leq \bar{u}_{ijkp}^{t} tca_{ijkp}^{t}; \qquad \forall i,j,k,p,t \tag{4.29}$$

$$X_{ijlk}^{t} \leq \bar{y}_{ijlk}^{t} \ell_{ij,l+1}^{t}; \qquad \forall i,j,l=1,...,L_i-1,k,t \tag{4.30}$$

$$X_{ijlk}^{t} \geq \bar{y}_{ijlk}^{t} \ell_{ijl}^{t}; \qquad \forall i,j,l=1,...,L_i-1,k,t \tag{4.31}$$

$$X_{ijL_ik}^{t} \leq \bar{y}_{ijL_ik}^{t} M; \qquad \forall i,j,k,t \tag{4.32}$$

$$X_{ijL_ik}^{t} \geq \bar{y}_{ijL_ik}^{t} \ell_{ijL_i}^{t}; \qquad \forall i,j,k,t \tag{4.33}$$

$$X_{ijlk}^{t}, Q_{ijkp}^{t}, ILP_{jk}^{t}, \overline{ILP}_{jk}^{t}, S_1, S_2 \geq 0; \qquad \forall i,j,l,k,p,t \tag{4.34}$$

Let $\boldsymbol{\delta^1}...\boldsymbol{\delta^5}$,$\boldsymbol{\delta^6}$,$\boldsymbol{\delta^7}$,$\boldsymbol{\delta^8}$,$\boldsymbol{\delta^9}$,$\boldsymbol{\delta^{10}}...\boldsymbol{\delta^{16}}$ be vectors of dual variables associated with constraints (4.4)-(4.8), (4.11), (4.13), (4.23), (4.24) and (4.27)-(4.33), respectively. Then, the DSP that provides an upper bound for the single objective problem (4.22)-(4.25) can be derived as follows:

$$\begin{aligned}
Max \quad Z^{DSP} = & \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t}\delta_{jk}^{1,t} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t}\delta_{jk}^{2,t} - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} apr_{jk}^{t}\delta_{jk}^{3,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} apd_{jk}^{t}\delta_{jk}^{4,t} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} lbr_{jk}^{t}\delta_{jk}^{5,t} - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{i=1}^{m} cap_{ij}^{t}\delta_{ij}^{6,t} + \\
& (TE^{min} + v\Delta\varepsilon_{TE})\delta^8 + (SR^{min} + v\Delta\varepsilon_{SR})\delta^9 - \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} (\bar{h}_{jk}^{t} sts_{jk}^{t} + dem_{jk}^{t} sph_j^{t})\delta_{jk}^{10,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} (dem_{jk}^{t} - (1-\bar{h}_{jk}^{t}) msh_{jk}^{t})\delta_{jk}^{11,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P} \bar{u}_{jk}^{t} tca_{ijkp}^{t}\delta_{ijkp}^{12,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} \bar{y}_{ijlk}^{t}\ell_{ij,l+1}^{t}\delta_{ijlk}^{13,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} \bar{y}_{ijlk}^{t}\ell_{ijl}^{t}\delta_{ijlk}^{14,t} -
\end{aligned}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\bar{y}_{ijL_ik}^{t}M\delta_{ijk}^{15,t}+\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\bar{y}_{ijL_ik}^{t}\ell_{ijL_i}^{t}\delta_{ijk}^{16,t} \tag{4.35}$$

s.t.

$$\begin{aligned}-rep_{ij}^{t}\delta_{jk}^{3,t}+otd_{ij}^{t}\delta_{jk}^{4,t}+rwe_{ij}\delta_{jk}^{5,t}-\delta_{ij}^{6,t}+\delta_{ijk}^{7,t}+[pre_{ij}+(\sum_{p=1}^{P}fse_{ijkp}^{t}/tca_{ijkp}^{t})]\delta^{8}+\\ [(\xi_1 var_k-\xi_2 los_k)/pca_k^t]\delta^{9}-\delta_{ijlk}^{13,t}+\delta_{ijlk}^{14,t}-\delta_{ijk}^{15,t}+\delta_{ijk}^{16,t}\leq ppr_{ijl}^{t}+\\ (\sum_{p=1}^{P}fsc_{ijkp}^{t}/tca_{ijkp}^{t});\qquad \forall i,j,l,k,t\end{aligned} \tag{4.36}$$

$$-\delta_{jk}^{1,t}+\delta_{jk}^{1,t+1}+\delta_{jk}^{2,t+1}+hoe_{jk}^{t}\delta^{8}-sph_j\delta_{jk}^{10,t+1}+\delta_{jk}^{11,t+1}\leq hoc_{jk}^{t};\qquad \forall j,k,t<T \tag{4.37}$$

$$-\delta_{jk}^{1,t}+hoe_{jk}^{t}\delta^{8}\leq hoc_{jk}^{t};\qquad \forall j,k,t=T \tag{4.38}$$

$$-\delta_{jk}^{1,t+1}+\delta_{jk}^{2,t}+sph_j\delta_{jk}^{10,t+1}-\delta_{jk}^{11,t+1}\leq shc_{jk}^{t};\qquad \forall j,k,t<T \tag{4.39}$$

$$\delta_{jk}^{2,t}\leq shc_{jk}^{t};\qquad \forall j,k,t=T \tag{4.40}$$

$$\delta_{jk}^{1,t}+\delta_{jk}^{2,t}-\delta_{ijk}^{7,t}+vse_{ijkp}^{t}\delta^{8}-sph_j\delta_{jk}^{10,t}+\delta_{jk}^{11,t}-\delta_{ijkp}^{12,t}\leq vsc_{ijkp}^{t};\qquad \forall i,j,k,p,t \tag{4.41}$$

$$\delta^{8}\leq\theta \tag{4.42}$$

$$-\delta^{9}\leq\theta \tag{4.43}$$

$$\delta_{jk}^{1,t},\delta_{jk}^{2,t},\delta_{jk}^{3,t},\delta_{jk}^{4,t},\delta_{jk}^{5,t},\delta_{ij}^{6,t},\delta_{jk}^{10,t},\delta_{jk}^{11,t},\delta_{ijkp}^{12,t},\delta_{ijlk}^{13,t},\delta_{ijlk}^{14,t},\delta_{ijk}^{15,t},\delta_{ijk}^{16,t}\geq 0;\qquad \forall i,j,l,k,p,t \tag{4.44}$$

$$\delta_{ijk}^{7,t},\delta^{8},\delta^{9}\in\mathbb{R};\qquad \forall i,j,k,t \tag{4.45}$$

Based on the theory of duality, if the DSP is unbounded, then PSP is infeasible. In such a circumstance, Δ_r denotes the set of extreme rays of the polyhedron formed by constraints (4.36)-(4.45). Otherwise, if the DSP has a bounded optimum solution, then Δ_p represents the set of extreme points of such a polyhedron.

Now according to the solution of the DSP, the MP that provides a lower bound for the single objective problem (4.22)-(4.25) at each iteration is attained as follows:

$$Min\quad Z^{MP}=\Gamma+\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K}Y_{ijlk}^{t}con_{ijk}^{t} \tag{4.46}$$

s.t. (12),(15),(18)

$$
\begin{aligned}
\Gamma \geq & \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t}\bar{\delta}_{jk}^{1,t} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t}\bar{\delta}_{jk}^{2,t} - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} apr_{jk}^{t}\bar{\delta}_{jk}^{3,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} apd_{jk}^{t}\bar{\delta}_{jk}^{4,t} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} lbr_{jk}^{t}\bar{\delta}_{jk}^{5,t} - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{i=1}^{m} cap_{ij}^{t}\bar{\delta}_{ij}^{6,t} + \\
& (TE^{min} + v\Delta\varepsilon_{TE})\bar{\delta}^{8} + (SR^{min} + v\Delta\varepsilon_{SR})\bar{\delta}^{9} - \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(H_{jk}^{t} sts_{jk}^{t} + dem_{jk}^{t} sph_{j}^{t})\bar{\delta}_{jk}^{10,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(dem_{jk}^{t} - (1 - H_{jk}^{t}) msh_{jk}^{t})\bar{\delta}_{jk}^{11,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P} U_{jk}^{t} tca_{ijkp}^{t}\bar{\delta}_{ijkp}^{12,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ij,l+1}^{t}\bar{\delta}_{ijlk}^{13,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ijl}^{t}\bar{\delta}_{ijlk}^{14,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t} M\bar{\delta}_{ijk}^{15,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t}\ell_{ijL_i}^{t}\bar{\delta}_{ijk}^{16,t}; \quad (\bar{\boldsymbol{\delta}}^{\mathbf{1}}, ..., \bar{\boldsymbol{\delta}}^{\mathbf{16}}) \in \Delta_p
\end{aligned} \tag{4.47}
$$

$$
\begin{aligned}
0 \geq & \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t}\hat{\delta}_{jk}^{1,t} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t}\hat{\delta}_{jk}^{2,t} - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} apr_{jk}^{t}\hat{\delta}_{jk}^{3,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} apd_{jk}^{t}\hat{\delta}_{jk}^{4,t} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K} dem_{jk}^{t} lbr_{jk}^{t}\hat{\delta}_{jk}^{5,t} - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{i=1}^{m} cap_{ij}^{t}\hat{\delta}_{ij}^{6,t} + \\
& (TE^{min} + v\Delta\varepsilon_{TE})\hat{\delta}^{8} + (SR^{min} + v\Delta\varepsilon_{SR})\hat{\delta}^{9} - \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(H_{jk}^{t} sts_{jk}^{t} + dem_{jk}^{t} sph_{j}^{t})\hat{\delta}_{jk}^{10,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(dem_{jk}^{t} - (1 - H_{jk}^{t}) msh_{jk}^{t})\hat{\delta}_{jk}^{11,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P} U_{jk}^{t} tca_{ijkp}^{t}\hat{\delta}_{ijkp}^{12,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ij,l+1}^{t}\hat{\delta}_{ijlk}^{13,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ijl}^{t}\hat{\delta}_{ijlk}^{14,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t} M\hat{\delta}_{ijk}^{15,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t}\ell_{ijL_i}^{t}\hat{\delta}_{ijk}^{16,t}; \quad (\hat{\boldsymbol{\delta}}^{\mathbf{1}}, ..., \hat{\boldsymbol{\delta}}^{\mathbf{16}}) \in \Delta_r
\end{aligned} \tag{4.48}
$$

$$
Y_{ijlk}^{t}, H_{jk}^{t}, U_{ijkp}^{t} \in \{0, 1\}; \qquad \forall i, j, l, k, p \tag{4.49}
$$

$$
\Gamma \in \mathbb{R} \tag{4.50}
$$

In the MP, constraint (4.47) is an optimality cut in which $\bar{\delta}_{jk}^{1}, ..., \bar{\delta}_{ijk}^{16}$ are the optimum values of the dual variables in the bounded DSP. In the latter constraint, Γ is a free continuous variable

that is used to underestimate total cost. In addition, constraint (4.48) represents a feasibility cut in which $\hat{\delta}^1_{jk}, ..., \hat{\delta}^{16}_{ijk}$ are the extreme rays of the unbounded DSP.

In the classical BDA as presented by Algorithm 1, the values of binary variables Y^t_{ijlk}, H^t_{jk}, $U^t_{ijkp} \in \{0,1\}; \forall i,j,l,k,p$ are initially set to some feasible solutions. Afterwards, at each iteration of the algorithm, the DSP is solved using the solution of the MP for binary variables. If the DSP is bounded, then the optimality cut (4.47) is applied into the MP using an extreme point of Δ_p corresponding to the optimum solution obtained. Otherwise, an extreme ray of Δ_r is exploited to formulate the feasibility cut (4.48) to be added into the MP.

Algorithm 1. The classical BDA

1: $\boldsymbol{Y}$, $\boldsymbol{H}$, $\boldsymbol{U} \leftarrow$ initial feasible solutions, $z^{upper} \leftarrow \infty$, $z^{lower} \leftarrow -\infty$
2: **while** $(z^{upper} - z^{lower}) > \varepsilon$ **do**
3: Solve DSP
4: **if** the DSP is unbounded **then**
5: Add the feasibility cut (4.48) to the MP
6: **else**
7: Add the optimality cut (4.47) to the MP
8: $z^{upper} \leftarrow z^{DSP}$
9: **end if**
10: Solve the MP
11: $z^{lower} \leftarrow z^{MP}$
12: **end while**

The classical BDA as described above is known to have quite slow convergence (Geoffrion and Graves 1974). Therefore, several algorithmic enhancements are presented in the following subsections in order to accelerate the classical BDA.

4.4.2.1 Valid inequalities

One of the reasons for slow convergence of the classical BDA is the weak lower bounds provided by the MP at primary iterations of the algorithm (Saharidis et al. 2011). To avoid this inefficiency, some valid inequalities are derived and added to the MP in order to restrict the feasible region and produce high quality solutions. Therefore, it is expected that the defined valid inequalities reduce the number of feasibility cuts (4.48) during the execution of solution

procedure.

According to Proposition 3.2, the valid inequality (3.51) is still non-redundant (facet-defining) and thus as strong as possible for constraint (4.12). Hence, one valid inequality is derived as follows:

$$\sum_{i,l\in\dot{N}} Y^t_{ijlk} + \sum_{i,l\notin\dot{N}} \theta^t_{ijlk} Y^t_{ijlk} \leq |\dot{N}|-1; \qquad \forall j,k,t \tag{4.51}$$

where

$$\theta^t_{ijlk} = |\dot{N}|-nsu^t_{jk}; \qquad \forall j,k,t \& i,l \notin \dot{N}$$

In addition, constraint (3.52) implies that the selected suppliers provide sufficient capacity to all raw materials in purchasing firms.

$$dem^t_{jk} \leq \sum_{i=1}^{m}\sum_{l=1}^{L_i} cap^t_{ij} Y^t_{ijlk}; \qquad \forall j,k,t \tag{4.52}$$

4.4.2.2 Pareto-optimal cuts

The PSP is typically degenerate because of its customary network structure. Accordingly, the DSP might have multiple optimum solutions (Magnanti and Wong 1981). It implies that different Benders optimality cuts can be generated from a set of alternative optimum solutions. Let $\eta(.)$ denote the parts of the objective function in the DSP that are independent of the binary decision variables. In this case, according to the dominating cut definition (Magnanti and Wong 1981), the optimality cut generated by the dual solution vector $\dot{\boldsymbol{\delta}}^{\boldsymbol{T}}$ dominates that build by the dual solution vector $\tilde{\boldsymbol{\delta}}^{\boldsymbol{T}}$ if and only if

$$\begin{aligned}
&\eta(\dot{\boldsymbol{\delta}}^{\mathbf{1}},...,\dot{\boldsymbol{\delta}}^{\mathbf{9}}) - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(H^t_{jk} sts^t_{jk} + dem^t_{jk} sph^t_j)\dot{\delta}^{10,t}_{jk} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(dem^t_{jk} - (1-H^t_{jk}) msh^t_{jk})\dot{\delta}^{11,t}_{jk} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P} U^t_{jk} tca^t_{ijkp}\dot{\delta}^{12,t}_{ijkp} - \\
&\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y^t_{ijlk}\ell^t_{ij,l+1}\dot{\delta}^{13,t}_{ijlk} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y^t_{ijlk}\ell^t_{ijl}\dot{\delta}^{14,t}_{ijlk} - \\
&\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y^t_{ijL_ik} M\dot{\delta}^{15,t}_{ijk} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y^t_{ijL_ik}\ell^t_{ijL_i}\dot{\delta}^{16,t}_{ijk} \geq \eta(\tilde{\boldsymbol{\delta}}^{\mathbf{1}},...,\tilde{\boldsymbol{\delta}}^{\mathbf{9}}) - \\
&\sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(H^t_{jk} sts^t_{jk} + dem^t_{jk} sph^t_j)\tilde{\delta}^{10,t}_{jk} + \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(dem^t_{jk} - (1-H^t_{jk}) msh^t_{jk})\tilde{\delta}^{11,t}_{jk} -
\end{aligned}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P} U_{jk}^{t} tca_{ijkp}^{t}\tilde{\delta}_{ijkp}^{12,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ij,l+1}^{t}\tilde{\delta}_{ijlk}^{13,t} +$$
$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ijl}^{t}\tilde{\delta}_{ijlk}^{14,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t} M\tilde{\delta}_{ijk}^{15,t} +$$
$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t}\ell_{ijL_i}^{t}\tilde{\delta}_{ijk}^{16,t}$$

for all $\boldsymbol{Y}$, $\boldsymbol{H}$ and $\boldsymbol{U}$ with strict inequality for at least one point. An optimality cut that cannot be dominated by any other cut is called a Pareto-optimal cut. Adding Pareto-optimal cuts accelerates convergence of the BDA. Let Υ denote the polyhedron defined by the following constraints: $\sum_{i=1}^{m}\sum_{l=1}^{L_i} Y_{ijlk}^{t} \leq nsu_{jk}^{k}; \forall j,k,t$, $\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{k=1}^{K} U_{ijkp}^{t} \leq z_j^p TmK; \forall j,p$, $\sum_{l=1}^{L_i} Y_{ijlk}^{t} \leq 1; \forall i,j,k,t$, $0 \leq Y_{ijlk}^{t} \leq 1; \forall i,j,l,k,t$, $0 \leq H_{jk}^{t} \leq 1; \forall j,k,t$ and $0 \leq U_{ijkp}^{t} \leq 1; \forall i,j,k,p,t$. In addition, let $ri(\Upsilon)$ indicate the relative interior of Υ. In this case, a Pareto-optimal cut can be obtained by solving the following auxiliary dual problem:

$$\begin{aligned} Max \quad & Z^{DSP\_AUX} = \eta(\boldsymbol{\delta^1},...,\boldsymbol{\delta^9}) - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(\bar{h}_{jk}^{0,t} sts_{jk}^{t} + dem_{jk}^{t} sph_{j}^{t})\delta_{jk}^{10,t} + \\ & \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(dem_{jk}^{t} - (1-\bar{h}_{jk}^{0,t})msh_{jk}^{t})\delta_{jk}^{11,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P}\bar{u}_{jk}^{0,t} tca_{ijkp}^{t}\delta_{ijkp}^{12,t} - \\ & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K}\bar{y}_{ijlk}^{0,t}\ell_{ij,l+1}^{t}\delta_{ijlk}^{13,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K}\bar{y}_{ijlk}^{0,t}\ell_{ijl}^{t}\delta_{ijlk}^{14,t} - \\ & \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\bar{y}_{ijL_ik}^{0,t} M\delta_{ijk}^{15,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\bar{y}_{ijL_ik}^{0,t}\ell_{ijL_i}^{t}\delta_{ijk}^{16,t} \end{aligned} \tag{4.53}$$

s.t. (36)-(45)

where $\bar{\boldsymbol{y}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{h}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{u}}^{\boldsymbol{0}} \in ri(\Upsilon)$. The optimality cut generated by the optimum solution of the above auxiliary dual problem is a Pareto-optimal cut in $ri(\Upsilon)$. It is worth mentioning that Pareto-optimal cuts, which are non-dominated, would be generated using $\bar{\boldsymbol{y}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{h}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{u}}^{\boldsymbol{0}} \in ri(\Upsilon)$ because the description of the convex hull formed by the MPs constraints is not already known and that it is quite difficult to find out $\bar{\boldsymbol{y}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{h}}^{\boldsymbol{0}}$ and $\bar{\boldsymbol{u}}^{\boldsymbol{0}}$ in $ri(\Upsilon)$.

4.4.2.3 Local branching

As stated before, one of the main reasons for slow convergence of the classical BDA is computational complexity of the MP, which is an integer programming problem with new optimality and feasibility cuts appended in each iteration. The newly added cuts make the MP even more

complicated, whereas the DSP is a linear programming model that can be typically solved immediately. Therefore, it is required to spend a significant amount of time to solve the MP particularly at middle and terminating iterations of BDA. To overcome this pitfall, a near optimum solution of the MP is attained at each iteration of BDA in lieu of acquiring its optimum solution (Geoffrion and Graves 1974). To implement such an idea, a local branching approach, which was initially developed by Fischetti and Lodi (2003) to deal with complex integer programming problems, was proposed by Rei et al. (2009) to efficiently solve complex MP in BDA. In local branching methodology, a reasonable neighborhood around the previous solution is specified to confine the region of feasible solutions in the MP. The current restricted MP is then solved to either optimality or a good feasible solution. Consequently, a significant amount of time is potentially saved under this methodology because the original complicated MP is now solved within a restricted region of feasible solutions.

Let $\varsigma^1 = (\bar{\boldsymbol{y}}^{\boldsymbol{1}}, \bar{\boldsymbol{h}}^{\boldsymbol{1}}, \bar{\boldsymbol{u}}^{\boldsymbol{1}})$ be the MPs optimum solution at the current iteration. Then, the region of its feasible solutions can be divided into two sub-regions in the next iteration by introducing the following disjunction:

$$\Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{1}}, \bar{\boldsymbol{h}}^{\boldsymbol{1}}, \bar{\boldsymbol{u}}^{\boldsymbol{1}}) \leq \kappa \vee \Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{1}}, \bar{\boldsymbol{h}}^{\boldsymbol{1}}, \bar{\boldsymbol{u}}^{\boldsymbol{1}}) \geq \kappa + 1 \tag{4.54}$$

where Δ denotes a proper distance function and κ is a reasonably-valued positive integer number. In this case, the reduced MP that is restricted to a sub-region by adding the left branching cut can efficiently be solved by a commercial solver. To implement local branching procedure, Hamming distance metric is used in the present paper. Let $\tilde{Y} = \{i, j, l, k, t : Y^t_{ijlk} = 1\}$, $\tilde{H} = \{j, k, t : H^t_{jk} = 1\}$ and $\tilde{U} = \{i, j, k, p, t : U^t_{ijkp} = 1\}$. Then, the extended representation of the left branching cut is given as follows:

$$\begin{aligned} \sum_{i,j,l,k,t \in \tilde{Y}} (1 - Y^t_{ijlk}) + \sum_{i,j,l,k,t \notin \tilde{Y}} Y^t_{ijlk} + \sum_{j,k,t \in \tilde{H}} (1 - H^t_{jk}) + \sum_{j,k,t \notin \tilde{H}} H^t_{ijlk} + \\ \sum_{i,j,k,p,t \in \tilde{U}} (1 - U^t_{ijkp}) + \sum_{i,j,k,p,t \notin \tilde{U}} U^t_{ijkp} \leq \kappa \end{aligned} \tag{4.55}$$

If $\varsigma^2 = (\bar{\boldsymbol{y}}^{\boldsymbol{2}}, \bar{\boldsymbol{h}}^{\boldsymbol{2}}, \bar{\boldsymbol{u}}^{\boldsymbol{2}})$ is assumed to be the restricted MPs solution on the left sub-region, then one of the following situations might happen in practice (Jeihoonian et al. 2016):

1. if ς^2 is an optimum solution of the current subproblem obtained within the predetermined time limit, then the left branching cut is substituted with the right branching one, i.e. $\Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{1}}, \bar{\boldsymbol{h}}^{\boldsymbol{1}}, \bar{\boldsymbol{u}}^{\boldsymbol{1}}) \geq \kappa+1$, and ς^2 becomes the new reference point for local branching procedure. In this case, local branching proceeds with solving the new local branching subproblem.

2. If the current subproblem is infeasible, i.e. no ς^2 can be acquired, then the left branching cut is replaced by the right branching one and the regions size of feasible solutions in the current subproblem is increased by $\lceil \kappa/2 \rceil$, i.e. $\kappa \leftarrow \kappa + \lceil \kappa/2 \rceil$. Afterwards, local branching continues with solving the new local branching subproblem.

3. If ς^2 is a feasible solution improving the objective function within the prespecified time limit, it is regarded as the new reference point for local branching procedure. In addition, ς^2 is eliminated from the region of feasible solutions by adding $\Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{2}}, \bar{\boldsymbol{h}}^{\boldsymbol{2}}, \bar{\boldsymbol{u}}^{\boldsymbol{2}}) \leq \kappa$ and $\Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{2}}, \bar{\boldsymbol{h}}^{\boldsymbol{2}}, \bar{\boldsymbol{u}}^{\boldsymbol{2}}) \geq 1$ to the MP.

4. if ς^2 is a feasible solution that does not improve the objective function within the predetermined time limit, then it is removed from the region of feasible solutions and κ is augmented by unit. In other words, the branching cuts $\Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{2}}, \bar{\boldsymbol{h}}^{\boldsymbol{2}}, \bar{\boldsymbol{u}}^{\boldsymbol{2}}) \leq \kappa + 1$ and $\Delta(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \bar{\boldsymbol{y}}^{\boldsymbol{2}}, \bar{\boldsymbol{h}}^{\boldsymbol{2}}, \bar{\boldsymbol{u}}^{\boldsymbol{2}}) \geq 1$ are added to the MP.

4.4.2.4 Knapsack cut

If a reasonably tight upper bound is available from the DSP, then the knapsack cut initially developed by Santoso et al. (2005) can be derived, which has a substantial impact on obtaining the MPs solution. Since the optimum values of objective functions in the DSP and MP provide the upper and lower bounds respectively at each iteration, it is concluded that $z^{upper} \geq Z^{MP}$. Therefore, based on the current inequality, objective function (4.46) and constraint (4.47), the following knapsack cut is formulated to be added to the MP:

$$
\begin{aligned}
& z^{upper} - [\eta(\boldsymbol{\delta}^{\boldsymbol{1}}, ..., \boldsymbol{\delta}^{\boldsymbol{9}}) - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(H_{jk}^{t} sts_{jk}^{t} + dem_{jk}^{t} sph_{j}^{t})\bar{\delta}_{jk}^{10,t} + \\
& \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{K}(dem_{jk}^{t} - (1 - H_{jk}^{t})msh_{jk}^{t})\bar{\delta}_{jk}^{11,t} - \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{p=1}^{P} U_{jk}^{t} tca_{ijkp}^{t}\bar{\delta}_{ijkp}^{12,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ij,l+1}^{t}\bar{\delta}_{ijlk}^{13,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L_i-1}\sum_{k=1}^{K} Y_{ijlk}^{t}\ell_{ijl}^{t}\bar{\delta}_{ijlk}^{14,t} - \\
& \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t} M\bar{\delta}_{ijk}^{15,t} + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} Y_{ijL_ik}^{t}\ell_{ijL_i}^{t}\bar{\delta}_{ijk}^{16,t}] \\
& \geq \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{v}\sum_{l=1}^{L_i}\sum_{k=1}^{K} Y_{ijlk}^{t} con_{ijk}^{t}
\end{aligned}
\tag{4.56}
$$

With respect to all the presented enhancements, Algorithm 2 outlines the enhanced BDA, i.e. En-BDA. In the current accelerated BDA, it should be noted that the MP now includes

the expressions (4.46)-(4.52), (4.56), (4.12), (4.15) and (4.18). Furthermore, ζ^c denotes the non-negative parameter defined for a convex combination that updates the core point in generating the Pareto-optimal cuts. The current non-negative parameter is usually set to 0.5, which has led to satisfactory results in most cases.

Algorithm 2. En-BDA

1: $\boldsymbol{Y}$, $\boldsymbol{H}$, $\boldsymbol{U} \leftarrow$ initial feasible solutions, $z^{upper} \leftarrow \infty$, $z^{lower} \leftarrow -\infty$
2: $\bar{\boldsymbol{y}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{h}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{u}}^{\boldsymbol{0}} \leftarrow$ initial points in $ri(\Upsilon)$, $\zeta^c = 0.5$
3: **while** $(z^{upper} - z^{lower}) > \varepsilon$ **do**
4: Solve the auxiliary DSP
5: Add the Pareto-optimal cut (4.47) to the MP
6: Solve the MP
7: $z^{lower} \leftarrow z^{MP}$
8: Solve the DSP
9: **if** the DSP is unbounded **then**
10: Add the feasibility cut (4.48) to the MP
11: $\bar{\boldsymbol{y}}^{\boldsymbol{0}} \leftarrow \zeta^c \bar{\boldsymbol{y}}^{\boldsymbol{0}} + \varepsilon$, $\bar{\boldsymbol{h}}^{\boldsymbol{0}} \leftarrow \zeta^c \bar{\boldsymbol{h}}^{\boldsymbol{0}} + \varepsilon$, $\bar{\boldsymbol{u}}^{\boldsymbol{0}} \leftarrow \zeta^c \bar{\boldsymbol{u}}^{\boldsymbol{0}} + \varepsilon$
12: **else**
13: Add the optimality cut (4.47) to the MP
14: $z^{upper} \leftarrow z^{DSP}$
15: $\bar{\boldsymbol{y}}^{\boldsymbol{0}} \leftarrow \zeta^c \bar{\boldsymbol{y}}^{\boldsymbol{0}} + (1-\zeta^c)\bar{\boldsymbol{y}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{h}}^{\boldsymbol{0}} \leftarrow \zeta^c \bar{\boldsymbol{h}}^{\boldsymbol{0}} + (1-\zeta^c)\bar{\boldsymbol{h}}^{\boldsymbol{0}}$, $\bar{\boldsymbol{u}}^{\boldsymbol{0}} \leftarrow \zeta^c \bar{\boldsymbol{u}}^{\boldsymbol{0}} + (1-\zeta^c)\bar{\boldsymbol{u}}^{\boldsymbol{0}}$
16: $\boldsymbol{\zeta^1} \leftarrow (\bar{\boldsymbol{y}}^{\boldsymbol{*}}, \bar{\boldsymbol{h}}^{\boldsymbol{*}}, \bar{\boldsymbol{u}}^{\boldsymbol{*}})$
17: Implement the local branching procedure
17: **end if**
12: **end while**

4.4.3 DEA super efficiency to choose the preferable solution

For each particular value of v, one Pareto optimal solution for the original SSSOA problem is attained by solving the single objective programming model (4.22)-(4.25) using En-BDA algorithm. Therefore, a set of $\vartheta + 1$ Pareto optimal solutions is given to the decision maker in order to select the final preferable solution. In this section, a decision support tool based on DEA super efficiency is elucidated, which enables to choose the preferable solution with respect to the super efficiency of all purchasing firms. In other words, the preferable solution is a Pareto

optimal solution for which DEA super efficiency of all purchasing firms is maximum.

DEA initially proposed by Charnes et al. (1978) is a nonparametric multi-factor productivity analysis model, which evaluates the relative efficiencies of a homogenous set of decision-making units (DMUs). Although DEA is a beneficial tool for measuring the relative efficiency, it often recognizes too many DMUs as efficient. This deficiency known as low discrimination power is due to self-evaluation nature of DEA method and becomes more intense when the number of DMUs is relatively small in comparison with the total number of inputs and outputs. There are numerous approaches in the literature to cope with low discrimination power (Adler et al. 2002). In the present paper, the super efficiency ranking method proposed by Li et al. (2007) is applied, which is always a feasible model and conquers some drawbacks of other approaches, as follows:

$$Min \qquad h_0^s = 1 + \frac{1}{n}\sum_{j=1}^{n}\frac{s_{j2}^+}{R_j^-} \tag{4.57}$$

s.t.

$$\sum_{v=1}^{\vartheta+1}\lambda_v \bar{x}_{jv} + s_{j1}^- - s_{j2}^+ = \bar{x}_{j0}; \qquad \forall j \tag{4.58}$$

$$\sum_{v=1}^{\vartheta+1}\lambda_v \bar{o}_{rv} - s_r^+ = \bar{o}_{r0}; \qquad \forall r \tag{4.59}$$

$$\lambda_v, s_{j1}^-, s_{j2}^+, s_r^+ \leq 0; \qquad \forall v, j, r \tag{4.60}$$

where $\bar{o}_{rv}$ is the amount of output r from DMU v, while $\bar{x}_{jv}$ denotes the amount of input j to DMU v. In this model, h_0^s is DEA super efficiency score of DMU_0 and R_j^- is the maximum amount of all inputs j, i.e. $R_j^- = max_v(\bar{x}_{jv})$. Constraint (4.58) allows input j of DMU_0 to increase by s_{j2}^+ or decrease by s_{j1}^-. Constraint (4.59) implies that output r of DMU_0 is permitted to increase by at most s_r^+. DMU_0 is super efficient if optimal value of the objective function in this model is greater than one. On the other hand, a super efficiency score that is smaller than one denotes the respective DMU is not super efficient. Consequently, the above DEA super efficiency model can be solely solved to rank all DMUs (Li et al. 2007). If there are $\vartheta+1$ DMUs to be evaluated, this model needs to be solved $\vartheta+1$ times in order to determine the super efficiency score of all DMUs under study.

In order to implement DEA super efficiency model (4.57)-(4.60) to choose the preferable solution among all previously attained Pareto optimal solutions, it is essential to define proper DMUs and their inputs and outputs. In the following, it is explained how to specify an individual DMU and derive its inputs and outputs in a unique way.

DMU determination- In this study, the set of all purchasing firms is regarded as one individual DMU that is evaluated in different states. A state is indeed one Pareto optimal solution

derived. In other words, instead of evaluating different DMUs, one individual DMU that is the set of all combined purchasing firms is evaluated in different $\vartheta+1$ conditions. Contrary to previous studies in the literature, this helps derive DEA super efficiency of aggregated purchasing firms.

Inputs characterization- The inputs are defined as physical raw materials transported from all suppliers to the individual DMU. Let $\psi_v; v = 0, ..., \vartheta$ denote the v-th Pareto optimal solution. Obviously, each Pareto optimal solution ψ_v contains an amount of raw material j in price level l provided by supplier i to purchasing firm k at time period t, i.e. $\psi_v = \{(X^t_{ijlk})_v; \forall i, j, l, k, t\}; v = 0, ..., \vartheta$. In this case, the following expression

$$\bar{x}_{jv} = \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K}(X^t_{ijlk})_v; \qquad \forall j, v \tag{4.61}$$

aggregates the order quantities of each raw material j transported from all suppliers in all price levels during all time periods to the individual DMU in each Pareto optimal solution. In other words, $\bar{x}_{jv}$ characterizes input j (physical raw material j) of the individual DMU with respect to the Pareto optimal solution v in DEA super efficiency model (4.57)-(4.60).

Output characterization- The outputs are defined as the end products manufactured by purchasing firms. Based on the available historical data, a properly defined production function is utilized to estimate the production level of end products for each purchasing firm. For each Pareto optimal solution ψ_v, the Cobb-Douglas production function gives the production level of end product r in purchasing firm k, i.e. o_{rkv}, as follows:

$$o_{rkv} = \alpha_{rk}\prod_{j=1}^{n}[\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{l=1}^{L_i}(X^t_{ijlk})_v]^{\beta_{jk}}; \qquad \forall r, k, v \tag{4.62}$$

where α_{rk} and β_{jk} are estimated based on available historical data. In this case, the amount of end product r of the individual DMU with respect to the Pareto optimal solution ψ_v in DEA super efficiency model (4.57)-(4.60) is derived as:

$$\bar{o}_{rv} = \sum_{k=1}^{K} o_{rkv}; \qquad \forall r, v \tag{4.63}$$

Therefore, $\vartheta+1$ DMUs (one individual DMU in $\vartheta+1$ different conditions) are characterized, each of which has n inputs and s outputs. Figure 4.2 illustrates the defined single DMU and its inputs and outputs for some Pareto optimal solution ψ_v. It also depicts inputs and outputs of each purchasing firm for the same Pareto optimal solution.

Remark 1: That the inputs and outputs are aggregated using equations (4.61) and (4.63) in

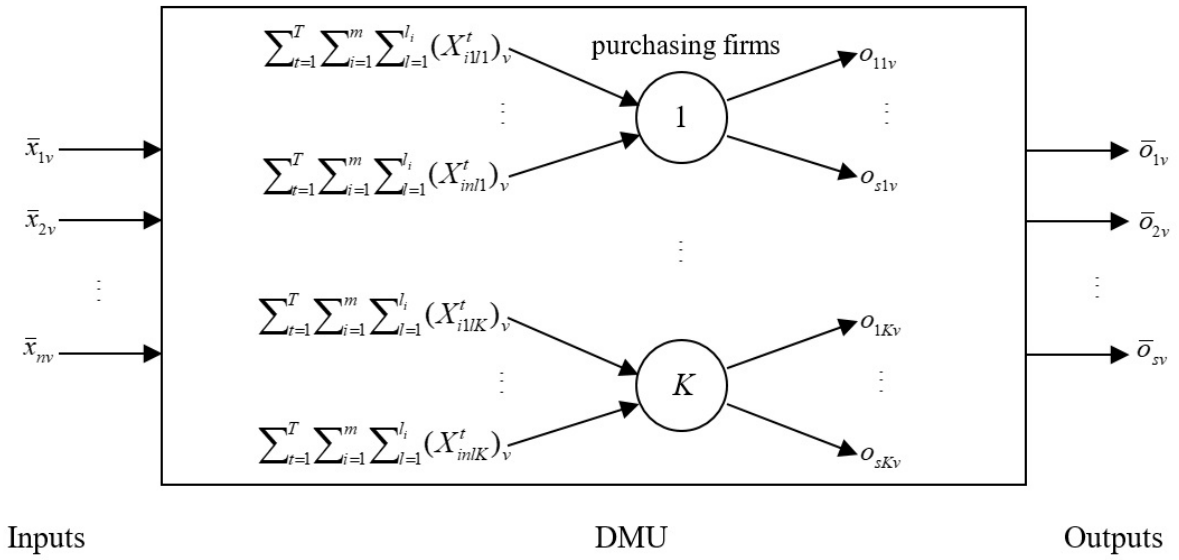


Figure 4.2: The single DMU of all purchasing firms with its inputs and outputs.

this study is inspired from the aggregate model proposed by Kao and Liu (2014) and Park and Park (2009).

Remark 2: n inputs and s outputs for the individual DMU are attained using equations (4.61) and (4.63), respectively, in $\vartheta + 1$ different conditions. To implement a DEA model, it is recommended that the number of DMUs be at least 3 times the total number of inputs and outputs (Charnes et al. 1978). Hence, the magnitude of in problem (21) is determined so as to have $\vartheta \geq 3(n + s) - 1$ satisfied.

4.5 Empirical study

An empirical study is presented in this section to elucidate how the SSSOA model and solution procedure may be implemented in a practical context. In this regard, the proposed approach is employed for a manufacturing and engineering company in automotive industry, which is the sole manufacturer of three types of automobile transmission systems in Iran, i.e. $K = 1$ and $s = 3$. The purpose of this empirical study is to evaluate four potential suppliers for two required raw materials in time horizon of two years, i.e. $m = 4$, $n = 2$ and $T = 2$, in such a way that not only the company's cost, CO2 emission and social responsibility are optimized, but also it operates in its most efficient condition. Because of the economic troubles encountered recently and lack of the required manufacturing resources, the issue of efficiency is of great importance for this company.

The required raw materials are in fact the forged parts of two shafts which we depict as

the primary shaft (PS) and secondary shaft (SS). Due to the complex chemical combinations and high-tech manufacturing processes required for these materials, the company succeeded in identifying only four potential domestic suppliers across the country, each of which has two price levels for each shaft, $L_i = 2; \forall i$. Furthermore, three types of transportation modes included a flatbed truck, box truck and tractor unit to ship forged parts from suppliers to the manufacturer, i.e. $P = 3$. This manufacturing company produces three types of transmission systems; automatic transmission (AT), manual transmission (MT) and continuous variable transmission (CVT). In the manufacturing process, one PS and one SS are assembled together in each type of transmission systems. All parameter values are defined as presented in supplementary material. These data are derived from historical records, which are mainly available in production planning and control department, purchasing department and quality control department of this manufacturing company.

Moreover, it suffices to have $\vartheta \geq 14$ in the present empirical study to satisfy $\vartheta \geq 3(n+s)-1$. Therefore, ϑ is arbitrarily set to 14 and subsequently the individual DMU, which is indeed the single manufacturer, is evaluated in 15 different conditions.

4.5.1 Results

Using the procedure described in Section 4.3.1, the minimum and maximum values of *TE* and *SR* objective functions are calculated as presented in Table 4.1.

Table 4.1: Minimum and maximum values of total emission (TE) and social responsibility (SR) objective functions.

Objective function	Minimum	Maximum
TE	110,155.1	218,892.2
SR	9.15	10.27

A strong valid inequality for constraint (4.12) was derived as inequality (4.51). In this regard, $\dot{N}$ is defined in this paper as a subset of $\bar{N}$ with $|\dot{N}| = 6$. Although 28 of such subsets, i.e. C_6^8, can be characterized, only 4 arbitrary cover inequalities associated with constraint (4.12) are defined as follows:

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} + Y^t_{3j2k} \leq 5; \qquad \forall j, k, t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} + Y^t_{4j1k} \leq 5; \qquad \forall j, k, t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} + Y^t_{4j2k} \leq 5; \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j2k} + Y^t_{4j1k} \leq 5; \qquad \forall j,k,t$$

Therefore, strong valid inequality for constraint (4.12) are defined as follows:

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} + Y^t_{3j2k} + 3(Y^t_{4j1k} + Y^t_{4j2k}) \leq 5; \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} + Y^t_{4j1k} + 3(Y^t_{3j2k} + Y^t_{4j2k}) \leq 5; \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j1k} + Y^t_{4j2k} + 3(Y^t_{3j2k} + Y^t_{4j1k}) \leq 5; \qquad \forall j,k,t$$

$$Y^t_{1j1k} + Y^t_{1j2k} + Y^t_{2j1k} + Y^t_{2j2k} + Y^t_{3j2k} + Y^t_{4j1k} + 3(Y^t_{3j1k} + Y^t_{4j2k}) \leq 5; \qquad \forall j,k,t$$

The En-BDA is implemented in Python 3.5 with GUROBI 3.5.2 optimizer on a computer with CPU of 2.5 GHz and 6.0 GB RAM, where the relative optimality gap is set to 0.01, i.e. $\varepsilon = 0.01$. For $v = 0, ..., 14$ in the single objective model (4.22)-(4.25), Table 4.2 presents the optimum order quantities of 15 different Pareto optimal solutions. As stated before, the individual manufacturer is evaluated in these 15 Pareto optimal solutions as though 15 different DMUs with distinct inputs and outputs are considered. For each Pareto optimal solution derived, the value of the corresponding objective function is also given in this table, where the trade-offs among objective functions are observed in the sense that improving TC and SR objective functions result in worsening TE objective function.

Table (4.3) shows the available amounts of forged PS and SS (physical inputs) for this manufacturing company calculated using equation (4.61). As stated before, the Cobb-Douglass production function helps derive the production level of end transmission systems (physical outputs). Appropriate production functions are characterized based on a set of historical records for forged PS and SS, and the respective production levels of manufactured transmission systems in this company as follows:

$$AT = 34.734 PS^{0.316} SS^{0.414} \tag{4.64}$$

$$MT = 32.466 PS^{0.294} SS^{0.439} \tag{4.65}$$

$$CVT = 27.295 PS^{0.250} SS^{0.490} \tag{4.66}$$

If the current production functions are applied to the amounts of forged PS and SS presented in Table 4.3, then the corresponding production levels for three transmission systems in each Pareto optimal solution are computed as also given in Table 4.3. The current physical inputs and outputs in Table 4.3 are applied to evaluate DEA super efficiency of this manufacturing company based on model (4.57)-(4.60). DEA super efficiency score of this company in 15 different conditions are also presented in Table 4.3.

Table 4.2: Optimal order quantities and the associated values of objective functions for each Pareto solution.

Order quantity ($\times 10^3$)	v														
	0	1	2	3	4	5	6	7	8*	9	10	11	12	13	14
$(X^1_{1111})_v$	0.00	0.00	0.00	0.00	0.00	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90
$(X^2_{1111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{1121})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{1121})_v$	0.00	0.00	0.00	0.00	33.06	36.27	47.89	45.24	41.08	40.53	40.53	40.53	40.53	40.53	40.53
$(X^1_{1211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{1211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{1221})_v$	24.70	37.15	40.67	39.55	39.91	39.63	42.10	42.10	42.10	42.10	42.10	42.10	42.10	42.10	42.10
$(X^2_{1221})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{2111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{2111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{2121})_v$	20.80	20.80	20.80	20.80	20.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{2121})_v$	22.60	41.20	41.20	41.20	41.20	41.20	32.89	38.78	41.20	41.20	41.20	41.20	41.20	41.20	41.20
$(X^1_{2211})_v$	24.70	3.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{2211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{2221})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{2221})_v$	40.74	40.74	40.74	40.74	40.00	40.00	40.00	40.00	40.00	32.27	35.50	38.73	40.00	48.39	48.90
$(X^1_{3111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{3111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{3121})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{3121})_v$	0.00	0.00	27.02	55.25	26.00	26.00	26.00	26.00	30.97	31.87	31.87	31.87	31.87	31.87	31.87
$(X^1_{3211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{3211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{3221})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{3221})_v$	0.00	0.00	0.00	0.00	17.16	15.60	22.80	24.09	24.09	34.70	34.70	34.70	34.70	24.50	0.00
$(X^1_{4111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{4111})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{4121})_v$	65.00	65.00	65.00	65.00	65.00	65.90	65.90	65.90	65.90	65.90	65.90	65.90	65.90	65.90	65.90
$(X^2_{4121})_v$	65.00	48.79	25.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{4211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^2_{4211})_v$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(X^1_{4221})_v$	49.40	57.70	58.13	59.25	58.89	59.17	56.70	56.70	56.70	56.70	56.70	56.70	56.70	56.70	56.70
$(X^2_{4221})_v$	59.00	59.00	59.00	59.00	42.00	43.58	36.30	35.00	35.00	35.00	35.00	35.00	36.96	42.00	69.22
TC ($\times 10^3$)	88,257	88,811	89,077	91,602	94,065	96,761	99,520	102,382	105,265	108,337	111,446	114,555	117,665	120,834	124,122
TE	110,155	117,922	125,689	133,456	141,223	148,990	156,757	158,362	159,293	161,997	162,969	163,941	164,817	164,358	162,480
SR	9.172	9.231	9.311	9.390	9.470	9.549	9.629	9.709	9.788	9.868	9.948	10.028	10.107	10.186	10.266

Table 4.3: The physical inputs and outputs of the manufacturing company and its super efficiency score in each Pareto optimal solution.

v	Available forged shafts (Inputs)		Transmission systems (Outputs)			Super efficiency
	PS	SS	AT	MT	CVT	
0	173,400.0	198,542.1	245,247.9	238,488.0	219,681.3	1.2284
1	175,793.1	198,542.1	246,312.4	239,451.0	220,435.4	1.1234
2	179,023.1	198,542.1	247,733.7	240,736.2	221,441.1	1.1167
3	182,252.9	198,542.1	249,137.4	242,005.0	222,433.1	1.1100
4	186,060.1	197,964.8	250,468.2	243,169.4	223,267.0	1.0033
5	189,272.6	197,982.2	251,836.0	244,405.8	224,234.2	1.0144
6	192,582.5	197,902.2	253,177.0	245,611.1	225,163.6	1.1720
7	195,826.6	197,888.0	254,509.4	246,812.5	226,097.9	1.2299
8*	**199,056.6**	**197,887.9**	**255,828.5**	**248,002.4**	**227,024.5**	**1.3124**
9	199,400.0	200,774.4	257,507.1	249,710.6	228,739.7	1.1285
10	199,400.0	204,004.3	259,214.1	251,466.2	230,535.4	1.0339
11	199,400.0	207,234.1	260,905.3	253,206.3	232,316.7	1.0000
12	199,400.0	210,464.0	262,581.2	254,931.2	234,083.9	1.0000
13	199,400.0	213,693.9	264,242.0	256,641.4	235,837.3	1.0000
14	199,400.0	216,923.6	265,888.1	258,337.0	237,577.2	1.0000

We observe although all 15 different conditions of the manufacturing company are Pareto optimal solutions of SSSOA model for which the company is absolutely indifferent, its DEA super efficiency score fluctuates significantly. Moreover, it turns out that the super efficiency model (4.57)-(4.60) has aptly discriminated different states of the company. Therefore, it can be declared by considering Table 4.3 that the manufacturing company is efficient in state 8. Referring to the column 8 of Table 4.2, the set of suppliers selected at each period of time to provide each forged shaft can be determined. In addition, the amounts of forged *PS* and *SS* assigned to each selected supplier can be determined in this efficient state.

4.5.2 Evaluating BDA performance

As explicated before, the En-BDA is developed in this paper is used to cope with the large-scale property of the proposed single objective programming model (4.22)-(4.25). However, the influence of En-BDA in decreasing solution time of the empirical study in this paper might not be completely observed because the problem explored does not have significantly high dimension. Hence, in this section, five substantially larger instances of the single objective programming model (4.22)-(4.25) are generated, where the precise quantity of all the parameters is characterized by Monte Carlo sampling method implemented on particular uniform distribution functions with predetermined intervals. Each instance is solved three times in order to examine the performance of En-BDA more accurately. The performance of En-BDA is compared with that of the classical BDA and GUROBI 3.5.2 within a time limit of 6 hours. The dimensions of generated instances with respect to the number of integer variables, the number of total variables and the number of constraints are given in Table 4.4, which present how large the developed single objective programming model can be in practice for commonplace supply networks.

Table 4.4: Dimensions of the generated test problems.

Instance	m	n	s	K	L_i	T	P	# of integer variables	# of total variables	# of constraints
1	15	8	6	4	2	6	5	20,352	40,908	33,981
2	25	12	6	4	2	12	5	101,376	203,344	166,673
3	35	15	8	4	2	12	8	252,720	506,179	364,985
4	45	22	12	6	2	18	8	1,071,576	2,145,554	1,533,889
5	50	30	18	8	2	24	10	3,461,760	6,929,314	4,690,385

The uniform distribution functions are defined in such a way that the proposed model retains its feasibility and the obtained results sound reasonable. In this regard, if $U \sim (a, b)$ denotes a uniform random variable in the interval (a, b), then Table 4.5 presents the uniform distribution

Table 4.5: Characterization of various parameters in test problems.

Parameter	Value	Parameter	Value
dem^t_{jk} (ton)	$U \sim (145, 200)$	pca^t_k (ton)	$U \sim (380, 500)$
ppr^t_{ijl}(\$)	$U \sim (210, 270)$	tca^t_{ijkp} (ton)	$U \sim (38, 50)$
fsc^t_{ijk}($\times 10^2$\$)	$U \sim (1.8, 2.2)$	msh^t_{jk} (kg)	$U \sim (55, 85)$
vsc^t_{ijk}(\$)	$U \sim (5, 10)$	lbr_{jk}	$U \sim (5, 8)$
hoc^t_{jk}(\$)	$U \sim (210, 230)$	rwe_{ij}	$U \sim (0.05, 0.50)$
shc^t_{jk}(\$)	$U \sim (660, 750)$	fse^t_{ijk} (kg)	$U \sim (160, 430)$
sts^t_{jk} (m3)	$U \sim (210, 320)$	vse^t_{ijk} (kg)	$U \sim (0.07, 0.4)$
sph_j (m3)	$U \sim (0.001, 0.002)$	hoe^t_{jk}($\times 10^{-5}$ kg)	$U \sim (31, 41)$
rep^t_{ij}(%)	$U \sim (4, 9)$	pre_{ij} (kg)	$U \sim (0.16, 0.2)$
otd^t_{ij}(%)	$U \sim (92, 100)$	con_{ijk}($\times 10^3$\$)	$U \sim (10, 18)$
apr^t_{jk}(%)	$U \sim (8, 12.5)$	los_k	$U \sim (0.4, 1)$
apd^t_{jk}(%)	$U \sim (58, 60)$	var_k	$U \sim (0.5, 1.1)$
cap^t_{ij} (ton)	$U \sim (420, 820)$		

functions that are used to generate the quantity of the corresponding parameters. Furthermore, $nsu^t_{jk} = \lceil m/2 \rceil + 1; \forall j, k, t$, $\kappa = \lceil (m + K + T)/2 \rceil + 1$ for local branching, $\ell_{ij1} = 80$ tons $\forall i, j, t$ and $\ell_{ij2} = 190$ tons $\forall i, j, t$, in all instances. In each instance, five valid inequalities are also derived with $|\dot{N}| = nsu^t_{jk} + 2$.

Table 4.6 summarizes the computational results obtained from solving each instance by En-BDA, classical BDA and GUROBI, where the elapsed time in hours, the number of iterations and the value of total cost objective function are also reported. The column entitled GAP in classical BDA indicates the relative difference between lower and upper bounds that are derived within the predetermined time limit.

In addition, the bold numbers represent the optimum value of the total cost objective function obtained within the time limit. According to Table 4.6, the following insightful findings can be drawn:

- During the time limit of 6 hours, GUROBI is only capable of solving instances 1 and 2 to optimality. However, it fails to find the optimum solution of the remaining instances within the dedicated time limit. Therefore, for such instances, the best value of total cost objective function acquired by GUROBI after 6 hours is presented.

- For instances 1, 2 and 3, the classical BDA outperforms GUROBI with respect to the solution time. In particular, for instances 1 and 2 for which both the classical BDA and GUROBI are able to find the optimum solutions, the solution time of GUROBI is greater than that of the classical BDA by 90.3% on average. In addition, the classical BDA finds the optimum solution of instance 3 in 4.508 hours on average, whereas GUROBI fails to

Table 4.6: Comparison results of the classical BDA, En-BDA and Gurobi

Instance	Run	Classical BDA				En-BDA			Gurobi	
		Time (h)	# of iterations	Cost	GAP (%)	Time (h)	# of iterations	Cost	Time (h)	Cost
1	1	2.198	1658	**95,368,889**	≤ 1	2.045	1369	**95,368,889**	4.267	**95,368,889**
	2	2.307	1246	**102,201,008**	≤ 1	2.419	913	**102,201,008**	4.118	**102,201,008**
	3	2.493	1599	**88,911,297**	≤ 1	2.127	1052	**88,911,297**	4.483	**88,911,297**
2	1	2.811	1983	**111,584,669**	≤ 1	3.011	1283	**111,584,669**	5.632	**111,584,669**
	2	2.749	2028	**114,498,706**	≤ 1	2.651	1390	**114,498,706**	5.719	**114,498,706**
	3	2.800	2095	**99,100,611**	≤ 1	2.612	1326	**99,100,611**	5.012	**99,100,611**
3	1	4.567	2796	**115,218,044**	≤ 1	3.159	1417	**115,218,044**	≥ 6	116,823,719
	2	4.218	3014	**116,074,183**	≤ 1	2.917	1864	**116,074,183**	≥ 6	117,041,621
	3	4.739	3154	**116,523,330**	≤ 1	2.648	1809	**116,523,330**	≥ 6	117,910,465
4	1	≥ 6	4019	124,321,083	4.95	3.364	2127	**123,085,284**	≥ 6	124,581,340
	2	≥ 6	3729	125,473,180	11.63	3.415	1953	**124,381,928**	≥ 6	125,908,443
	3	≥ 6	3843	125,049,775	5.11	3.291	2026	**123,592,068**	≥ 6	124,893,071
5	1	≥ 6	3559	130,811,079	8.17	3.718	1973	**128,419,734**	≥ 6	131,098,157
	2	≥ 6	4035	130,227,164	4.26	3.822	2119	**127,645,129**	≥ 6	130,853,294
	3	≥ 6	3384	130,649,382	9.54	3.904	2214	**127,273,844**	≥ 6	130,419,063

do so. However, it is observed that the classical BDA is unable to solve instances 4 and 5 within the time limit of 6 hours. Thats why the gap between the lower and upper bounds obtained in the latter instances is greater than the optimality gap of 1%.

- All instances can be successfully solved by En-BDA in the time limit of 6 hours. Particularly, the solution time of En-BDA for instances 1 and 2 is on average 3.3% (96.6%) less than that of the classical BDA (GUROBI). In addition, the solution time of En-BDA for instance 3 is on average 33.0% less than that of classical BDA. While neither the classical BDA nor GUROBI are able to solve instances 4 and 5 to optimality in the specified time limit, En-BDA solves them in 3.357 hours and 3.814 hours on average, respectively. It is also worth mentioning that the average relative gap between total cost reported by En-BDA and the classical BDA (GUROBI) for instances 4 and 5 (3, 4 and 5), for which the classical BDA (GUROBI) fails to find the optimum solution, is equal to 1.61% (1.55%) that values \$2,022,279 (\$1,923,959). This implies that the best feasible solutions obtained by the classical BDA and GUROBI within 6 hours are far from the optimum solutions derived by En-BDA.

  With respect to the number of iterations, Figure 4.3 demonstrates the average number of iterations when solving each instance using the classical BDA and En-BDA. In summary, it is concluded that the developed computational enhancements are significantly able to improve the solution time and the number of iterations of En-BDA in comparison with the classical BDA and GUROBI.

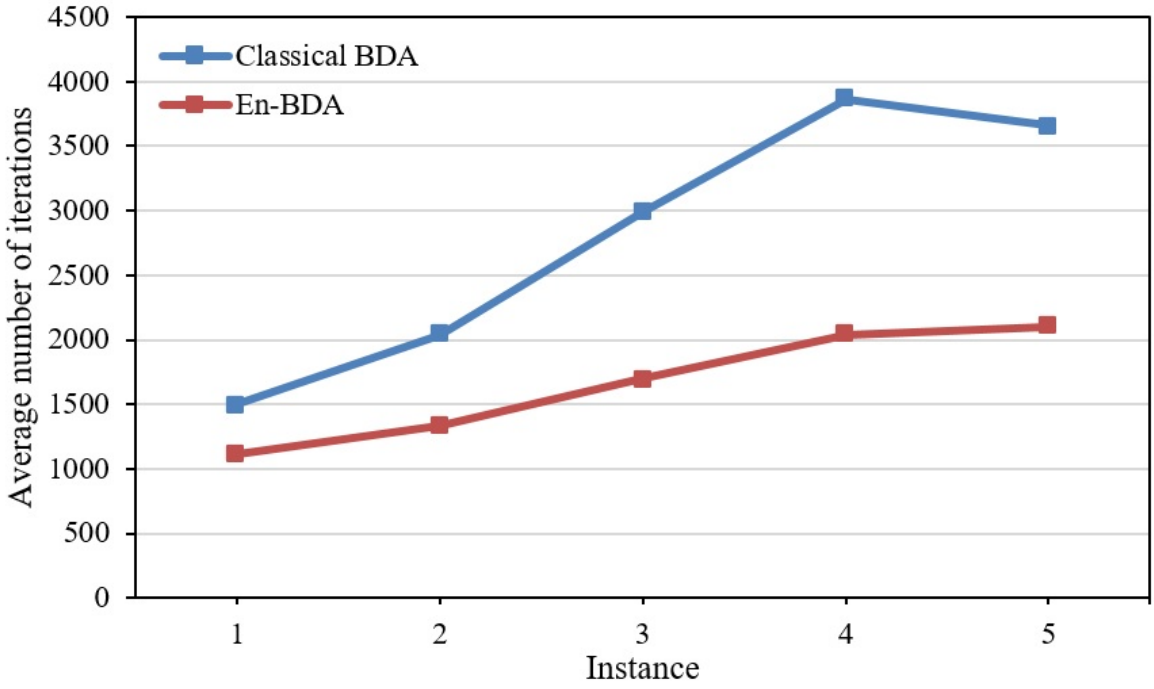


Figure 4.3: Average number of iterations for test instances.

4.6 Appendix

Indexes

i: Index of suppliers; $i = 1, ..., m$

j: Index of raw materials; $j = 1, ..., n$

k: Index of purchasing firms; $k = 1, ..., K$

r: Index of end products; $r = 1, ..., s$

l: Index of price levels by suppliers; $l = 1, ..., L_i$

p: the index of transportation modes; $p = 1, ..., P$

t: Index of period; $t = 1, ..., T$

Parameters

dem^t_{jk}: required demand of raw material j in purchasing firm k in period t.

rep^t_{ij}: rejection percentage of raw material j provided by supplier i in period t.

ppr^t_{ijl}: unit purchasing price of raw material j at price level l provided by supplier i in period t.

fsc^t_{ijkp}: fixed shipment cost of raw material j shipped from supplier i to purchasing firm k by transportation mode p in period t.

vsc^t_{ijkp}: variable shipment cost of unit raw material j shipped from the supplier i to purchasing

firm k by transportation mode p in period t.

msh_{jk}^{t}: maximum amount of shortage for raw material j at purchasing firm k in period t.

sts_{jk}^{t}: maximum available storing space for raw material j in purchasing firm k in period t.

nsu_{jk}^{t}: maximum number of suppliers hired by purchasing firm k to provide raw material j in period t.

otd_{ij}^{t} : on-time delivery percentage of raw material j provided by supplier i in period t.

apd_{jk}^{t}: minimum acceptable percentage for on-time delivery of raw material j to purchasing firm k in period t.

apr_{jk}^{t}: maximum acceptable percentage for rejecting raw material j in purchasing firm k in period t.

cap_{ij}^{t}: capacity of supplier i to provide raw material j in period t.

fse_{ijkp}^{t}: fixed shipment CO2 emission of raw material j shipped from supplier i to purchasing firm k by transportation mode p in period t.

vse_{ijkp}^{t}: variable shipment CO2 emission of raw material j shipped from supplier i to purchasing firm k by transportation mode p in period t.

hoe_{jk}^{t}: holding CO2 emission of each unit of raw material j in purchasing firm k in period t.

con_{ijk}: fixed contract cost of supplier i with purchasing firm k for raw material j.

z_{j}^{p}: 1 if raw material j can be transported by transportation mode p; otherwise 0.

lbr_{jk}: lower bound of rating value on raw material j for purchasing firm k.

rwe_{ij}: rating weight of supplier i for providing raw material j.

tca_{ijkp}^{t}: capacity of transportation mode p for raw material j from supplier i to purchasing firm k at time period t.

hoc_{jk}^{t}: holding cost of each unit of raw material j in purchasing firm k in period t.

sph_{j}: required space for holding a unit of raw material j.

ℓ_{ijl}^{t}: upper limit of purchase volume for raw material i in supplier j at price level l in period t; $\ell_{ij1}^{t} \leq \ell_{ij2}^{t} \leq ...\ell_{ijL_i}^{t}$.

shc_{jk}^{t}: shortage cost of each unit of raw material j in purchasing firm k in period t.

pca_{k}^{t} : production capacity of purchasing firm k in period t.

L_i: number of price levels offered by supplier i.

pre_{ij}: processing CO2 emission of material j by supplier i.

los_{k}: lost days caused by works damages in purchasing firm k.

var_{k}: variable number of created jobs at purchasing firm k.

M: a big positive number.

Decision variables

X_{ijlk}^{t}: order quantity of raw material j at price level l provided by supplier i for purchasing firm k in period t.

Y_{ijlk}^{t}: 1 if supplier i provides raw material j at price level l to purchasing firm k in period t; Otherwise 0.

Q_{ijkp}^{t}: amount of raw material j transported from supplier i to purchasing firm k by using the transportation mode p at time period t.

H_{jk}^{t}: if raw material j is held by purchasing firm k in period t; Otherwise, i.e. shortage condition, 0.

U_{ijkp}^{t}: 1 if raw material j is transported from supplier i to purchasing firm k by transportation mode p at time period t; Otherwise 0.

ILP_{jk}^{t}: inventory level of material j at the end of period t in purchasing firm k.

$\overline{ILP}_{jk}^{t}$: shortage level of material j at the end of period t in purchasing firm k.

Chapter 5

Conclusions and Future Research

In chapter 2 of this dissertation, we developed an integrated multi-objective multi-period MINLP model to incorporate DEA efficiency in planning a generic sustainable closed loop supply chain network, in which all sustainability aspects are considered. To accomplish this target, we first formulated a bi-objective output-oriented DEA model to derive the efficiency score of all DMUs (located facilities in this context) simultaneously. In the current DEA model, physical input, current expense and labor force constitute the input factors, whereas the total amount of outgoing materials/products defines the output factor of each facility in each period. As the original MINLP model was non-convex, we linearized the bilinear terms in DEA efficiency constraints by piecewise McCormick envelopes approach in the first step of the developed solution procedure. In order to cope with the elevated dimension of the obtained relaxed MILP model, we then added a set of valid inequalities in the second step of solution procedure. Finally, we obtained an aggregated single objective MILP model using a weighted sum fuzzy aggregation function. The computational experiment demonstrated the impact of incorporating DEA efficiency into sustainable supply chain design process, where the orientation type of DEA model played a key role in comprehending the obtained results. Particularly, we observed increasing DEA efficiency of facilities results in raising total profit generated through supply chain network (economic aspect). However, there exist trade-offs between DEA efficiency and both total CO2 emission (environmental aspect) and social responsibility (social responsibility aspect).

In chapter 3, we elaborated a new integrated multi-objective multi-period multi-perishable product MINLP model with stochastic demand in the presence of shortage and discount conditions for supplier selection and order allocation problem, where all sustainability aspects as well as the purchasing firms efficiency were considered. We then proposed a novel solution procedure to first deal with the non-convexity issue of the developed model using piecewise McCormick envelopes approach. After adding a couple of valid inequalities to improve the solution time, we

attained a deterministic equivalent of the original stochastic programming model by means of chance constrained programming. Ultimately, we acquired a single aggregated objective function using a fuzzy multi-objective programming.

In chapter 4, we addressed the sustainable supplier selection and order allocation problem through development of a comprehensive multi-period, multi-product multi-objective MILP model that also took price discount, multimodal transportation and shortage condition into account, in which all sustainability aspects were included based on the triple bottom line approach. We devised a hybrid solution procedure, which consisted of ε-constraint method, enhanced BDA and DEA. In particular, we converted the original multi-objective programming model to a single objective programming model using the ε-constraint method in the first step of the solution procedure. In this regard, total cost objective function was treated as the primary objective function, whereas total emission and social responsibility objective functions were transferred into the constraints set. Afterwards, the current single objective model was solved using BDA accelerated using a set of computational enhancements in order to attain one Pareto optimal solution. Since a Pareto optimal solution characterizes the physical raw materials transported from suppliers to purchasing firms, the production level of physical end products was derived by Cobb-Douglas production function. Using the obtained physical inputs and outputs, the DEA super efficiency of purchasing firms were evaluated for all Pareto optimal solutions generated. In this case, the final preferable solution, which indicated the set of selected suppliers and orders allocation, was a Pareto optimal solution associated with the greatest DEA super efficiency score. The developed BDA that was accelerated in this chapter by various computational enhancements had its own merits to emphasize when applying to large-scale instances. In particular, valid inequalities, Pareto-optimal cuts, local branching procedure and knapsack cut were incorporated towards devising En-BDA. The evaluation of its performance in comparison with the classical BDA and GUROBI revealed that En-BDA is significantly capable of solving large-scale instances of the developed programming model in this chapter within the predetermined time limit. It was observed that En-BDA could improve the average total cost \$2,022,279 and \$1,923,959 in instances that the classical BDA and GUROBI were not able to solve, respectively. On the other hand, with respect to the number of iterations, En-BDA could find the optimal solution in less number of iterations on average in comparison with the classical BDA.

5.1 Future research

We exploited the triple bottom line approach to formulate sustainability aspects in this dissertation, as it is the frequently used methodology in the literature in this context. However, Benjaafar et al. (2013) presented different formulations for considering carbon footprint in

supply chain models. It might be interesting to explore the formulations in the sustainability context and analyze how various formulations result in different results.

In addition, in chapters 2 and 3, we employed piecewise McCormick envelopes approach to linearize bilinear terms appeared in DEA efficiency related constraints. As explained earlier, there exist other approaches to deal with bilinear terms. Applying them and exploring their effects on the solution procedure in terms of solution time and obtained dimension of the relaxed MILP model would be another interesting area to investigate.

In chapter 3, in some cases, the uncertain demand might also be modeled as a fuzzy parameter with a proper membership function. A combination of fuzzy and stochastic uncertainties that is modeled as either a random fuzzy or fuzzy random demand is another possibility to consider in this study.

Moreover, in chapter 4, the proposed approach provided DEA supper-efficiency score of all purchasing firms over the entire time horizon. An interesting development of the present study is to propose a framework by which DEA super efficiency score of each purchasing firm in each period of time is obtained. In this case, the impact of suppliers performance on each purchasing firm with respect to sustainability criteria can be monitored. Furthermore, as clarified before, DEA model is a frontier-based methodology and consequently is sensitive to the quality of input and output data. Therefore, any noise existing in measured data can lead to inauthentic results when evaluating DMUs efficiency. In other words, precise information of inputs and outputs is important in the application of successful DEA model outputs. In chapter 4, the production level of end products (physical outputs) was derived by Cobb-Douglas production function based on formerly recorded data, which may not be as precise as desired. In such a circumstance, the inherent uncertainty is needed to be somehow taken into account. One way to present the uncertainty in data is to apply the membership function in fuzzy set theory proposed by Zadeh (1965). In this case, a fuzzy production function would be firstly built using recorded crisp inputs and fuzzy outputs, and then a fuzzy DEA super efficiency model would be developed to derive the super efficiency scores.

REFERENCES

[1] Adler, N., Friedman, L., Sinuany-Stern, Z. (2002). Review of ranking methods in the data envelopment analysis context. *European Journal of Operational Research*, 140(2), 249-265.

[2] Aktin, T., Gergin, Z. (2016). Mathematical modelling of sustainable procurement strategies: three case studies. *Journal of Cleaner Production*, 113, 767-780.

[3] Alumur, S.A., Nickel, S., Saldanha-da-Gama, F., Verter, V. (2012). Multi-period reverse logistics network design. *European Journal of Operational Research*, 220(1), 67-78.

[4] Amindoust, A., Ahmed, S., Saghafinia, A., Bahreininejad, A. (2012). Sustainable supplier selection: A ranking model based on fuzzy inference system. *Applied Soft Computing*, 12, 1668-1677.

[5] Andersen, P., Petersen, N.C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, 39, 1261-1264.

[6] Anstreicher, K.M. (2009). Semi-definite programming versus the reformulation-linearization technique for nonconvex quadratically constrained quadratic programming. *Journal of Global Optimization*, 43(23), 471-484.

[7] Azadi, M., Shabani, A., Khodakarami, M., Saen, R.F. (2014). Planning in feasible region by two-stage target-setting DEA methods: An application in green supply chain management of public transportation service providers. *Transportation Research Part E* (70), 324-338.

[8] Azadnia, A.H., Saman, M.Z.M., Wong, K.Y. (2015). Sustainable supplier selection and order lot-sizing: an integrated multi-objective decision-making process. *International Journal of Production Research*, 53(2), 383-408.

[9] Bai, C., Sarkis, J. (2010). Integrating sustainability into supplier selection with grey system and rough set methodologies. *International Journal of Production Economics*, 124, 252-264.

[10] Banker, R.D., Moray, R.C. (1986). Efficiency analysis for exogenously fixed input and output. *Operation Research*, 34(4), 513-518.

[11] Benders, J.F. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4, 238-252.

[12] Benjaafar, S., Li, Y., Daskin, M. (2013). Carbon footprint and the management of supply chains: insights from simple models. *IEEE Transactions on Automation Science and Engineering*, 10(1), 99-116.

[13] Bergamini, M.L., Aguirre, P., Grossmann, I.E. (2005). Logic-based outer approximation for globally optimal synthesis of process networks. *Computers and Chemical Engineering*, 29, 1914-1933.

[14] Boussofiane, A., Dyson, R.G., Thanassoulis, E. (1991). Applied data envelopment analysis. *European Journal of Operational Research*, 52, 1-15.

[15] Bouzembrak, Y., Allaoui, H., Goncalves, G., Bouchriha, H. (2013). A multi-modal supply chain network design for recycling waterway sediments. *International Journal of Environment and Pollution*, 51(1), 15-31.

[16] Brandenburg, M., Govindan, K., Sarkis, J., Seuring, S. (2014). Quantitative models for sustainable supply chain management: Developments and directions. *European Journal of Operational Research*, 233, 299-312.

[17] Branke, J., Deb, K., Miettinen, K., Slowinski, R. (2008). *Multi-objective optimization: Interactive and evolutionary approaches*. Berlin: Springer-Verlag.

[18] Buyukozkan, G., Cifci, G. (2011). A novel fuzzy multi-criteria decision framework for sustainable supplier selection with incomplete information. *Computers in Industry*, 62, 164-174.

[19] Cambini, R., Sodini, C. (2005). Decomposition methods for solving nonconvex quadratic programs via branch and bound. *Journal of Global Optimization*, 33, 313-336.

[20] Castro, P.M. (2015). Tightening piecewise McCormick relaxations for bilinear problems. *Computers and Chemical Engineering*, 72, 300-311.

[21] Castro, P.M., Teles, J.P. (2013). Comparison of global optimization algorithms for the design of water-using networks. *Computers and Chemical Engineering*, 52, 249-261.

[22] Celebi, D.C., Bayraktar, D. (2008). An integrated neural network and data envelopment analysis for supplier evaluation under incomplete information. *Expert Systems with Applications*, 35, 1698-1710.

[23] Charnes, A., Cooper, W.W., Huang, Z.M., Sun, D.B. (1990). Polyhedral cone-ratio DEA models with an illustrative application to large commercial banks. *Journal of Econometrics*, 46, 73-91.

[24] Charnes, A., Cooper, W.W. (1959). Chance-Constrained Programming. *Management Science*, 6(1), 73-79.

[25] Charnes, A., Cooper, W.W., Rhodes, E. (1978). Measuring efficiency of decision making units. *European Journal of Operational Research*, 2, 429-444.

[26] Chopra, S., Meindl, P. (2012). *Supply Chain Management Strategy, Planning, And Operation*. New Jersey: Pearson.

[27] Cruz, J.M. (2009). The impact of corporate social responsibility in supply chain management: multicriteria decision-making approach. *Decision Support Systems*, 48, 224-236.

[28] DEFRA (2012). *Sustainable business and resource efficiency*. available at: http://webarchive.nationalarchives.gov. uk/ 20130123162956/http:/www.defra.gov.uk/environment/economy/business-efficiency/.

[29] Dehghanian, F., Mansour, S. (2009). Designing sustainable recovery network of end-of-life products using genetic algorithm. *Resources, Conservation and Recycling*, 53(10), 559-570.

[30] Demirel, E., Demirel, N., Goken, H. (2016). A mixed integer linear programming model to optimize reverse logistics activities of end-of-life vehicles in Turkey. *Journal of Cleaner Production*, 112(3), 2101-2113.

[31] Devika, K., Jafarian, A., Nourbakhsh, V. (2014). Designing a sustainable closed-loop supply chain network based on triple bottom line approach: A comparison of metaheuristics hybridization techniques. *European Journal of Operational Research*, 235, 594-615.

[32] Dickson, G.W. (1966). An analysis of vendor selection systems and decisions. *Journal of Purchasing*, 2(1), 5-17.

[33] Dobler, D.W., Lee, L., Burt, N. (1990). *Purchasing and Materials Management: text and cases*. New York: McGraw-Hill.

[34] Dotoli, M., Falagario, M. (2012). A hierarchical model for optimal supplier selection in multiple sourcing contexts. *International Journal of Production Research*, 50(11), 2953-2967.

[35] Dunphy, D., Griffiths, A., Benn, S. (2003). *Organizational Change for Corporate Sustainability*. New York: Routledge Publication.

[36] Dyllick, T., Hockerts, K. (2002). Beyond the business case for corporate sustainability. *Business Strategy and the Environment*, 11(2), 130-141.

[37] Elkington, J. (1998). *Cannibals with forks: the triple bottom line of 21st century business*. Oxford: Capstone.

[38] European, Commission (2011). *A resource-efficient EuropeFlagship initiative under the Europe 2020 strategy. Communication from the Commission to the European Parliament, the Council*, the European Economic and Social Committee and the Committee of the Regions, Brussels.

[39] Fahimnia, B., Sarkis, J., Dehghanian, F., Banihashemi, N., Rahman, S. (2013). The impact of carbon pricing on a closed-loop supply chain: An Australian case study. *Journal of Cleaner Production*, 59, 210-225.

[40] Fahimnia, B., Jabbarzadeh, A., Ghavamifar, A., Bell, M. (2017). Supply chain design for efficient and effective blood supply in disasters. *International Journal of Production Economics*, 183, 700-709.

[41] Falagario, M., Sciancalepore, F., Costantino, N., Pietroforte, R. (2012). Using a DEA-cross efficiency approach in public procurement tenders. *European Journal of Operational Research*, 218, 523-529.

[42] Fallahpour, A., Olugu, E.U., Musa, S.N., Wong, K.Y., Noori, S. (2017). A decision support model for sustainable supplier selection in sustainable supply chain management. *Computers & Industrial Engineering*, 105, 391-410.

[43] Felfel, H., Ayadi, O., Masmoudi, F. (2016). A decision-making approach for a multi-objective multisite supply network planning problem. *International Journal of Computer Integrated Manufacturing*, 29(7), 754-767.

[44] Fischetti, M., Lodi, A. (2003). Local branching. *Mathematical programming*, 98(1-3), 23-47.

[45] Floudas, C.A. (1995). *Nonlinear and mixed-integer optimization: Fundamentals and applications*. New York: Oxford University Press.

[46] Geoffrion, A.M., Graves, G.W. (1974). Multicommodity distribution system design by benders decomposition. *Management Science*, 20(5), 822-844.

[47] Ghadimi, P., Ghassemi-Toosi, F., Heavey, C. (2017). A multi-agent systems approach for sustainable supplier selection and order allocation in a partnership supply chain. *European Journal of Operational Research*, In Press.

[48] Ghasemi, M.-R., Ignatius, J., Emrouznejad, A. (2014). A bi-objective weighted model for improving the discrimination power in MCDEA. *European Journal of Operational Research*, 233, 640-650.

[49] Gok, M.S., Altndag, E. (2015). Analysis of the cost and efficiency relationship: experience in the Turkish pay for performance system. *The European Journal of Health Economics*, 16, 459-469.

[50] Govindan, K., Khodaverdi, R., Jafarian, A. (2013). A fuzzy multi criteria approach for measuring sustainability performance of a supplier based on triple bottom line approach. *Journal of Cleaner Production*, 47, 345-354.

[51] Govindan, K., Rajendran, S., Sarkis, J., Murugesan, P. (2015). Multi criteria decision making approaches for green supplier evaluation and selection: a literature review. *Journal of Cleaner Production*, 98, 66-83.

[52] Grimm, J.H., Hofstetter J.S., Sarkis J. (2014). Critical factors for sub-supplier management: a sustainable food supply chains perspective. *International Journal of Production Economics*, 152, 159-173.

[53] Guo, P. (2009). Fuzzy data envelopment analysis and its application to location problems. *Information Sciences*, 179, 820-829.

[54] Gupta, P., Govindan, K., Mehlawat, M.K., Kumar, S. (2016). A weighted possibilistic programming approach for sustainable vendor selection and order allocation in fuzzy environment. *International Journal of Advanced Manufacturing Technology*, 86, 1785-1804.

[55] Ha, S.H., Krishnan, R. (2008). A hybrid approach to supplier selection for the maintenance of a competitive supply chain. *Expert Systems with Applications*, 34, 1303-1311.

[56] Hamdan, S., Cheaitou, A. (2017). Supplier selection and order allocation with green criteria: An MCDM and multi-objective optimization approach. *Computers and Operations Research*, 81, 282-304.

[57] Harraz, N.A., Galal, N.M. (2011). Design of sustainable end-of-life vehicle recovery network in Egypt. *Ain Shams Engineering Journal*, 2(3-4), 211-219.

[58] Hassini, E., Surti, C., Searcy, C. (2012). A literature review and a case study of sustainable supply chains with a focus on metrics. *International Journal of Production Economics*, 140(1), 69-82.

[59] Jafari-Songhori, M., Tavana, M., Azadeh, A., Khakbaz, M.H. (2011). A supplier selection and order allocation model with multiple transportation alternatives. *International Journal of Advanced Manufacturing Technology*, 52, 365-376.

[60] Jeihoonian, M., Kazemi-Zanjani, M., Gendreau, M. (2016). Accelerating Benders decomposition for closed-loop supply chain network design: Case of used durable products with different quality levels. *European Journal of Operational Research*, 251(3), 830-845.

[61] Kamath, K.R., Pakkala, T.P.M. (2002). A Bayesian approach to a dynamic inventory model under an unknown demand distribution. *Computers & Operations Research*, 29, 403-422.

[62] Kao, C., Liu, S.-T. (2014). Multi-period efficiency measurement in data envelopment analysis: The case of Taiwanese commercial banks. *Omega* 47, 90-98.

[63] Karsak, E.E., Dursun, M. (2014). An integrated supplier selection methodology incorporating QFD and DEA with imprecise data. *Expert Systems with Applications*, 41, 6995-7004.

[64] Karuppiah, R., Grossmann, I.E. (2006). Global optimization for the synthesis of integrated water systems in chemical processes. *Computers & Chemical Engineering*, 30, 650-673.

[65] Khadivi, M.R., Fatemi-Ghomi, S.M.T. (2012). Solid waste facilities location using of analytical network process and data envelopment analysis approaches. *Waste Management*, 32, 1258-1265.

[66] Klimberg, R.K., Ratick, S.J. (2008). Modeling data envelopment analysis (DEA) efficient location/allocation decision. *Computer & Operation Research*, 35, 457-474.

[67] Kolodziej, S., Castro, P.M., Grossmann, I.E. (2013). Global optimization of bilinear programs with a multiparametric disaggregation technique. *Journal of Global Optimization*, 57, 1039-1063.

[68] Kumar, D., Rahman, Z., Chan, F.T.S. (2017). A fuzzy AHP and fuzzy multi-objective linear programming model for order allocation in a sustainable supply chain: a case study. *International Journal of Computer Integrated Manufacturing*, 30(6), 535-551.

[69] Kuo, R.J., Lin, Y.J. (2012). Supplier selection using analytic network process and data envelopment analysis. *International Journal of Production Research*, 50(11), 2852-2863.

[70] Kuo, R.J., Wang, Y.C., Tien, F.C. (2010). Integration of artificial neural network and MADA methods for green supplier selection. *Journal of Cleaner Production*, 18(12), 1161-1170.

[71] Lau, K.H. (2013). Measuring distribution efficiency of a retail network through data envelopment analysis. *International Journal of Production Economics*, 146, 598-611.

[72] Li, X.-B., Reeves, G.R. (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research*, 115, 507-517.

[73] Li, P., Arellano-Garcia, H., Wozny, G. (2008). Chance constrained programming approach to process optimization under uncertainty. *Computers and Chemical Engineering*, 32, 25-45.

[74] Li, S., Jahanshahloo, G.R., Khodabakhshi, M. (2007). A super-efficiency model for ranking efficient units in data envelopment analysis. *Applied Mathematics and Computation*, 184, 638-648.

[75] Liu, J.S., Lu, L.Y.Y., Lu, W.-M., Lin, B.J.Y. (2013). Data envelopment analysis 19782010: A citation-based literature survey. *Omega*, 41, 3-15.

[76] Liu, B. 1999. *Uncertain Programming*. New York: Wiley.

[77] Liu, F., Ding, F.Y., Lall, V. (2000). Using data envelopment analysis to compare suppliers for supplier selection and performance improvement. *Supply Chain Management*, 5(3), 143-150.

[78] Luthra, S., Govindan, K., Kannan, D., Mangla, S.K., Garg, C.P. (2017). An integrated framework for sustainable supplier selection and evaluation in supply chains. *Journal of Cleaner Production*, 140(3), 1686-1698.

[79] Magnanti, T.L., Wong, R.T. (1981). Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria. *Operations Research*, 29(3), 464-484.

[80] Maiyar, L.M., Thakkar, J.J. (2017). A combined tactical and operational deterministic food grain transportation model: Particle swarm based optimization approach. *Computers & Industrial Engineering*, 110, 30-42.

[81] Matopoulos, A., Barros, A.C., van der Vorst, J. (2015). Resource-efficient supply chains: a research framework, literature review and research agenda. *Supply Chain Management*, 20(2), 218-236.

[82] Mavrotas, G. (2009). Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. *Applied Mathematics and Computation*, 213, 455-465.

[83] McCormick, G.P. (1976). Computability of global solutions to factorable nonconvex programs. Part I. Convex underestimating problems. *Mathematical Programming*, 10, 147-175.

[84] Mitropoulos, P., Mitropoulos, I., Giannikos, I. (2013). Combining DEA with location analysis for the effective consolidation of services in the health sector. *Computers & Operations Research*, 40, 2241-2250.

[85] Mogale, D.G., Dolgui, A., Kandhway, R., Kumar, S.K., Tiwari, M.K. (2017). A multi-period inventory transportation model for tactical planning of food grain supply chain. *Computers & Industrial Engineering*, 110, 379-394.

[86] Mogale, D.G., Kumar, S.K., Mrquez, F.P.G., Tiwari, M.K. (2017). Bulk wheat transportation and storage problem of public distribution system. *Computers & Industrial Engineering*, 104, 80-97.

[87] Mohajeri N., Amin, G.R. (2010). Railway station site selection using analytical hierarchy process and data envelopment analysis. *Computers & Industrial Engineering*, 59, 107-114.

[88] Mohammed, A., Wang, Q. (2017). The fuzzy multi-objective distribution planner for a green meat supply chain. *International Journal of Production Economics*, 184, 47-58.

[89] Moheb-Alizadeh, H., Rasouli, S.M., Tavakkoli-Moghaddam, R. (2011). The use of multi-criteria data envelopment analysis (MCDEA) for location-allocation problems in a fuzzy environment. *Expert Systems with Applications*, 38, 5687-5695.

[90] Monczka, R., Handfield, R., Giunipero, L., and Patterson, J, (2014). *Purchasing and Supply Chain Management*. Cincinnati, Southwestern Publishing.

[91] Mota, B., Gomes, M.I., Carvalho, A., Barbosa-Povoa, A.P. (2015). Towards supply chain sustainability: economic, environmental and social design and planning. *Journal of Cleaner Production*, 105(15), 14-27.

[92] Neto, J.Q.F., Bloemhof-Ruwaard, J.M., Nunen, J.A.E.E.V., Heck E.V. (2008). Designing and evaluating sustainable logistics networks. *International Journal of Production Economics*, 111, 195-208.

[93] Ninlawan, C., Seksan, P., Tossapol, K., Pilada, W. (2010). The implementation of green supply chain management practices in electronics industry, *Proceedings of the International Multi Conference of Engineers and Computer Scientists*, March 17-19, Hong Kong.

[94] Nuti, S., Daraio, C., Speroni, C., Vainieri, M. (2011). Relationships between technical efficiency and the quality and costs of health care in Italy. *International Journal for Quality in Health Care*, 23(3), 324-330.

[95] Orji, I.J., Wei, S. (2015). An innovative integration of fuzzy-logic and systems dynamics in sustainable supplier selection: A case on manufacturing industry. *Computers Industrial Engineering*, 88, 1-12.

[96] Park, K.S., Park, K. (2009). Measurement of multi-period aggregative efficiency. *European Journal of Operational Research*, 193, 567-580.

[97] Paydar, M.M., Babaveisi, V., Safaei, A.S. (2017). An engine oil closed-loop supply chain design considering collection risk. *Computers and Chemical Engineering*, 104, 38-55.

[98] Perez-Fortes, M., Lainez-Aguirre, J., Arranz-Piera, P., Velo, E., Puigjaner, L. (2012). Design of regional and sustainable bio-based networks for electricity generation using a multi-objective MILP approach. *Energy*, 44(1), 79-95.

[99] Pishvaee, M.S., Razmi, J., Torabi, S.A. (2014). An accelerated Benders decomposition algorithm for sustainable supply chain network design under uncertainty: A case study of medical needle and syringe supply chain. *Transportation Research Part E*, 67, 14-38.

[100] Pishvaeea, M.S., Razmi, J., Torabi, S.A. (2012). Robust possibilistic programming for socially responsible supply chain network design: A new approach. *Fuzzy Sets and Systems*, 206, 1-20.

[101] Punniyamoorthy, M., Mathiyalagan, P., Parthiban, P. (2011). A strategic model using structural equation modeling and fuzzy logic in supplier selection. *Expert Systems with Applications*, 38(1), 458-474.

[102] PwC. (2011). *Minerals and metals scarcity in manufacturing: the ticking time bomb. Sustainable Materials Management Report*, available at: www.pwc.com/en_GX/gx/sustainability/research-insights/assets/impact-of-minerals-metals-scarcity-on-business.pdf.

[103] Rahmaniani, R., Crainic, T.G., Gendreau, M., Rei, W. (2017). The Benders decomposition algorithm: A literature review. *European Journal of Operational Research*, 259, 801-817.

[104] Rei, W., Cordeau, J.-F., Gendreau, M., Soriano, P. (2009). Accelerating Benders decomposition by local branching. *INFORMS Journal on Computing*, 21(2), 333-345.

[105] Saen, R.F. (2007). A new mathematical approach for suppliers selection: Accounting for non-homogeneity is important. *Applied Mathematics and Computation*, 185(1), 84-95.

[106] Saen, R.F. (2008). Using super-efficiency analysis for ranking suppliers in the presence of volume discount offers. *International Journal of Physical Distribution Logistics Management*, 38(8), 637-651.

[107] Saen, R.F. (2010). Developing a new data envelopment analysis methodology for supplier selection in the presence of both undesirable outputs and imprecise data. *International Journal of Advanced Manufacturing Technology*, 51, 1243-1250.

[108] Saharidis, G.K., Boile, M., Theofanis, S. (2011). Initialization of the Benders master problem using valid inequalities applied to fixed-charge network problems. *Expert Systems with Applications*, 38, 6627-6636.

[109] Santibanez-Aguilar, J.E., Gonzalez-Campos, J.B., Ponce-Ortega, J.M., Serna-Gonzalez, M., El-Halwagi, M.M. (2014). Optimal planning and site selection for distributed multiproduct biorefineries involving economic, environmental and social objectives. *Journal of Cleaner Production* 65(0), 270-294.

[110] Santoso, T., Ahmed, S., Goetschalckx, M., Shapiro, A. (2005). A stochastic programming approach for supply chain network design under uncertainty. *European Journal of Operational Research*, 167, 96-115.

[111] Sarkis, J., Dhavale, D.G. (2015). Supplier selection for sustainable operations: A triple-bottom-line approach using a Bayesian framework. International Journal of Production Economics, 166, 177-191.

[112] Scott, J., Ho W., Dey P.K., Talluri S. (2015). A decision support system for supplier selection and order allocation in stochastic, multi-stakeholder and multi-criteria environments. *International Journal of Production Economics*, 166, 226-237.

[113] SCU (2012). *Science for Environment Policy In-depth Report: Resource Efficiency Indicators. University of the West of England, Bristol*, available at: http://ec.europa.eu/science-environment-policy.

[114] Seiford, L.M. (1994). *A DEA bibliography (19781992)*. In A. Charnes, W. W. Cooper, A. Lewin, & L. Seiford (Eds.), *Data Envelopment Analysis: Theory, Methodology and Applications*. Boston: Kluwer Academic Publishers.

[115] Seiford, L.M., Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation. *European Journal of Operational Research*, 142, 16-20.

[116] Seuring, S., Mller, M. (2008). From a literature review to a conceptual framework for sustainable supply chain management. *Journal of Cleaner Production*, 16(15), 1699-1710.

[117] Sexton, T.R., Silkman, R.H., Hogan, A.J. (1986). *Data envelopment analysis: Critique and extensions*. In R.H. Silkman (Ed.), *Measuring Efficiency: An Assessment of Data Envelopment Analysis*. San Francisco: Jossey-Bass.

[118] Shalke, P.N., Paydar, M.M., Hajiaghaei-Keshteli, M. (2017). Sustainable supplier selection and order allocation through quantity discounts. *International Journal of Management Science and Engineering Management*, 12, 1-13.

[119] Song, W., Xu, Z., Liu, H-.C. (2017). Developing sustainable supplier selection criteria for solar air-conditioner manufacturer: An integrated approach. *Renewable and Sustainable Energy Reviews*, 79, 1461-1471.

[120] Talluri, S., Narasimhan, R., Nair, A. (2006). Vendor performance with supply risk: A chance-constrained DEA approach. *International Journal of Production Economics*, 100, 212-222.

[121] Tang, C.S., Zhou, S. (2012). Research advances in environmentally and socially sustainable operations. *European Journal of Operational Research*, 223, 585-594.

[122] Thomas, P., Chan, Y., Lehmkuhl, L., Nixon, W. (2002). Obnoxious-facility location and data-envelopment analysis: A combined distance-based formulation. *European Journal of Operational Research*, 141, 495-514.

[123] Thompson, R.G., Langemeier, L.N., Lee, C.T., Thrall, R.M. (1990). The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics*, 46, 93-108.

[124] Torabi, S.A., Hassini E. (2008). An interactive possibilistic programming approach for multiple objective supply chain master planning. *Fuzzy Sets and Systems*, 159 (2), 193-214.

[125] Trapp, A.C., Sarkis, J. (2016). Identifying Robust portfolios of suppliers: a sustainability selection and development perspective. *Journal of Cleaner Production*, 112, 2088-2100.

[126] Tuzkaya, G., Gulsun, B., Onsel, S. (2011). A methodology for the strategic design of reverse logistics networks and its application in the Turkish white goods industry. *International Journal of Production Research*, 49(15), 4543-4571.

[127] ster, H., Easwaran, G., Akali, E., etinkaya, S. (2007). Benders decomposition with alternative multiple cuts for a multi-product closed-loop supply chain network design model. *Naval Research Logistics*, 54, 890-907.

[128] WCED (World Commission on Environment and Development). (1987). *Our common future*. Oxford: Oxford University Press. Weber, C.A. (1996). A data envelopment analysis approaches to measuring vendor performance. *Supply Chain Management*, 1(1), 28-39.

[129] Weber, C.A., Current, J.R., Benton, W.C. (1991). Vendor selection criteria and methods. *European Journal of Operational Research*, 50, 2-18.

[130] Weber, C.A., Current, J.R., Dessai, A. (1998). Non-cooperative negotiation strategies for vendor selection. *European Journal of Operational Research*, 108, 208-223.

[131] Weber, C.A., Current, J.R., Dessai, A. (2000). An optimization approach to determining the number of vendors to employ. *Supply Chain Management*, 5(2), 90-98.

[132] Willis, H.T., Huston, R.C., Pohlkamp, F. (1993). Evaluation measures of just in time supplier performance. *Production and Inventory Management Journal*, 34(2), 1-5.

[133] Wolsey L.A. (1998). *Integer Programming*. Wiley: New York.

[134] Wu, D. (2009). Supplier selection: A hybrid model using DEA, decision tree and neural network. *Expert Systems with Applications*, 36, 9105-9112.

[135] Xu J., Li B., Wu D. (2009). Rough data envelopment analysis and its application to supply chain performance evaluation. *International Journal of Production Economics*, 122, 628-638.

[136] You F., Tao L., Graziano, D., Snyder, S. (2012). Optimal design of sustainable cellulosic biofuel supply chains: Multiobjective optimization coupled with life cycle assessment and input-output analysis. *AIChE Journal*, 58(4), 1157-1180.

[137] Yousefi, S., Soltani, R., Saen, R.F., Pishvaee, M.S. (2017). A robust fuzzy possibilistic programming for a new network GP-DEA model to evaluate sustainable supply chains. *Journal of Cleaner Production*, 166, 537-549.

[138] Yue, D., Slivinsky, M., Sumpter, J., You, F. (2014). Sustainable design and operation of cellulosic bioelectricity supply chain networks with life cycle economic, environmental, and social optimization. *Industrial & Engineering Chemistry Research*, 53(10), 4008-4029.

[139] Zadeh, L.A. (1965). Fuzzy Sets. *Information and Control*, 8, 338-353.

[140] Zhalechian, M., Tavakkoli-Moghaddam, R., Zahiri, B., Mohammadi, M. (2014). Sustainable design of a closed-loop location-routing-inventory supply chain network under mixed uncertainty. *Transportation Research Part E*, 89, 182-214.

[141] Zhou, X., Pedrycz, W., Kuang, Y., Zhang, Z. (2016). Type-2 fuzzy multi-objective DEA model: An application to sustainable supplier evaluation. *Applied Soft Computing*, 46, 424-440.

[142] Zimmer, K., Frohling, M., Schultmann, F. (2016). Sustainable supplier managementa review of models supporting sustainable supplier selection, monitoring and development. *International Journal of Production Research*, 54(5), 1412-1442.

[143] Zimmermann, H.J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1, 45-55.