Failure and Non-Failure Probability of a Specific Pressure Vessel

M. Prodan
Motor Columbus Consulting Engineers Inc., Parkstr. 27, CH-5401 Baden, Switzerland

Summary

The analysis presented in this paper was dominated by deterministic design calculations. In addition to the deterministic calculations a probabilistic analysis of the failure quotients was performed. The failure probabilities considered were of the order of magnitude of 10^{-15} for one single critical event per year or 10^{-9} through 10^{-7} during the usage over an extremely long time period. Simple engineering approaches were used, a number of idealizations was made and there was a lack of information concerning the input data. Therefore, the probabilistic results should be regarded as associated with uncertainties.

1. Introduction

The study of the structural reliability has received particular attention in the last few years. With the probabilistic approach, in addition to deterministic design calculations, one searches for a feedback to the actual performance of structures under service and specified, i. e. known as extraordinary, conditions. Since most structures do not fail, the actual margin to failure is generally not known.

In the following, a practical example will be shown of how to apply probabilistic methods for the reliability analysis of a specific vessel.

2. Vessel Description

The vessel shall be used for the storage of hazardous materials, 1500 m subterranean over a period of some hundreds of years. It has a length of 2000 mm (cylinder with top and bottom), external diameter 940 mm and wall thickness of 250 mm (200 mm for the mechanical layout plus 50 mm corrosion allowance), see Fig. 1. Spheroidal cast iron or cast steel is assumed for the vessel material. The number of vessels to be manufactured and used in the near future is of the order of magnitude of a few hundred.

3. Imaginable Failure and Damaging Mechanisms

3.1 Corrosion

Theoretical and experimental chemical investigations have shown that the loss of wall thickness by corrosion during the usage period of some hundred years can be safely compensated with a corrosion allowance of 50 mm.

3.2 Buckling

Under normal, i. e. most probable imaginable conditions, the vessels will be stored in an underground disposal chamber - like a cavern - and there will be no external mechanical loads acting on them. In the case of a water flooding of the disposal chamber one can assume for a worst case a 1500 m high water column. This could lead to an external hydrostatic pressure of 150 bar on the vessels. The double value of this, i. e. $p = 300$ bar, was assumed as the hypothetical loading of the external vessel wall. The reason for the higher assumed pressure was the soak pressure of the wet surrounding stone bentonite. Further, from deterministic calculations it is concluded that a hypothetical buckling can be regarded as the dominant failure mechanism. The lowest value of the allowable external pressure was calculated as $p_b = 400$ bar according to buckling conditions reduced by the usual safety factors.

At this point we can make the following remarks from a deterministic point of view. Firstly, a simple criterion $p = 300 \text{ bar} \leq p_b = 400 \text{ bar}$ can be regarded as fulfilled. The value $p_b = 400$ bar is based on conservative calculations. The value $p = 300$ bar is also conservative in relation to the hydrostatic external pressure as explained above. However, the external pressure can be imagined as caused not only by water pressure and soak pressure of the wet bentonite, but also by earth layers or stone pressure due to geological factors (e. g. strong earthquakes leading to a destroyed disposal chamber). The probability of such a scenario is, however, very small, i. e. practically out of the question.

In this context it should be mentioned that there is a scarcity of geological data presently available for immediate use in a structural reliability analysis.

For several reasons we consider this situation to be unsatisfactory.

4. Probability of Failure by Buckling

4.1 Failure Quotient

The failure quotient was calculated from the convolution of two distributions, see Fig. 2. Both the external pressure and the corresponding material property of the critical buckling load were assumed to be normally distributed variables. With the notations of Fig. 2, and referring to the deterministic results of paragraph 3.2, the following input data were chosen:

$R_0 = 2000$ bar	$(5 \times (p_b = 400$ bar$))$
$S_0 = 220$ bar	$(< p = 300$ bar$)$
$\sigma_R = 220$ bar	(standard deviations, numerical values arbitrarily selected on the basis of considerations on the strength of the material).
$\sigma_S = 50$ bar	

The failure quotient p_f was calculated with readily available standard cumulative normal distribution function values from the tables in Ref. [1] as follows:

$$p_f = \Phi\left(-\frac{R_o - S_o}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) = \Phi\,(-\,7.9) = 0.15 \cdot 10^{-14}. \tag{1}$$

As an illustration of some consequences or of the correctness of the input assumptions for the calculation of p_f, the following function values F_r and F_s (distribution functions or the integrals of the probability density functions - see equation (4) -) may be considered:

$$\begin{aligned}
F_r\ (R = 2000\ \text{bar}) &= 0.5 \\
F_r\ (R = 1200\ \text{bar}) &= 1.6 \cdot 10^{-4} \\
F_r\ (R = 400\ \text{bar}) &= 1.5 \cdot 10^{-13} \\
F_s\ (S = 300\ \text{bar}) &= 5.5 \cdot 10^{-2} = \frac{1}{18}\ .
\end{aligned} \tag{2}$$

The quotient p_f can be interpreted as the failure probability per one event, which could be critical. Let us extend this interpretation, due to the lack of better information, so that p_f is the failure probability per event and year.

Furthermore, it should be emphasized that the calculations (1) and (2) are in full agreement with the general equations

$$p_f = F_\nu(1) = P\ (R \leqslant S) = \int_0^\infty F_r(x) \cdot f_s(x)\ dx \tag{3}$$

and

$$F_r(x) = P\ (0 < R \leqslant x) = \int_0^x f_r(\xi) d\xi \tag{4a}$$

further

$$F_s(x) = P\ (x \leqslant S < \infty) = \int_x^\infty f_s(\xi) d\xi \tag{4b}$$

assuming normal distributions. p_f is calculated from the convolution of $F_r(x)$ and $f_s(x)$, see equations (3) and (4a) and Fig. 2.

4.2 Time Dependent Failure Probability

The general equation for the time-dependent failure probability would have the form

$$F_1(t) = 1 - \left[\int_0^\infty \sum_{r=0}^\infty p_r(t) \left\{ F_s(x) \right\}^r \cdot f_r(x)\ dx \right] \tag{5}$$

The probability that the actual life of the vessel will exceed the required life time of e. g. 1000 years is then $1 - F_1(t)$. This is the so-called non-failure probability.

Instead of equation (5), we used the simplified equations (6) and (7) for the determination of the time-dependent failure probability

$$F_1(t) \approx 1 - (1 - p_f)^t \tag{6}$$

$$F_1(t\ = 1000 \text{ years}) \approx 0.28 \cdot 10^{-8}$$

$$F_1(t) \approx 1 - \exp\ (-p_f \cdot t) \tag{7}$$

$$F_1\ (t = 1000 \text{ years}) \approx 0.13 \cdot 10^{-6}$$

So far a distinction has to be made between the simplifications with the equations (1) through (4), and the equations (5) through (7) respectively. The first simplifications are more justifiable, i. e. they surely do not exceed legitimate limits. The use of equations (6) and (7) is in our case less explicable. The additional conditions for a transition from equation (5) to the equations (6) and (7) are discussed in Ref. [1] and [2]. These conditions are not completely fulfilled in our case. Therefore, equation (5) should be used.

For equation (5) we have no information on the essential functions leading to this equation, e. g. $p_r(t)$. Furthermore, for a complex application, such as ours in this paper, closed-form solutions do not exist for equation (5). However, our final alternative was the following: We could use a very simple, non-time-consuming method, or nothing at all. We think that the use of equations (6) and (7), even beyond their rigorous validation limits, was a better choice than no method. Additionally, two important conditions supporting the use of equations (6) and (7) are in our case more or less fulfilled, namely:

- relatively smaller deviations of R than those of S (i. e. $\sigma_R/R_0<\sigma_S/S_0$, the rigorous condition would be $\sigma_R = 0$)
- the essential events are "unusual events" (e. g. strong earthquakes in middle Europe or events like the formation of the Alps before 200 Million years).

5. Conclusion

In many cases there is a clear need for probabilistic structure reliability analyses. The objective of this paper was to demonstrate by means of an actual example, how far the simplest reliability procedures can be used for the solution of practical problems. Regarding the very low failure probabilities 10^{-15}, 10^{-9} and 10^{-7}, and the fact that the thick-walled vessel seems to be very safe in the given case, but also because of deterministic reasons, it is questionable whether some alternative distributions should be used for further analyses or not. The results of such advanced procedures could be regarded with less caution and uncertainty. However, apart from developing better methods and models, it should be remembered that there would still be a scarcity of sufficient input data due to the nature of very safe constructions.

Acknowledgements

For their critical reading of this paper I wish to thank Mr. N. Jaussi, Motor-Columbus, Baden, and Dr. J. C. Radon, Imperial College, London. Thanks to Dr. Wick and Mr. Bineck, Steag Ltd., Essen, for cooperation in designing of the vessel, and to Mrs. M. Wright who typed the entire manuscript.

References

/ 1 / Schuëller, G. I., Einführung in die Sicherheit und Zuverlässigkeit von Tragwerken, W. Ernst & Sohn, Berlin/München, 1981.

/ 2 / Thoft-Christensen, P. / M. J. Baker, Structural Reliability Theory and Its Applications. Springer-Verlag, Berlin, Heidelberg, New York, 1982.

Figure 1: - Geometry of the investigated vessel

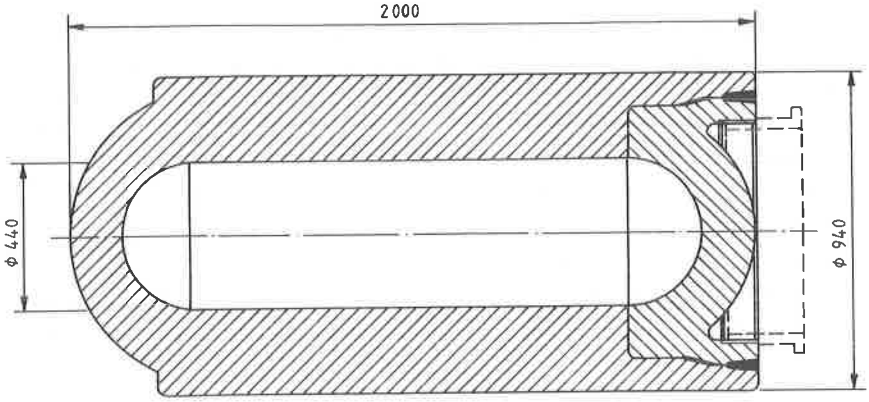
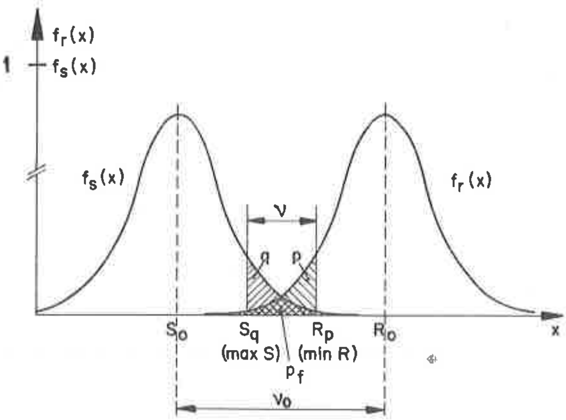


Figure 2: - Probability density functions of the load and the structural resistance with related notations



S_o - Mean value of the load, e. g. external pressure

R_o - Mean value of the resistance, e. g. buckling load

S_q - "Maximum" load acc. to the deterministic approach

R_p - "Minimum" resistance acc. to the deterministic approach

p, q - Fractions (see shaded areas)

$\nu_o=R_o/S_o$ - Safety factor acc. to the mean values

$\nu=R_p/S_q$ - Safety factor acc. to min R/max S

p_f - Failure probability

$f_s(x)$ - Probability density function of the load

$f_r(x)$ - Probability density function of the resistance