Theory and Implementation of Analytical Tools to Calculate Response Changes in Equipment Previously Evaluated by Testing

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Abstract

In situ modal testing directly provides a subset of the system's frequencies and mode shapes, avoiding what can be a difficult process of analytically developing the unique mass and stiffness matrices. As the full set of eigendata of a structural system theoretically contains an infinite number of frequencies and mode shapes, only a subset of that data is obtainable either analytically or through testing. The modal data resulting from these tests may be used for seismic equipment qualification studies. This paper addresses the development and application of a computer program to calculate in-cabinet response spectra and to account for future modifications to the mass and stiffness of the control panel without the need of developing a finite element model. The program is named EDASP for Equipment Dynamic Analysis Software Package. EDASP computes in-cabinet response spectra based on given floor motion and dynamic properties. The software also has the capability to calculate new dynamic properties based upon the old dynamic properties and given changes to the mass [ΔM] and stiffness [ΔK] of the cabinet.

As an example of the application of this software, consider the problem of adding a new subcomponent to an existing control panel. Such a modification will change the mass and stiffness of the panel, and this may invalidate the existing seismic qualification. The proposed software will generate the information to requalify the panel and to specify the seismic requirements for the new subcomponents. To accomplish this evaluation, a user of this software will first define the changes in mass and stiffness caused by the proposed modification which is not a trivial calculation. Once so defined, the change in mass, [ΔM], and change in stiffness [ΔK] are input to the proposed program, and other pertinent information -- current mode shapes and frequencies of the control panel, damping ratios, and floor motion time-histories -- are input from an existing data base. The program will calculate the new mode shapes, frequencies, and response by considering the changes specified by [ΔM], [ΔK], and structural damping.

EDASP uses these new dynamic properties and calculates modal participation factors to determine a seismic time-history of response and response spectrum at the new subcomponent location, and this spectrum can be used by the equipment vendor or testing lab to qualify the new subcomponent. The software also calculates new response spectra at existing subcomponent locations, and these can be compared to existing seismic documentation for adequacy.

The purpose of this paper is to discuss the development and capabilities of the EDASP program, particularly in relation to existing requirements in the commercial nuclear power industry.

1. Introduction

This paper discusses the development and theory of computer software to calculate in-cabinet response spectra and account for future modifications to the mass and stiffness of Class 1E assemblies. The program is called EDASP for Equipment Dynamic Analysis Software Package. Class 1E equipment assemblies in nuclear power plants are subject to constant physical change due to addition and removal of electrical devices. These changes are alterations to system parameters; specifically changes in mass and often stiffness. Since the eigenvalues and eigenvectors are changed by these alterations, the qualification documentation is affected. Specifically, the response characteristics may change affecting the calculated (or measured) amplified response spectra used as input for devices (generally known as the RRS), or the structural integrity documentation.

A computer program has been developed which can predict the effect of such changes. Furthermore, the program calculates the new dynamic properties without the need of a computer finite element model. New dynamic properties are calculated based on the original dynamic properties by considering changes in the mass and stiffness matrices. As such, the software operates on the equations of motion. When the generalized equations of motion are transformed into modal coordinates, the changes (additions or deletions) in mass and stiffness may be incorporated directly into the equations. An eigenvalue solution may then be performed on the new equations of motion resulting in new (altered state) frequencies and mode shapes.

As an example of the application of EDASP, consider the problem of adding a new subcomponent to an existing control panel. Such a modification will change the mass and stiffness of the panel, and this may invalidate the existing seismic qualification. The software will generate the information to requalify the panel and specify the seismic requirements for the new subcomponents. To accomplish this evaluation, the user will first define the changes in mass and stiffness caused by the proposed modification in terms of the appropriate degrees-of-freedom of the mass and stiffness matrices of the control panel. Once so defined, the change in mass $[\Delta M]$, and change in stiffness $[\Delta K]$ are input to the program, and other pertinent information -- current mode shapes and frequencies of the control panel, damping ratios, and floor motion time-histories -- are input from the data base. The program will calculate the new mode shapes and frequencies by considering the changes specified by $[\Delta M]$ and $[\Delta K]$. The theoretical background for this calculation is found in the following section.

The software will use these new dynamic properties and calculated modal participation factors to determine an acceleration history of response and its response spectrum at the new subcomponent location, and this spectrum can be used by the equipment vendor or testing lab to qualify the new subcomponent. The software will also calculate new response spectra at other existing subcomponent locations, and these can be compared to existing seismic documentation for adequacy.

This paper presents the basic theory for the program in the next section. An example will then be presented to explain the manner in which the software will work.

2. Theory

As previously stated, EDASP operates on the equations of motion where the changes in mass and stiffness may be introduced after the coordinate transformation.

The equations of motion of the undamped structural system may be written as:

$$[M]\ddot{x} + [K]x = \underline{f} \qquad (1)$$

where [M] and [K] are the mass and stiffness matrices, $\underline{X}$ is the vector of displacement in physical coordinates, and $\underline{f}$ is the vector of applied forces. In general, [M] and [K] contain non-zero off off diagonal terms, thus equation (1) represents a set of coupled, second order differential equations.

Using the coordinate transformation $\underline{X} = [\phi] Z$, Equation (1) may be rewritten in modal coordinates as:

$$[\,^{\diagdown}I_{\diagdown}]\ddot{\underline{z}} + [\,^{\diagdown}\omega^2_{\diagdown}]\underline{z} = [\phi]^T\underline{f} \quad , \qquad (2)$$

where $[\,^{\diagdown}I_{\diagdown}]$ is the identity matrix, and $[\,^{\diagdown}\omega^2_{\diagdown}]$ and $[\phi]$ contain the frequencies (eigenvalues) and mode shapes (eigenvectors) which constitute the solutions to:

$$\left([K] - \omega^2[M]\right)\phi = 0 \qquad (3)$$

The mode shapes are assumed to be normalized so that:

$$[\phi]^T[M][\phi] = [\,^{\diagdown}I_{\diagdown}] \text{ and } [\phi]^T[K][\phi] = [\,^{\diagdown}\omega^2_{\diagdown}] \quad . \qquad (4)$$

Note that when the equations of motion are in modal coordinates, a set of uncoupled second order differential equations results, thus easing the solutuion process.

In situ testing directly provides a subset of the system's frequencies and mode shapes, avoiding what can be a difficult process of analytically developing the mass and stiffness matrices required to solve Equation (3). As the full set of eigendata of a structural system theoretically contains an infinite number of frequencies and mode shapes, only a subset of that data is obtainable either analytically or through testing.

When the goal is to conduct a response analysis, the required size of the subset is determined by the spatial distribution and the frequency content of the applied forces. In the case when the set of modal data of a system is to be used to predict the modal data of that system after modification, the requirements on the original set of modal data are more stringent.

The modal data of the modified system will have to meet the aforementioned requirements developed by an examination of the applied forces. The modal data of the original system must span a sufficiently large space so that linear combinations of the original mode shapes will approximate what are expected to be the significant mode shapes of the modified system.

Given that an adequate set of modal data for the original system exists, prediction of the modal data of the modified system proceeds as follows:

$$\left([M] + [\Delta M]\right)\ddot{\underline{x}} + \left([K] + [\Delta K]\right)\underline{x} = \underline{f} \tag{5}$$

where [M] and [K] are the mass and stiffness matrices of the original system and [ΔM] and [ΔK] are the desired mass and stiffness modifications.

Again, using the coordinate transformation:

$$\underline{x} = [\phi]\,\underline{z} \quad , \tag{6}$$

Equation (5) can be rewritten as:

$$[\phi]^T\left([M] + \Delta M\right)[\phi]\ddot{\underline{z}} + [\phi]^T\left([K] + [\Delta K]\right)[\phi]\underline{z} = [\phi]^T\underline{f} \tag{7}$$

Note that the mode shapes of the original system are used in Equaton (7). In effect, the assumption is made that the original mode shapes are adequate to describe the behavior of the modified system.

Equation (7) can be further reduced to:

$$\left([\diagdown I \diagdown] + [\phi]^T[\Delta M][\phi]\right)\ddot{\underline{z}} + \left([\diagdown \omega^2 \diagdown] + [\phi]^T[\Delta K][\phi]\right)\underline{z} = [\phi]^T\underline{f} \quad , \tag{8}$$

or more simply:

$$[M^*]\ddot{\underline{z}} + [K^*]\underline{z} = \underline{f}^* \tag{9a}$$

where,

$$[M^*] = \left([\diagdown I \diagdown] + [\phi]^T[\Delta M]\,[\phi]\right) \tag{9b}$$

$$[K^*] = [\diagdown \omega^2 \diagdown] + [\phi]^T[\Delta K][\phi] \tag{9c}$$

Equation (9a) represents the equations of motion of the modified system, written in the modal coordinates of the original system. Note that Equation (9) can be written once modal data for the original system are obtained and the mass and stiffness modifications are analytially defined.

Finally, the modal data of the modified system is obtained through the solution of the eigenvalue problem.

$$\left([K^*] - (\omega^*)^2 [M^*]\right) \underline{\phi}^* = 0 \quad . \tag{10}$$

Note that the above development does not include any damping effects. Theoretically, damping can be included through the use of a velocity dependent term in Equation (1). Decoupling of the equations of motion with non-complex mode shapes would, however, occur only if the damping matrix was proportional to a linear combination of the mass and stiffness matrices. In practice, it is not usually possible to define the damping matrix. As a result, the eigenanalysis is performed ignoring damping. At the point at which a response analysis is conducted, a damping term is introduced at the moal level. From Equation (2), the undamped equation of motion for the i^{th} mode is:

$$\ddot{z}_i + \omega_i^2 z_i = \underline{\phi}_i^T \underline{f} \quad , \tag{11}$$

$$\ddot{\underline{z}}_i + 2S_i \omega_i \dot{z}_i + \omega_i^2 z_i = \phi_i^T \underline{f} \quad . \tag{12}$$

where S_i is the fraction of critical damping assumed for the i^{th} mode. Damping values are selected based on judgement and past engineering experience.

In addition to the equipment's natural frequencies and mode shapes, one further set of informtion is required to conduct a seismic analysis. That inforamtion is the structure's modal participation factors. The participation factors describe the distribution of mass in the structure. EDASP allows for three options for determining the particpation factors. First, the user can input a mass matrix. This matrix would be developed by calculating the structure's weight and appropriately distributing those weights among the data points in the structural model. In the second option, the software will assume a uniform mass distribution. When either of these first two options are chosen, the software will check the mass orthogonality conditions and output the results to the user. The third and final option will make direct use of the mass orthogonality conditions to determine the participation factors. To execute this option, the user will provide n(n-1)/2 -- where n is the number of know modes -- linearly independent mass distributions of arbitrary scale. The software will scale these mass distributions so that mass orthogonality conditions are satisifeid, and participation factors will then be calculated.

3. Example Problem

As an example of the application of EDASP, consider a control panel which has been tested either by low amplitude in situ modal techniques or a shaking-table during which it was well instrumented. Figure 1 shows a measurement point grid for such a hypothetical case. The numbered measurement points shown represent discrete locations on the structures at which measurements were taken. As such, the response of each point is relative to every other point on the structure for each normal mode stored. Because of this relationship, a change or changes can be postulated at various points on the equipment and their effect on the equipment can be computed.

If we consider the control panel in Figure 1, let us assume it is an existing panel at a nuclear power plant for which a modal survey using in situ testing techniques has been conducted. Furthermore, the panel contains Class 1E devices at or very near points 3, 10 and 14. As such, those devices have been qualified using the techniques of IEEE 344-1975. Therefore, documentation exists showing that the test response spectrum (TRS) envelopes the required response spectrum (RRS) at locations 3, 10, and 14, respectively.

As is the case with any important electrical assembly (panel) changes are made to the panel frequently. In this example, we will assume that a twelve (12) pound controller is added at point 11. The first need is to define the RRs at point 11, considering the inclusion of its 12 lb. weight. The user inserts the [ΔM] matrix into the software which is a null matrix with the exception of mass quantity at location 11 corresponding to 12 lbs. of weight. The software recalls the base acceleration history and original modal data set from the data base and performs its algorithm. The RRS, which is the amplified response at point 11 to be used to test and qualify the controller, is output. Note that it now includes the effect of the controller's 12 lb. weight. Since the user wants to assess the impact of this change (addition of 12 lb. controller) to the panel, the user requests the new RRS at points 3, 10, and 14. The program also has an option to store the TRS of any and all 1E devices in the cabinet such that an immediate comparison of new RRS to existing TRS can be made at all 1E device locations.

Let us consider the possibility that either the RRs to which the controller will be subject or RRS/TRS comparison at other locations is not favorable. In such an instance, the user can postulate the device location at any other point on the panel where there is available space. this process can be continued until an optional location is found and as such becomes a sensitivity evaluation of the panel. Once a suitable location for the controller is found, the new modal data set and new RRS/TRS are stored in the data base.

Another example of the capability of the software is its ability to predict new dynamic response based on changes in stiffness. Referring again to Figure 1, consider the addition of structural members to the control panel to reduce local amplification and structurally strengthen the panel. In this case, a stiffness analysis determining the additional stiffness provided by the new structural members is performed. This calculation is performed externally to the program. A static condensation is performed to relate these new elemental stiffnesses to the degrees-of-freedom (specifically, the adjacent measurement points) on the grid. The affected measurement points are 17 through 24. The [ΔK] matrix is inserted into the program and as before new dynamic properties (frequencies and mode shapes) are output. Furthermore, new RRs and RRS/TRS comparisons can be made anywhere on the panel. When the user is satisfied with the postulated change, the new modal properties and RRS/TRS comparisons are stored.

4. Conclusion

The EDASP computer program is a valuable tool to predict new response properties due to physical changes on Class 1E assemblies. It is a means of maintaining Class 1E documentation current for panels which have been modally or shake-table tested but for which no analytical model has been developed. This program offers a convenient means of tracking and documenting all changes made to electrical equipment and is a valuable tool to meet the requirements of the governing standards, such as, the IEEE 344 Standard.