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PARAMETER IDENTIFICATION OF A STRESS TRIAXIALITY-DEPENDENT PLASTIC DAMAGE MODEL FOR CONCRETE

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ABSTRACT

Studies on parameter identification of a plasticity-based damage constitutive model for concrete are presented in this paper. Differences between the stress-strain curves experimentally obtained and the ones numerically obtained at local level were used as the objective functional of a regularized least square method. For solving the inverse problem, the solution of the direct problem at local level obtained by a driver subroutine was embedded in the iterative solution procedure for the inverse problem proposed in this paper. The sensitivity matrix was calculated numerically by the finite difference method. Numerical examples are given for fitting the numerically obtained stress-strain curves on to a set of experimental results. Parameter identifications under various loading cases, which include uniaxial extension and triaxial compression with different confining stresses, were performed. Results indicate that the numerical scheme is stable and has reasonable accuracy.

Keywords: Inverse analysis, parameter identification, plasticity, damage.

1. INTRODUCTION

Constitutive parameter identification at local level has been investigated by several researchers (Furukawa, Sugata, Yoshimura, Hoffman, 2002; Bolzon, Fedele, Maier, 2002; Mahnken,, Stein, 1997) in recent years. The necessity of the study on this topic comes from the fact that laboratory test can only be done with a specified structure but never with a material point. For homogeneous materials, the behaviour of a material point can be nicely approximated by the experimental results with an elaborately designed structure (i.e. a specimen). But for heterogeneous materials, such as concrete, the behaviour of a material point can not be reasonably approximated by the experimental results of a specimen. The testing results of a Representative Volume Element of concrete can sometimes be regarded as a material property. But this is not suitable for the analysis of fracture and damage of concrete structures.

The method used in the inverse analysis at material level by different researchers varies depending on the different purposes in practice. The so-called Kalman Filter is used by Bolzon, Fedele, Maier (2002) for identification of material parameters related to cohesive crack in a composite material. A gradient-based optimization algorithm is used by Mahnken and Stein (1997) for parameter identification of the Gurson ductile plastic damage model. The Regularized Least Square method is used by Shen, Cen and Xu (1995) for the identification of strength parameters of geo-materials.

In the following sections, the direct plastic damage model used in the calculation is introduced first; formulation of an inverse analysis of a constitutive model is presented afterwards. The least square method is adopted in order to obtain a good fit of the calculated stress-strain curve onto the measured ones.

Three parameters, among which two parameters are used for describing scalar damage evolution and one parameter for triaxiality-dependent plastic hardening, were chosen as unknowns to be identified. Numerical validations have been performed for triaxial compression of a cubic specimen under various confinement pressures. The results indicate that the numerical scheme proposed here is valid for the identification of plastic damage parameters for material properties of concrete-like materials.

2. DIRECT MODEL OF AN ELASTO-PLASTIC DAMAGE MODEL： FORMULATION OF THE DIRECT MODEL

The plasticity-based isotropic damage model proposed by Lee and Fenves, (1998) is a model which specifically concerns the stress-triaxiality dependent plastic hardening. The principal characters of the model are: firstly, the damage evolution is not only connected to the increase in plastic strain, but also influenced explicitly by the elastic strain; secondly, the damage evolution is coupled with an increase in plastic strain; thirdly, the stress-triaxiality-dependent plastic damage loading condition is defined in this model. The generalized Drucker-Prager criterion for plastic loading, together with its plastic potential for the non-associated plastic flow rule, is referred to by this model.

The fundamental relationships of the plasticity-based damage model are listed in Eqn.(1) as:

$$\begin{cases} \tilde{\sigma}_{ij} = \dfrac{\sigma_{ij}}{1-D}, \quad \tilde{\varepsilon}^{e}_{ij} = (1-D)\varepsilon^{e}_{ij}, \quad \tilde{E}^{0}_{ijkl} = \dfrac{E_{ijkl}}{(1-D)^2} \\ \sigma_{ij} = E^{0}_{ijkl}(1-D)^2\left(\varepsilon_{kl} - \varepsilon^{p}_{kl}\right) \end{cases} \tag{1}$$

where σ_{ij} and ε_{ij} are total stress and strain tensors respectively, superscript *p* stands for plastic and *e* represents elastic quantities, overhead tilde ~ represents the quantities for fictitious net materials, *D* represents the isotropic damage variable, and E^{0}_{ijkl} is the elasticity tensor of the intact material.

The stress triaxiality-dependent plastic damage loading condition is defined in the effective stress space in the following form:

$$\tilde{f} = \alpha_F \tilde{I}_1 + \tilde{J}_2^{1/2} - \left[k + k_\infty\left(1 - e^{-b\gamma\lambda}\right)\right] = 0 \le 0 \tag{2}$$

and the damage plastic potential function in the following form:

$$F = \tilde{Q} + \frac{Y^2}{2S\gamma}(1-D) \tag{3}$$

The plastic part of the potential, i.e. $\tilde{Q}$, is given in the effective stress space as:

$$\tilde{Q} = \alpha_Q \tilde{I}_1 + \tilde{J}_2^{1/2} - \left[k + k_\infty\left(1 - e^{-b\gamma\lambda}\right)\right] \tag{4}$$

where *k* is initial shear strength constant, k_∞ is the strain hardening limit of the fictitious net material, which corresponds to infinite equivalent plastic strain, i.e. $\lambda \to \infty$, and α_F is a material constant designed for pressure-sensitivity properties, parameter *b* is a model constant, and α_Q is the dilatancy constant for the non-associated flow rule if $\alpha_Q \ne \alpha_F$. The stress triaxiality is defined as:

$$\gamma = \left|\frac{I_1/\sqrt{3}}{\sqrt{2J_2}}\right|, \quad J_2 \ne 0 \tag{5}$$

Normality rules are adopted for plastic strain increment and damage evolution:

$$\dot{\varepsilon}^{p}_{ij} = \dot{\lambda}\frac{\partial F}{\partial \sigma_{ij}}, \quad \dot{D} = \dot{\lambda}\frac{Y}{S\gamma}(1-D) \tag{6}$$

where $\dot{\lambda}$ is the inelastic multiplier. The parameters used in this model are: $E, \nu, k, \alpha_F, \alpha_Q, k_\infty$ and b mainly for plasticity, and S particularly for damage evolution. Stress triaxiality γ is a special variable introduced in this model.

The elastoplastic damage loading condition for a given strain increment $\Delta\varepsilon_{ij}$ can be expressed conceptually in the effective stress space as

$$\tilde{f} = \tilde{f}^0 + \frac{\partial \tilde{f}}{\partial(\Delta\lambda)} \cdot \Delta\lambda \le 0 \tag{7}$$

where $\tilde{f}^0$ is the value of yielding function at the starting effective stress state $\tilde{\sigma}^0_{ij}$. The formulation of Newton-Raphson iteration equation between $\Delta\lambda$ and $\tilde{f}$ is formed as:

$$\Delta\lambda = \Delta\lambda_0 - \tilde{f}_0 \left(\frac{\partial \tilde{f}}{\partial \lambda} \right)^{-1} \tag{8}$$

where $\tilde{f}^0$ is the value of yielding function at the starting effective stress state $\tilde{\sigma}^0_{ij}$.

A driver subroutine proposed in Shen, Shen and Zhou (2005) has been embedded in the program developed in this study. The program is designed for the purpose of parameter identification of a 3-dimensional constitutive model at local level, i.e. for a material point only. Its principle can be explained as follows: a mixed loading condition is applied with $\varepsilon_{11} = \varepsilon_{11}(t)$, and $\sigma_{22} = \sigma_{33} = const$, which means that a strain loading will be applied incrementally in the axial direction under a constant stress confinement in the other two directions. The strain loading is applied quasi-elastically in the axial direction, while the self-equilibrium mechanism at this material point will result in variation of lateral strains nonlinearly in order to keep the lateral confinement constant.

3. FORMULATION OF NUMERICAL SCHEME FOR INVERSE ANALYSIS

Because strain loading is adopted in the numerical calculation of the direct model, the difference between the numerical results of the stress component in axial direction and the one obtained experimentally will be used as the object function to be minimized in the procedure of identification. The numerical scheme is formulated as follows: Assuming that x_i^r is the approximate value of unknown vector x_i obtained from the r-th iteration, σ_j^{*r} is the collective stress tensor which collects stress components directly calculated from x_i^r at all the measurement points, and $\overline{\sigma}_j^*$ is the collective vector which collects all measured stress values of the experimental loading history. For the experimental result of the stress-strain history which has 100 loading increments, the total number of components of σ_j^* is 100.

A truncated Taylor's series expansion of the stress vector σ_i^{*r} around σ_i^{*r-1} can be given in the following form,

$$\begin{aligned} \sigma_i^{*r} &= \sigma_i^{*r-1} + \frac{\partial \sigma_i^*}{\partial x_j}(x_j^r - x_j^{r-1}) \\ &= \sigma_i^{*r-1} + Y_{ij}(x_j^r - x_j^{r-1}) \end{aligned} \tag{9}$$

where Y_{ij} is the sensitivity coefficient tensor,

$$Y_{ij} = \frac{\partial \sigma_i^*}{\partial x_j} \tag{10}$$

For a problem in which the experimental results of the stress-strain history has 100 loading increments and the number of parameters to be identified is 10, the number of components of tensor Y_{ij} is 100×10=1000.

The regularized objective function for the parameter identification with the Least Square Method can be written as follows:

$$T = \left(\sigma_i^* - \overline{\sigma}_i^*\right)^2 + \left(x_i^r - x_i^{r-1}\right)^T R_{ij} \left(x_i^r - x_i^{r-1}\right) \tag{11}$$

where R_{ij} is a positive definite regularization matrix. Using the minimization condition, the following approximate value of the unknowns can be obtained:

$$x_i^r = x_i^{r-1} + \left(Y_{ij}Y_{jk} + R_{ik}\right)^{-1} Y_{kl}\left(\sigma_l^{*r-1} - \overline{\sigma}_l^*\right) \quad (12)$$

As it is assumed that matrix R_{ik} is positive definite, it is easy to get the inverse of the matrix $\left(Y_{ij}Y_{jk} + R_{ik}\right)$. So a stable solution can always be obtained.

4. NUMERICAL VALIDATIONS

In this section, numerical validation of the identification procedure is performed for the afore-mentioned plastic damage model under uniaxial compression. The constitutive behaviour for compressions under various hydrostatic stress confinements are performed with the parameter obtained with uniaxial compression without stress confinement.

With reference to the existing literature, the following values of material parameters were adopted in the calculation:

E=3×104MPa, ν=0.2, αF=0.15, αQ=0.25, k=0.2MPa

Parameter b, S, and k_∞ were set as the objective unknowns and were identified with experimental data from Lee and Fenves (1998).

1. Inverse Analysis of the Material Property with Regularized Least Square Method.

After a series of trial calculations, the value of the diagonal component of **R** corresponding to the 3 unknowns were taken as 0.4, 0.4, 0.2 respectively in the process of inverse computation. The fluctuation of the objective parameters is rather small, thus convergence is favoured.

The selection of the step length of the difference was carried out. Approximating derivative with finite difference means approximating tangent direction with secant direction. Theoretically, the less the step length of the difference calculation, the nearer the secant direction will be to tangent direction. But in the process of numerical calculation, there exists a certain tolerable error. If the step length of the difference is too small, it could occur that the tolerance error of calculation will exert more effect on the declining direction of the objective function than the approximated difference calculation does. Sometimes, it will even make the objective function not decline. After some trial calculations, the step length of the difference was taken as 0.01.

Table 2 The convergence situation of the normalized unknown parameters.

	x_1(S)	x_2(b)	$x_3(k_\infty)$(MPa)	$\frac{\left\lVert \overline{\mathbf{x}}^r - \overline{\mathbf{x}}^{r-1} \right\rVert}{\left\lVert \overline{\mathbf{x}}^{r-1} \right\rVert} \times 100\,\%$	$\frac{T^r}{T_0} \times 100\,\%$
Initial value	5×10^{-5}	80	180.d0,		
Lower bound	1×10^{-5}	50	100		
Upper bound	5×10^{-4}	200	300		
Iter. No. r=1	8.86×10^{-5}	82.3	176.6	19.9	19.2
r=2	1.21×10^{-4}	76.6	182.4	7.1	6.9
r=3	1.19×10^{-4}	125.7	160.5	3.48	24.5
r=4	7.92×10^{-5}	141.2	158.3	23.2	13.4
r=5	1.13×10^{-4}	148.6	140.7	6.1	5.26
r=6	5.01×10^{-5}	69.4	175.9	1.5	6.67
r=7	4.96×10^{-5}	82.3	187.4	6.8	4.82

Because the difference between the quantities of different objective variables is significant, it is necessary to introduce the normalized objective variable during iteration. There is

$$\overline{x}_i = \frac{x_i}{x_i^{(0)}} \quad (13)$$

where x_i is the un-normalized unknown variable, $x_i^{(0)}$ is the initial value of the unknowns, and $\bar{x}_i$ is the normalized (dimension-less) vector of unknown variable.

The dual convergence criterion was adopted for the convergence of he objective variable and the decline of the objective function in the process of calculation. The convergence tolerable error of objective variables is set at 5%, the convergence tolerance for objective functional is 10%.

Table 2 shows the convergence situation of the objective functional. Figure 1 shows the comparison curve from the numerical results of a stress-strain curve with its experimental data from Lee and Fenves (1998). Figure 2 shows the damage evolution corresponding to the strain loading.

It is seen in figure 1 that the numerically predicted response of $\varepsilon_{11} - \sigma_{11}$ is in reasonably good accordance with the test results introduced in Lee and Fenves (1998) at both the pre-peak and post-peak loading stages. The $\varepsilon_{22} - \sigma_{11}$ curve shows the dilatancy property of the proposed model. In figure 2, the damage evolution behaviour obtained numerically tends asymptotically to its limit value of 1.0.

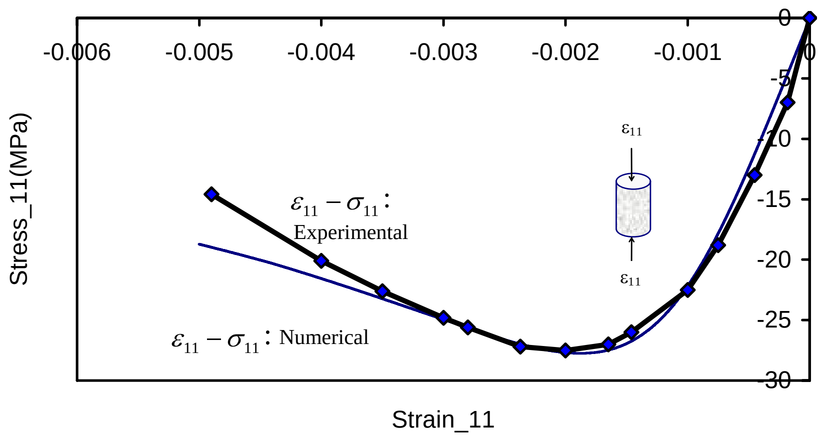

Figure 1. Stress-strain behaviour under uniaxial compression: the diamond marks represent experimental values (after Lee and Fenves, 1998), the thin solid line represents numerical result.

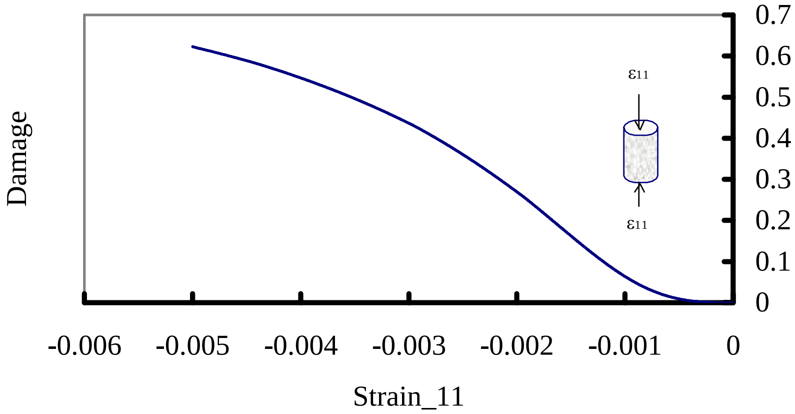

Figure 2. Damage evolution under uniaxial compression: numerical result.

2. Compression with Confinements

The behaviour of the model for concrete under hydrostatic confinement is an important consideration as it indicates pressure-sensitivity behaviour. In the numerical tests performed in this study, the hydrostatic confinement, i.e. $\sigma_m \mathbf{I}$, is applied before strain loading is applied in the 11-direction. Figure 3 shows the variation of the stress-strain response caused by the confinement of the stress-strain behaviour and damage evolution response, with the other parameters kept unchanged. As the increment of confinement increases, the softening phenomena become weaker and weaker.

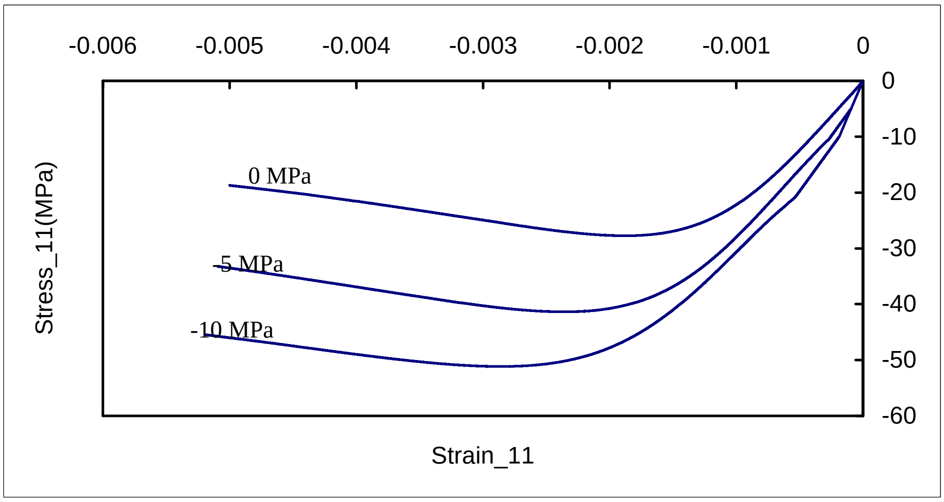

Figure 3 Influence of confinement: stress-strain behaviour with various stress confinements (σ_m =0, -5, -10 MPa).

5. CONCLUDING REMARKS

The Regularized Least Square method was adopted in the numerical scheme for the parameter identification of a plastic damage model for concrete. Numerical results of inverse analysis by the loading case of uniaxial compression indicate the validity of the proposed numerical scheme for the identification of 3 parameters related to plastic damage properties.

Due to the limitations of the experimental results, it is still not possible to embed the influence of the stress triaxiality in the computer program of the inverse analysis in this study. It will be a task for a further investigation.

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